Hybrid Steering of Wheeled Wheels

Muhammad Adeel Awan, David Purdy and Amer Hameed

Abstract This paper is focussed to assess the performance of a low cost steer-by-wire system developed in the Vehicle Dynamics Laboratory at Cranfield University. The steering system plays an important role in vehicle's safety by influencing its handling performance and therefore considered as a critical vehicle system. The performance of a steer-by-wire system is predominantly linked to its actuators' behaviour. This paper contains the findings of experiments carried out to measure the actuators' response. As the steering commands can be represented by step, sinusoidal and combination of such inputs therefore the actuators are subjected to similar sort of inputs with varying frequencies and amplitudes. In the later part a controller is developed to improve the performance of the steering system using the torque vectoring technique.

Keywords Hybrid steer · Steer-by-wire · Yaw-control · LQR · Vehicle model

Nomenclature

α_f, α_r	Front and rear tyre slip angles respectively
δ_f	Front steering angle of the vehicle
$\dot{\omega}_{fl}\dot{\omega}_{fr}\dot{\omega}_{rr}\dot{\omega}_{rl}$	Angular velocities of the front left, front right, rear right and rear left wheels respectively

F2012-G06-005

M. A. Awan (✉) · D. Purdy · A. Hameed
Cranfield University, Cranfield, UK
e-mail: m.awan@cranfield.ac.uk

D. Purdy
e-mail: d.j.purdy@cranfield.ac.uk

A. Hameed
e-mail: a.hameed@cranfield.ac.uk

SAE-China and FISITA (eds.), *Proceedings of the FISITA 2012 World Automotive Congress*, Lecture Notes in Electrical Engineering 198,
DOI: 10.1007/978-3-642-33795-6_30,

$\sum F_x$	Sum of forces acting on the vehicle in x-direction,
$\sum F_y$	Sum of forces acting on the vehicle in y-direction,
$\sum M_z$	Sum of moments acting on the vehicle in z-direction,
$\tau_{fl}\tau_{fr}\tau_{rr}\tau_{rl}$	Torque acting on the front left, front right, rear right and rear left wheels respectively
a	Distance between the front axle and the vehicle's centre of mass
b	Distance between the rear axle and the vehicle's centre of mass
F_d	Air drag force,
$F_{x_{fw}}F_{x_{rw}}$	Longitudinal tyre forces at front and rear respectively
$F_{y_{fw}}F_{y_{rw}}$	Lateral tyre forces at front and rear respectively
I_z	Moment of inertia about z-axis
m	Mass of the vehicle,
r	Yaw rate of the vehicle
$T_F T_R$	Track width at front and rear respectively
$R_{fl}R_{fr}R_{rr}R_{rl}$	Effective radius of the front left, front right, rear right and rear left wheels respectively
u	Longitudinal velocity of the vehicle
v	Lateral velocity of the vehicle
$C_{\alpha f}C_{\alpha r}$	Cornering stiffness of the front and rear tyre respectively

1 Introduction

Since the inception of high speed automobiles, the automotive engineers are in a constant pursuit to enhance their manoeuvrability and stability. The earlier versions of automobiles were steered by a tiller, which was replaced by a steering wheel. The steering wheel is connected to the wheels by means of mechanical linkages and transfers the driver's commands to the wheels through these linkages. This effort to enhance the steering system of a vehicle by introducing the steering wheel dates back to 1893 when a Frenchman, name Alfred Vacheron, introduced the steering wheel to his race car [1]. These efforts to improve the vehicle stability and steerability continued on and resulted in the form of anti-lock brake system (ABS), differential braking, yaw control, traction control, and active steering system, which made their way to the production models. Among these performance enhancement systems active steering phenomenon, being the least matured, still lures the researchers.

