

# Model Based Nonlinear Controller Design for Fuel Rail System of GDI Engine

Pengyuan Sun, Baiyu Xin, H. Chen and J. Li

**Abstract** The precise control of rail pressure in GDI engines is an important issue. To reduce the workload of calibration and enhance the robustness in automotive product development process, a model-based controller design method is presented in this paper. A control-oriented fuel rail system nonlinear dynamics model, involving the high pressure pump, the fuel rail and the injectors, is established. The backstepping technique is used to derive a nonlinear rail pressure controller for the simplified model. The simulation results with MATLAB/Simulink demonstrate the effectiveness of the proposed control scheme, and control precise and response satisfy the design requirements.

**Keywords** GDI engine · Fuel rail high pressure control · Backstepping algorithm

## 1 Introduction

The gasoline direct injection (GDI) technology has been widely used in the internal combustion engine to meet the increased environmental requirements and demands on decreased fuel consumption. The unique working mode of GDI engine that the fuel injection happens in the cylinder directly, determines its economy and emissions [1]. The fuel injection system based on the Fuel Rail architecture is a

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key device in GDI engine. The stable rail pressure makes GDI engine easy to control precisely the fuel injection. So the fuel rail pressure control becomes one of main tasks in GDI engine.

Many research studies have been carried out on the rail pressure control topic. For instance, a model reference adaptive control algorithm based on a common rail (CR) mean value model is proposed to reduce the residual pressure in the rail [2], and the results are satisfactory. The author of the paper [3] proposed an injection pressure regulation to stabilize the fuel pressure in the CR fuel line. The experimental results with the closed loop performance confirm the effectiveness of the control algorithm in the mean value rail pressure model. In the paper [4], the author identifies the second-order CR system model through the experiments data, and a CR pressure robust controller is designed and analyzed under the QFT control theory. A feed-forward based fuzzy PID controller is developed for the CR pressure control in the paper [5]. In the paper [6], a CR fuel system is modeled by system identification theory, and the model validation is then carried out with experimental data. A rule modelling method is provided for the diesel engine, and a slide controller is derived for the rail pressure control [7]. However, the choice of the sliding surface depends on experience, and there are some differences between GDI engine and diesel engine in the fuel system. GDI engine works with unique features.

In the automotive product development process, model-based controller design methods have been widely accepted to reduce the workload in engineering calibration and improve the control performance. In this paper, a mathematical model of fuel rail system based on the structure of GDI engine is established. There are some nonlinear characteristics in the model. A nonlinear rail pressure controller is derived by backstepping technology with the simple model. The simulation results with Simulink validate the controller performances.

The paper is organized as follows. The math model of the GDI engine fuel rail system is briefly described in Sect. 2, including the general description of the fuel rail system's operational principle. And a nonlinear controller is derived by backstepping technology in detail in Sect. 3. Then simulation results are presented to validate the backstepping controller performances in Sect. 4. Section 5 gives the conclusions.

## 2 The Math Model of the Fuel Rail System

As one of the most important parts in GDI engine fuel system, the fuel rail system's structure and characteristics make the injection pressure up to 150~200 bar. The pressure is independent of the engine speed to ensure a good spray atomization in a low engine speed. The basic structure of the GDI fuel rail system includes a pressure control valve, a high pressure pump, a fuel rail, the injectors, a rail pressure sensor and the Electronic Control Unit (ECU). Take CA4GA1T1 engine, made by FAW, as an instance, the structure diagram of the GDI fuel system is shown in Fig. 1.

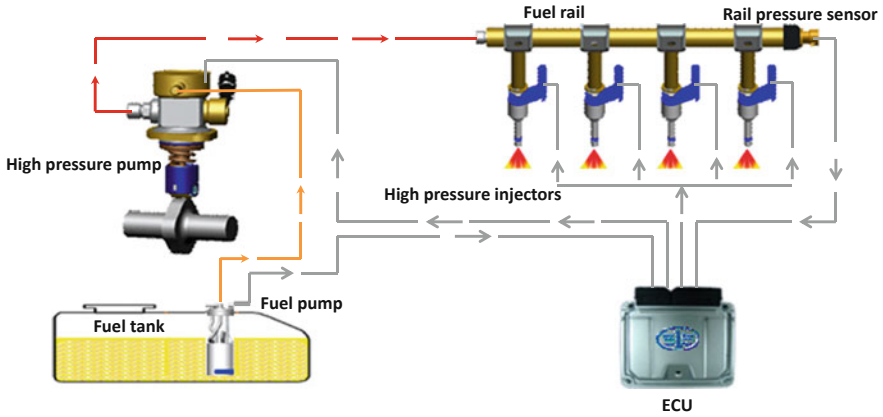


Fig. 1 The diagram of the fuel rail system

The low pressure pump generates about  $3 \sim 5 \text{ kg/cm}^2$  fuel pressure. The fuel flows into the high pressure pump passed by the pressure control valve. The fuel pressure is raised up to  $50 \sim 120 \text{ kg/cm}^2$  by the high pressure pump. The fuel rail is a fuel container made by aluminum alloy to absorb the pulse of the high pressure fuel. The injectors are connected with the rail and get a high injection pressure. The ECU gets the real-time rail pressure from the rail pressure sensor as a feedback signal and sends the order to the pressure control valve for rail pressure control. The other function of the ECU is to issue the fuel injection pulse width commands for injection. The high pressure pump, a pressure sensor, a pressure limiting valve and injectors are installed next to the fuel rail. The pressure limiting valve prevents the fuel rail from the damage by excessive pressure. The high pressure pump is lubricated by gasoline. The outlet check valve of the high pressure pump ensures the system to work.

Considering compressibility of the fuel, the basic principle of modelling is shown in the following expression [7].

$$K_f = -\frac{dp}{dv/v} = \frac{dp}{d\rho/\rho} \tag{1}$$

where  $K_f$  is the bulk modulus of elasticity defined as the relationship between the density and pressure,  $\rho$  is the fuel density,  $p$  is the pressure of the fuel and the volume is defined as  $v$ .

The relationship between the volume change and the pressure change can be got from Eq. (1),

$$\frac{dp}{dt} = -\frac{K_f}{v} \cdot \frac{dv}{dt} \tag{2}$$

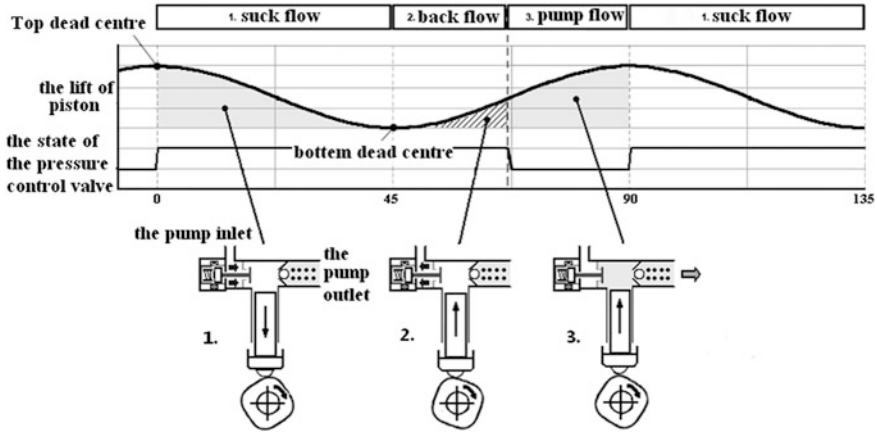


Fig. 2 The principle of the high pressure pump in GDI engine

And

$$\frac{dv}{v} = \frac{dv_m}{dt} - q_{in} + q_{out} \tag{3}$$

Where  $dv/v$  takes into account the intake and the outtake flows  $q_{in}$  and  $q_{out}$ , and the volume changes  $dv_m/dt$  due to the motion of mechanical parts. According to the energy conservation law, the fuel flows can be calculated as the following expression.

$$q = \text{sgn}(\delta p) \cdot c_d \cdot A_0 \cdot \sqrt{\frac{2|\delta p|}{\rho}} \tag{4}$$

Where  $A_0$  is the interested orifice section,  $c_d$  is discharge coefficient defined as the ratio of actual and ideal flows, which is decided by the shape of the cross section.  $\delta p$  is the pressure difference of the cross section on both sides.