An active steering system is one which uses an auxiliary arrangement to assist the driver in controlling the vehicle. The active steering system can have various forms; it can be a conventional steering arrangement assisted by an electric motor/hydraulic pump or an arrangement in which the steering wheel is not directly connected to the wheels and the wheels are steered by actuators that receive

Fig. 1 The test vehicle fitted with the steer-by-wire vehicle

commands through a microprocessor. The latter steering arrangement is termed as steer-by-wire system and is one of the advanced forms of the active steering systems. The active steering goes back to at least the sixties [2] when Kasselmann and Keranen [3] studied an active steering system in which a proportional feedback controller augmented the front steering angle by monitoring the vehicle yaw-rate with a gyro.

The motivation for this paper is to envisage a future vehicle with a low cost steer-by-wire system. This idea raises the need to explore the possibilities that allow the replacement of a conventional steering system with a steer-by-wire system. Due to its critical nature generally a steer-by-wire system contains high performance actuators. These high performance actuators/motors are costly and therefore do not provide a feasible solution for production vehicles. This paper investigates the possibility of using the low cost actuators as a part of the steer-by-wire system. In order to overcome the limited performance of these low cost actuators this paper explores the compensation by torque vectoring techniques. This provides control over the torque distribution to all wheels of a vehicle. Generally this technique is implemented by a mechanical differential; however, this work uses in-wheel electric motors to implement the torque vectoring mechanism as these are being considered for both civilian and military vehicles.

2 Experimental Setup

The test vehicle, on which the low performance steering system was fitted, is shown in Fig. 1. It is a dune buggy converted to include a steer-by-wire system comprising two low cost linear actuators, a potentiometer to convert steering commands into the respective voltages, a data acquisition (DAQ) module and a laptop for data manipulation and gathering. The front and rear McPherson strut suspension has been altered to accommodate four of such actuators to make the vehicle four wheels steerable. However, for this work the rear wheels have been

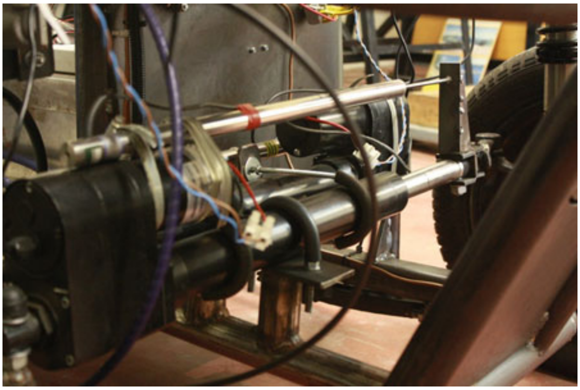


Fig. 2 Front steering actuator and LVDT replacing the conventional steering

kept fixed and only front two wheels were allowed to steer during the investigation. The actuator's performance data sheet, provided by the supplier, suggests that the actuator can travel at maximum speed of 45 mm/sec when a load of 2.3 kN is acting on it.

Figure 2 shows the steer-by-wire system replacing the conventional steering system. Due to physical limitations it was not possible to fit the actuators in-line to each other on the vehicle chassis. The kinematic arrangement on the right side of the vehicle was slightly modified to accommodate the actuator's offset and therefore both of the actuators travel in different lengths to steer the two wheels in parallel. The actuators were fitted with two calibrated LVDTs to measure the actuators' displacement during the experiments.

3 Actuator's Response

This section describes the experiments carried out to measure the response of the low cost actuators. As mentioned earlier the steering commands can be generally represented in the form of square and sinusoidal inputs. Therefore, the actuator was subjected to inputs of similar forms of various frequencies and amplitudes.

3.1 Square Wave Response

The layout diagram of the experimental set-up is shown in the Fig. 3. In order to investigate the actuators' response they were fed with an input in the form of a square wave.

Figure 4 shows the displacement versus time graph for the actuator. The data shown in the figure was collected when the actuator was commanded to move from one end to other at the frequency of 0.1 Hz. This frequency was chosen to allow the actuator to reach the end position fully and then move back.