Based on the above principle, the mathematical model of the GDI fuel rail system is established. To achieve the goals that the model can catch the fundamental physical aspects and the model is simple enough for control, a control-oriented model of a GDI fuel rail system is deduced.

### 2.1 The High Pump Model

There are many types of the high pump structure in fuel rail system. One of the types is cam-driven structure. The principle is shown in Fig. 2.

The cam is driven by the crankshaft of the engine in normal operation. The piston moves downward, when the cam moves to the pump bottom dead centre from the pump top dead centre. Due to the pressure difference, the fuel flows into the high pressure pump from its intake. As the check valve exists, there is no back-flow at the outlet of the high pressure pump. The fuel flows to the fuel rail from the high pump, when the pressure control valve keeps close and the cam runs to the pump top dead centre from the pump bottom dead centre. On the contrary, the fuel flows back to the low pressure circuit, if the pressure control valve is open. The pumped fuel volume is controlled by the control valve at the inlet of the high pressure pump.

The fuel flow pressure equation in the high pressure pump is shown as follows:

$$\dot{p}_p = \frac{K_f(p_p)}{v_p(\theta)} \left( \frac{dv_{mp}}{dt} + q_u - q_{pr} - q_0 \right) \quad (5)$$

Where  $q_u$  is the volume flow at the inlet of the high pressure pump,  $q_{pr}$  is the intake volume flow of the fuel rail, and  $q_0$  is the leakage fuel. The fuel volume change due to piston motion is  $dv_{mp}/dt$ .

$$\frac{dv_{mp}}{dt} = A_p \cdot \frac{dh_p}{dt} = A_p \cdot \omega_{rpm} \frac{dh_p}{d\theta} \quad (6)$$

And  $q_u$  and  $q_{pr}$  can be written in the form:

$$q_u = \text{sgn}(P_t - p_p) \cdot c_{tp} \cdot (U \cdot A_{tp}) \cdot \sqrt{\frac{2|P_t - p_p|}{\rho}} \quad (7)$$

$$q_{pr} = \text{sgn}(p_p - p_r) \cdot c_{pr} \cdot A_{pr} \cdot \sqrt{\frac{2|p_p - p_r|}{\rho}} \quad (8)$$

where  $U$  is the state of the pressure control valve.  $U = 0$  when the valve is closed contrarily,  $U = -1$  when the valve is open.  $c_{tp}$  and  $c_{pr}$  are the discharge coefficient of the inlet of the high pressure pump and the discharge coefficient of the rail inlet respectively.  $A_{tp}$  and  $A_{pr}$  are the interested orifice section of the high pressure pump inlet and the interested orifice section of the rail inlet. Ignore the change of the fuel density caused by the pump pressure variety. The pump pressure state can be rewritten as Eq. (9), and  $v_p(\theta) = V_p^0 - A_p \cdot h_p(\theta)$ .

$$\begin{aligned} \dot{p}_p = & \frac{K_f(p_p)}{v_p(\theta)} \left( A_p \cdot \omega_{rpm} \frac{dh_p}{d\theta} - \text{sgn}(P_t - p_p) \cdot c_{tp} \cdot (U \cdot A_{tp}) \cdot \sqrt{\frac{2|P_t - p_p|}{\rho}} \right. \\ & \left. - \text{sgn}(p_p - p_r) \cdot c_{pr} \cdot A_{pr} \cdot \sqrt{\frac{2|p_p - p_r|}{\rho}} - q_0 \right) \end{aligned} \quad (9)$$

## 2.2 The Fuel Rail

In short, the fuel rail is a fuel container with a certain volume. As a storage component, the main effect of the fuel rail is to make the hydraulic pressure stability, reduce the pressure fluctuation, and hold the fuel pressure. The rail pressure sensor and pressure limiting valve are beside the rail. To simplify the model, the volume of fuel injected  $q_{ri}$  is considered as a disturbance known. Ignore the tiny variety of the volume flow which is caused by pressure change. Then the model of fuel rail can be written in the following form.

$$\dot{p}_r = \frac{K_f(p_p)}{V_r} (q_{pr} - q_{ri}) \quad (10)$$

According Eq. (4), the rail model representation is

$$\dot{p}_r = \frac{K_f(p_r)}{V_r} \left( \text{sgn}(p_p - p_r) \cdot c_{pr} \cdot A_{pr} \cdot \sqrt{\frac{2|p_p - p_r|}{\rho}} - q_{ri} \right) \quad (11)$$

Where  $V_r$  is the volume of the fuel rail.

## 3 Design of a Backstepping Controller for the Fuel Rail System

According to the structure and the operation principle of the GDI fuel rail system, the block diagram of the rail pressure control is shown in Fig. 3. The reference rail pressure, which is decided by the engine working condition, and the real-time rail pressure are the inputs of the controller. The output of the controller affects the GDI fuel rail injection system. The system model should be simplified in order to design an appropriate control law. The injection quantity and the leakage of the high pressure pump are considered as the disturbance with uncertainty, and the impact of the fuel pressure on the volume bulk modulus is neglected here. Then the system model is stated as Eq. (12).

$$\begin{cases} \dot{p}_p = \frac{K_f}{v_p(\theta)} (A_p \cdot \omega_{rpm} \frac{dh_p}{d\theta} + \text{sgn}(P_t - p_p) \cdot c_{tp} \cdot (u \cdot A_{tp}) \cdot \sqrt{\frac{2|P_t - p_p|}{\rho}} \\ \quad - \text{sgn}(p_p - p_r) \cdot c_{pr} \cdot A_{pt} \cdot \sqrt{\frac{2|p_p - p_r|}{\rho}} - q_0) \\ \dot{p}_r = \frac{K_f}{V_r} (\text{sgn}(p_p - p_r)_{pr} \cdot A_{pt} \cdot \sqrt{\frac{2|p_p - p_r|}{\rho}} - q_{ri}) \end{cases} \quad (12)$$

Define  $x_1 = p_p$ ,  $x_2 = p_r$  and  $u = q_u$ . Taking the actual application into consideration, the ECU can control the pump only when  $x_1 > x_2$ , and when  $x_1 \leq x_2$ , the rail pressure will be reduced as engine runs. So the state-space representation is

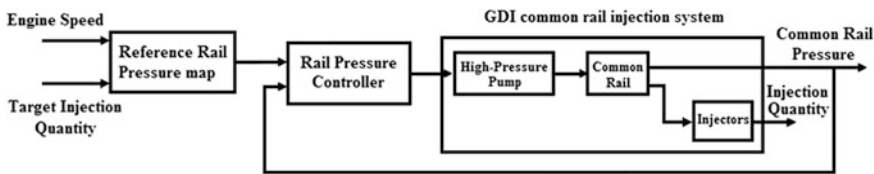


Fig. 3 The block diagram of the rail pressure control in the fuel rail injection system

$$\begin{cases} \dot{x}_1 = \frac{K_f}{v_p(\theta)} (A_p \cdot \omega_{rpm} \frac{dh_p}{d\theta} + u - a_{12} \cdot \sqrt{x_1 - x_2} - q_0) \\ \dot{x}_2 = \frac{K_f}{V_r} (a_{12} \cdot \sqrt{x_1 - x_2} - q_{ri}) \end{cases} \quad (13)$$

Where  $a_{11}$  and  $a_{12}$  are shown as

$$a_{11} = c_{tp} \cdot A_{tp} \cdot \sqrt{\frac{2}{\rho}}, a_{12} = c_{pr} \cdot A_{pr} \cdot \sqrt{\frac{2}{\rho}} \quad (14)$$

Because there are coupling between  $x_1$  and  $x_2$ , a new variable  $z$  is introduced as  $z = \sqrt{x_1 - x_2}$ .