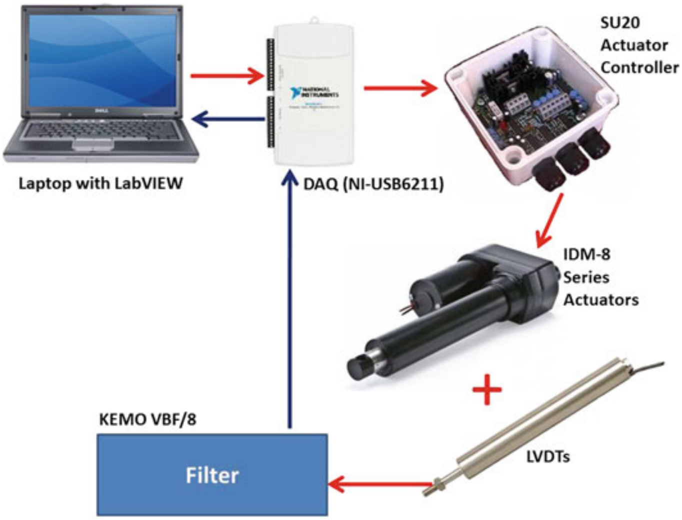


Fig. 3 Experimental set-up layout

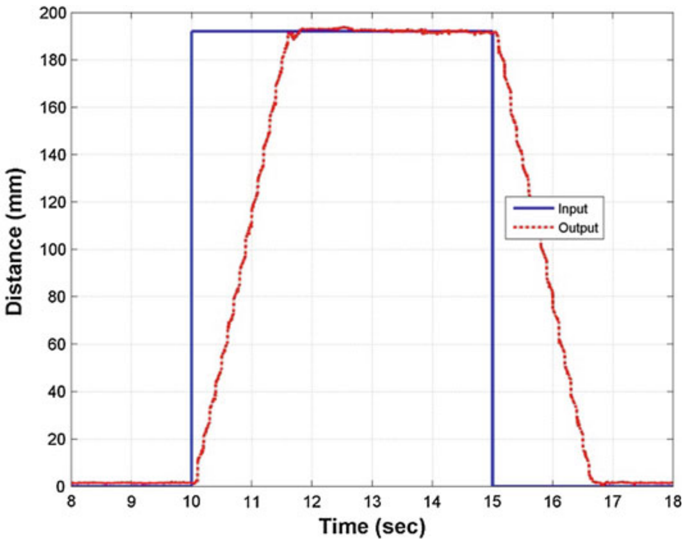


Fig. 4 Response of the actuator when subjected to the square wave input of 0.1 Hz

This experiment was repeated several times by varying input frequencies and amplitudes. The measured responses show a relatively simple behaviour consisting of a scaling factor to compensate for the steering kinematics and velocity saturation from the actuator. Using this analogy a Simulink model is developed that imitate the delayed actuator response. This model was validated by comparing its

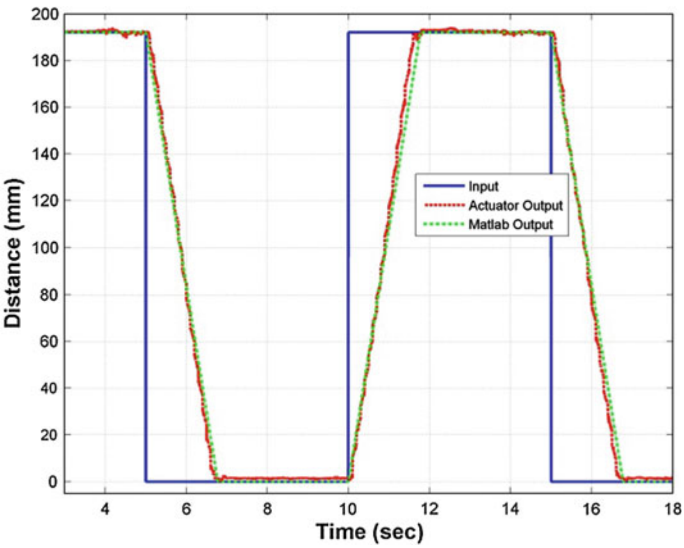


Fig. 5 Comparison of the Simulink model's response with the experimental output

output with that of the actuator, when subjected to an input of the same frequency and amplitude. This has been shown in Fig. 5, which compares the outputs of the Simulink model and the actuator when both are excited with a square wave of 0.1 Hz and maximum amplitude.

3.2 Sine Wave Response

In this set of experiments the actuator was driven by sine wave inputs of varying frequencies and amplitudes to identify its behaviour in manoeuvres representing lane-change or obstacle avoidance. Figure 6 shows the output comparison for the Simulink model and the actuator when subjected to a sine wave input having frequency of 0.2 Hz and maximum amplitude. These results show that the Simulink model of the steer-by-wire system represents the real system in an acceptable manner. The root mean square error value between the actual and simulated responses was within the range of 15–20 mm.

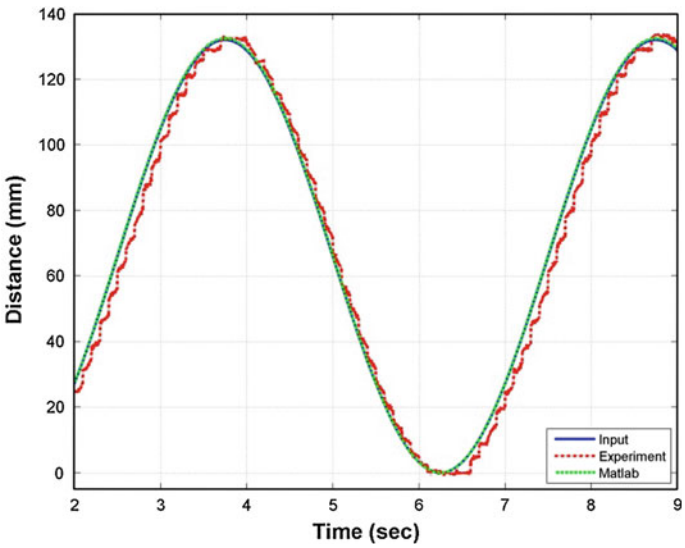
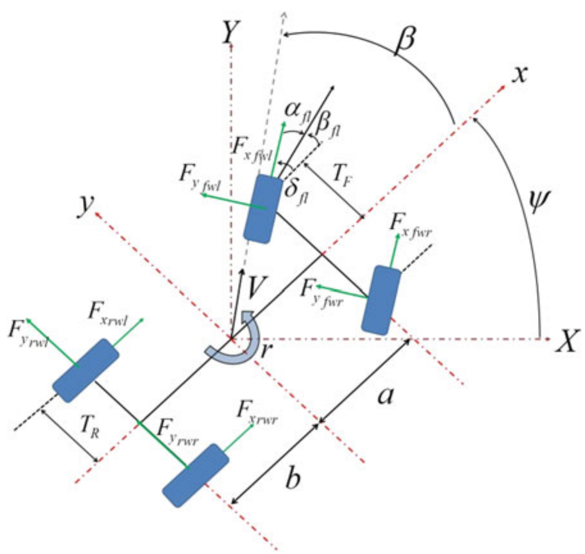


Fig. 6 Comparison of sine wave response of the Simulink model and actuator

Fig. 7 Vehicle model with seven degrees of freedom



4 Simulation and Analysis

4.1 Vehicle Model

In order to analyse the effect of the above discussed steer-by-wire system on the vehicle's handling performance, its Simulink model is integrated with a vehicle model, shown in Fig. 7, having seven degrees of freedom. This vehicle model is also developed using Matlab/Simulink and the degrees of freedom are vehicle's lateral, longitudinal and yaw motions along with the rotational degree of freedom for each tyre.