Then equations of state system can be written with the new variable  $z$  as

$$\begin{cases} \dot{z} = \frac{1}{2z} \left[ \frac{K_f}{v_p(\theta)} \cdot A_p \cdot \omega_{rpm} \frac{dh_p}{d\theta} + \frac{K_f}{v_p(\theta)} \cdot u - \left( \frac{K_f}{V_r} + \frac{K_f}{v_p(\theta)} \right) \cdot a_{12} \cdot z - \frac{K_f}{v_p(\theta)} \cdot q_0 + \frac{K_f}{V_r} \cdot q_{ri} \right] \\ \dot{x}_2 = \frac{K_f}{V_r} \cdot (a_{12} \cdot z - q_{ri}) \end{cases} \quad (15)$$

The whole system is divided into several sub-systems in backstepping technique. By building the state error, the virtual control input is designed for each sub-system. The stability of the system is ensured by Lyapunov theory. The derived process of the controller is simple. There are many success cases in the aerospace and process control areas [8].

So the backstepping technique is used to derive the nonlinear controller with a feed-forward controller for the problem stated earlier. According to the form of the system state equation, the fuel rail tracking error variance  $e_2$  is defined to be  $e_2 = x_2^* - x_2$ .

$$\dot{e}_2 = \dot{x}_2^* - \dot{x}_2 = \dot{x}_2^* - \frac{K_f}{V_r} (a_{12} \cdot z - q_{ri}) \quad (16)$$

As the first Lyapunov function,  $V_2$  is defined as  $V_2 = \frac{1}{2} e_2^2$ . If the virtual control  $z^*$  is selected as  $z^* = \frac{V_r}{K_f \cdot a_{12}} \cdot \dot{x}_2^* + \frac{k_1 \cdot V_r}{K_f \cdot a_{12}} \cdot x_2^* - \frac{k_1 \cdot V_r}{K_f \cdot a_{12}} \cdot x_2 + \frac{q_{ri}}{a_{12}}$ , and when  $z$  is equal to  $z^*$ , there is Eq. (17).

$$\dot{V}_2 = e_2 \cdot \left[ \dot{x}_2^* - \frac{K_f}{V_r} (a_{12} \cdot z - q_{ri}) \right] = -k_1 \cdot e_2^2 \leq 0 \quad (17)$$

And the system meets the Lyapunov stability condition. A new error variable  $e_1$  is selected as  $e_1 = z^* - z$ . The error  $e_1$  is rewritten in the form:

$$e_1 = \frac{V_r}{K_f \cdot a_{12}} \cdot \dot{x}_2^* + \frac{k_1 \cdot V_r}{K_f \cdot a_{12}} \cdot x_2^* - \frac{k_1 \cdot V_r}{K_f \cdot a_{12}} \cdot x_2 + \frac{q_{ri}}{a_{12}} - z \quad (18)$$

The system Lyapunov function is selected as the form  $V_1 = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2$ , and if the derivative of  $V_1$  is negative, the fuel rail system is stable.

$$\begin{aligned} \dot{V}_1 &= e_1 \cdot \dot{e}_1 + e_2 \cdot \dot{e}_2 = e_2 \left( \dot{x}_2^* - \frac{K_f}{V_r} \cdot a_{12} \cdot z + \frac{K_f \cdot q_{ri}}{V_r} \right) + e_1 \cdot \dot{e}_1 \\ &= -k_1 \cdot e_x^2 + e_1 \left( \frac{K_f \cdot a_{12}}{V_r} \cdot e_2 + \dot{e}_1 \right) \end{aligned} \quad (19)$$

When the control law is selected as (20), the condition  $e_1 \left( \frac{K_f \cdot a_{12}}{V_r} \cdot e_2 + \dot{e}_1 \right) = -k_2 \cdot e_1^2$  is met, and there is an equation likes Eq. (21) to make the system stable.