Applying the Newton's second law of motion, the equations of motion can be written as:

$$\sum F_x = F_{x_{fwl}} cos\delta_{fl} - F_{y_{fwl}} sin\delta_{fl} + F_{x_{fwr}} cos\delta_{fr} - F_{y_{fwr}} sin\delta_{fr} + F_{x_{rwl}} + F_{x_{rwr}} - F_d \tag{1}$$

$$\sum F_y = F_{x_{fwl}} sin\delta_{fl} + F_{y_{fwl}} cos\delta_{fl} + F_{x_{fwr}} sin\delta_{fr} + F_{y_{fwr}} cos\delta_{fr} + F_{y_{rwl}} + F_{y_{rwr}} \tag{2}$$

$$\begin{aligned} \sum M_z =& \left(F_{x_{fwl}} sin\delta_{fl} + *\Phi_{\theta_{\phi\chi\lambda}} \psi\omega\sigma\delta_{\phi\lambda} + \Phi_{\xi_{\phi\chi\rho}} \sigma\iota\nu\delta_{\phi\rho} + \Phi_{\theta_{\phi\chi\rho}} \psi\omega\sigma\delta_{\phi\rho}\right) \times \alpha \\ & - \left(F_{x_{fwl}} cos\delta_{fl} - *\Phi_{\theta_{\phi\chi\lambda}} \sigma\iota\nu\delta_{\phi\lambda} - \Phi_{\xi_{\phi\chi\rho}} \psi\omega\sigma\delta_{\phi\rho} + \Phi_{\theta_{\phi\chi\rho}} \sigma\iota\nu\delta_{\phi\rho}\right) \times T_F \\ & - \left(F_{y_{rwl}} + F_{y_{rwr}}\right) \times b - \left(F_{x_{rwl}} - F_{x_{rwr}}\right) \times T_R \end{aligned} \tag{3}$$

For wheels the equations of motion are:

$$\dot{\omega}_{fl} = \frac{1}{I_{fwl}}\left(\tau_{fl} - R_{fl} F_{x_{fwl}}\right) \tag{4}$$

$$\dot{\omega}_{fr} = \frac{1}{I_{fwr}}\left(\tau_{fr} - R_{fr} F_{x_{fwr}}\right) \tag{5}$$

$$\dot{\omega}_{rr} = \frac{1}{I_{rwr}}\left(\tau_{rr} - R_{rr} F_{x_{rwr}}\right) \tag{6}$$

$$\dot{\omega}_{rl} = \frac{1}{I_{rwl}}\left(\tau_{rl} - R_{rl} F_{x_{rwl}}\right) \tag{7}$$

The torque values in the Eqs. (4)–(7) are the output of the in-wheel electric motors, which are transferring the torque through a reducer as discussed by Esmailzadeh et al. [4].

4.2 Tyre Model

A simplified version of Dugoff's tyre model is used for this work. These equations are given in [5], which provides tyre forces in pure slip as well as combined slip scenario. The longitudinal and lateral tyre forces are written as:

$$F_x = -\frac{C_{S_x} S_x}{(1 - S_x)} f(\lambda)$$

$$F_x = -\frac{C_\alpha tan\alpha}{(1 - S_x)} f(\lambda)$$

where λ is a non-dimensional parameter related to tyre/road friction coefficient μ and normal load F_z. The Dugoff's model assumes the normal load to be uniform and this has been represented by the definition of λ. It is given by:

$$\lambda = \frac{\mu F_z (1 - S_x)}{2\left\{ (C_{S_x} S_x)^2 + (C_\alpha tan\alpha)^2 \right\}^{1/2}}$$

where

$$\mu = \mu_O (1 - A_S u_w [S_x^2 + (tan\alpha)^2]^{1/2}$$

The function $f(\lambda)$ is defined as:

$$f(\lambda) = \begin{cases} (2 - \lambda)\lambda, & \lambda < 1 \\ 1, & \lambda \geq 1 \end{cases}$$

In the above equations μ_O is the tyre road friction coefficient $u_w [S_x^2 + (tan\alpha)^2]^{1/2}$ is zero, A_S is the friction reduction factor and u_w is the velocity component in the wheel plane.