$$\begin{aligned} u &= \frac{v_p(\theta)}{K_f} \cdot \left[ -\frac{K_f}{v_p(\theta)} \cdot A_p \cdot \omega_{rpm} \cdot \frac{dh_p}{d\theta} - \frac{K_f}{V_r} \cdot q_{ri} + \frac{K_f}{v_p(\theta)} \cdot q_0 \right. \\ &\quad + \left( \frac{K_f}{V_r} + \frac{K_f}{v_p(\theta)} \right) \cdot a_{12} \cdot z + 2z \cdot \frac{K_f \cdot a_{12}}{V_r} \cdot (x_2^* - x_2) + 2z \cdot \frac{V_r}{K_f \cdot a_{12}} \cdot \ddot{x}_2^* \\ &\quad + 2z \cdot \frac{k_1 \cdot V_r}{K_f \cdot a_{12}} \cdot \dot{x}_2^* - 2z \cdot \frac{k_1 \cdot V_r}{K_f \cdot a_{12}} \cdot \dot{x}_2 + 2z \cdot \left( \frac{q_{ri}}{a_{12}} \right)' \\ &\quad \left. + 2z \cdot k_2 \left( \frac{V_r}{K_f \cdot a_{12}} \cdot \dot{x}_2^* + \frac{k_1 \cdot V_r}{K_f \cdot a_{12}} \cdot x_2^* - \frac{k_1 \cdot V_r}{K_f \cdot a_{12}} \cdot x_2 + \frac{q_{ri}}{a_{12}} - z \right) \right] \end{aligned} \quad (20)$$

$$\dot{V}_1 = -k_1 \cdot e_1^2 - k_2 \cdot e_2^2 \leq 0 \quad (21)$$

In order to make the controller applied in the actual fuel rail system, the final control law is collated as (22).

$$u = A(z) + K_p(x_2^* - x_2) + K_d(\dot{x}_2^* - \dot{x}_2) \quad (22)$$

The parameters of PD feedback control law are  $K_p$  and  $K_d$ .  $A(z)$  is the feed-forward.

$$A(z) = -A_p \cdot \omega_{rpm} \cdot \frac{dh_p}{d\theta} + q_0 + a_{12} \cdot z + \frac{v_p(\theta)}{V_r} \cdot (a_{12} \cdot z - q_{ri}) + 2z \cdot \frac{v_p(\theta) \cdot V_r}{K_f^2 \cdot a_{12}} \cdot \ddot{p}_r^* \quad (23)$$

$$K_p = 2z \cdot \frac{v_p(\theta)}{K_f} \cdot \left( \frac{K_f \cdot a_{12}}{V_r} + \frac{k_1 \cdot k_2 \cdot V_r}{K_f \cdot a_{12}} \right) \quad (24)$$

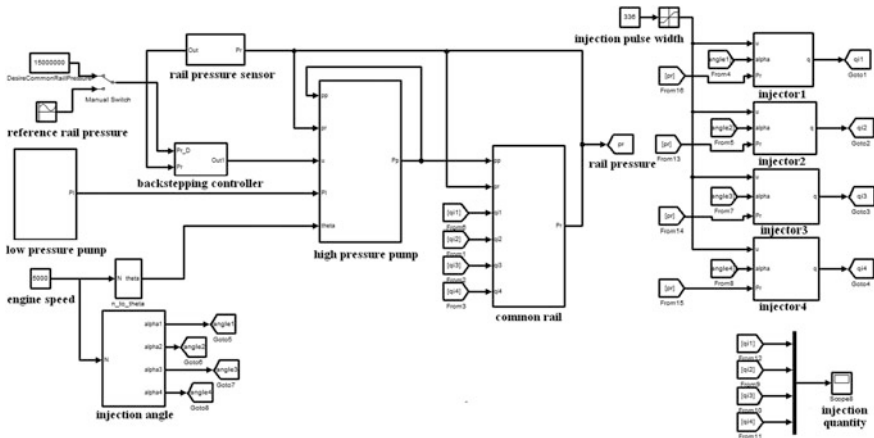
$$K_d = 2z \cdot \frac{v_p(\theta)}{K_f} \cdot \frac{(k_1 - k_2) \cdot V_r}{K_f \cdot a_{12}} \quad (25)$$

Considering the form of the system state equations, the dynamic fuel characteristics of the high pressure pump and the rail are included in the feed-forward.



**Table 1** The values table of the parameters in GDI fuel rail system

The volume of rail (m <sup>3</sup> )	$65.824 \times 10^{-6}$
The section of the inlet in high pressure pump (m <sup>2</sup> )	$8.553 \times 10^{-6} \times 0.75$
The section of the inlet in fuel rail (m <sup>2</sup> )	$12.566 \times 10^{-6} \times 0.65$
The section of the pump piston (m <sup>2</sup> )	$78.5398 \times 10^{-6}$
The max volume of high pressure pump (m <sup>3</sup> )	$0.27 \times 10^{-6}$
The leakage of the high pressure pump (m <sup>3</sup> /s)	0.0005
The pressure supplied by low pressure pump (bar)	5.7



**Fig. 4** The block diagram of the close-loop fuel rail system with backstepping controller

## 4 The Controller Performances

To test the rail pressure, a fuel rail plant model is established with Simulink software. And the parameters of the fuel rail system are shown in Table 1. With the parameters, the Simulink model works well and can reflect the actual fuel rail system characteristics to a certain extent.

Embed the backstepping rail pressure controller into Simulink system, as shown in Fig. 4.

Two different operating conditions by using Simulink software are simulated to test the control law.

### 4.1 Condition 1: Constant Pressure Tracking Test

Set the referenced rail pressure as 150 bar. The stable engine speed is 5000 rpm. The fuel injection pulse width is 2.2 ms, and 66°. The start angle of injection is

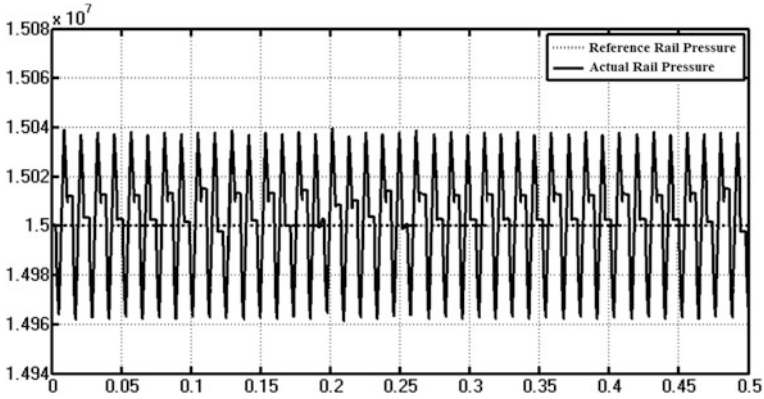


Fig. 5 The simulation result for the constant pressure tracking test 1

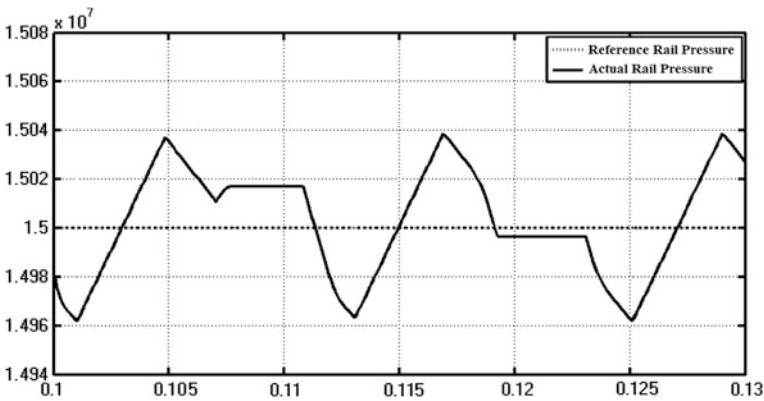


Fig. 6 The simulation result for the constant pressure tracking test 2

270° and the density of the gasoline is 0.73 kg/L. The controller parameters are set as  $k_1 = 5.1$ ,  $k_2 = 3.03$ . Within the 0.5 s simulation, the rail pressure can be stable at the set-point pressure by the control action. The results are shown in Figs. 5 and 6. The max error is less than 0.4 bar.

### 4.2 Condition 2: Sine Function Referenced Pressure Tracking Test

To verify the dynamic tracking performance of the nonlinear controller, a sine pressure tracking test is implemented. The signal sine has amplitude of 10, bias of 140 bar and frequency of 4.5 rad/s. In Figs. 7 and 8, the rail pressure is properly taken close to the set-point with a small fluctuation. The max error is less than 2

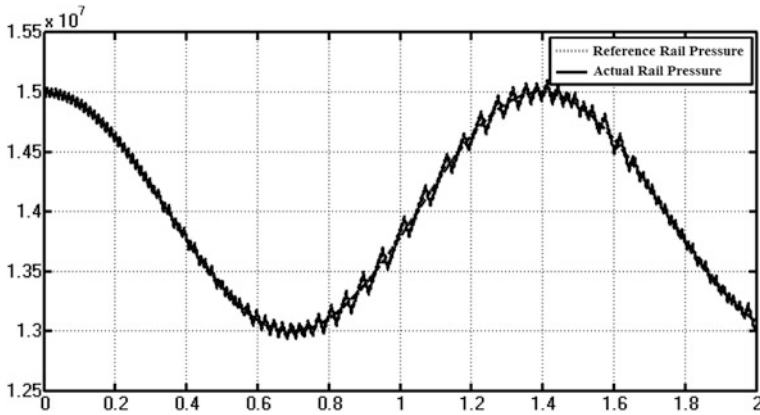


Fig. 7 The simulation result for the sine function referenced pressure tracking test 1

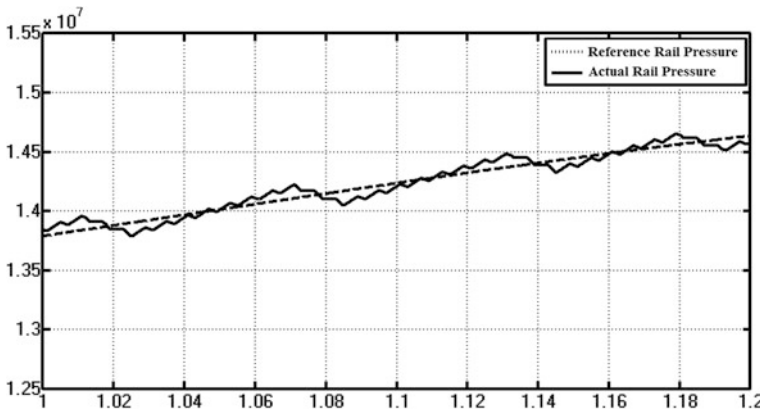


Fig. 8 The simulation result for the sine function referenced pressure tracking test 2

bar. The backstepping controller is still able to maintain the rail pressure close to the reference value.

From the results, the controller designed before has the performance to stabilize the rail pressure. The real-time bench test will be implemented at the next step for validating the controller real-time performance.

## 5 Conclusions

To meet the rail pressure control requirements in GDI fuel rail system, this paper presents a mathematical model (in the control-oriented) of a fuel rail system for GDI engine. The model is obtained by the main fluid dynamic. The parameters are

obtained from the real geometrical data of a real GDI engine. Then the rail pressure controller is derived by backstepping technology based on the simple model. The controller performance is validated preliminarily by the simulation. This method-based on model reduces the calibration work and the development cost of ECU.

For the application of this nonlinear rail pressure controller, the fuel pressure in high pressure pump is needed to be known. So an estimator for the pump pressure may be the next work in the future. Then the hardware in loop experiment will be finished for testing the on-line performances of the fuel rail controller with a real GDI fuel rail test bench.

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