4.3 Controller Design

The controller proposed in this paper is based on Linear Quadratic Regulator (LQR) theory, discussed in detail by Dutton et al. [6]. A two degrees of freedom bicycle model is used as the reference model for the controller design. The controller monitors the vehicle (7 DOF) yaw-rate and compares it with the reference yaw-rate generated by the bicycle model. The 2 DOF bicycle is derived in detail by Abe [7] and it can written in state-space form as follows:

$$\dot{x} = Ax + Bu$$

where

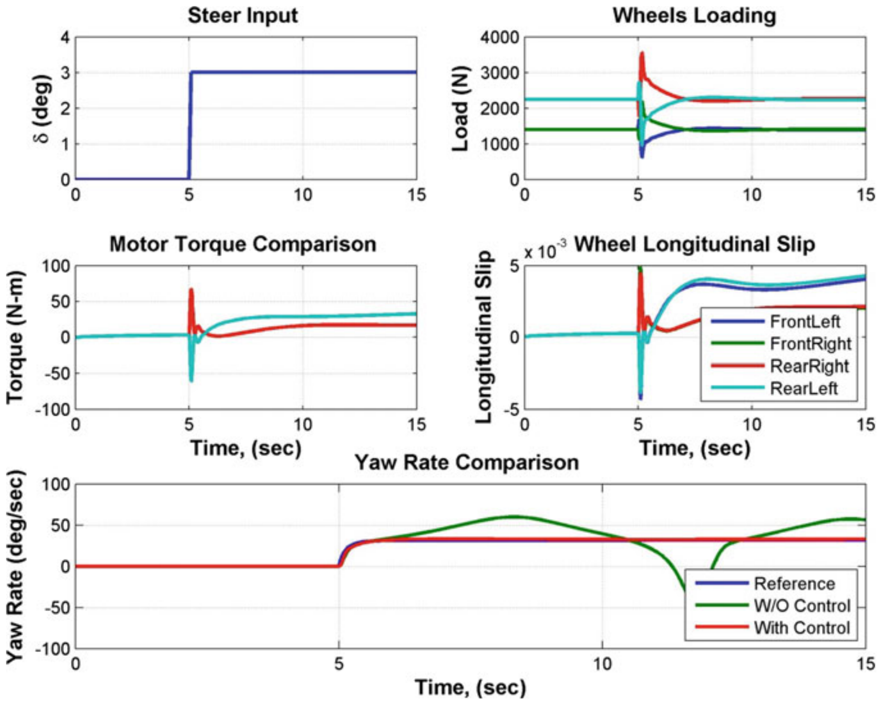


Fig. 8 Output recorded for a J-Turn manoeuvre at the speed of 65 km/h

$$A = \begin{bmatrix} -\dfrac{2(C_{\alpha f} + C_{\alpha r})}{mu} & \dfrac{2(C_{\alpha r}b + C_{\alpha f}a)}{mu} - u \\ \dfrac{2(C_{\alpha r}b + C_{\alpha f}a)}{I_z u} & -\dfrac{2(C_{\alpha f}a^2 + C_{\alpha r}b^2)}{I_z u} \end{bmatrix}, \quad B = \begin{bmatrix} \dfrac{2C_{\alpha f}}{m} \\ \dfrac{C_{\alpha f}a}{I_z} \end{bmatrix}$$

4.4 J-Turn Manoeuvre with a Constant Speed

The J-turn manoeuvre is a simple testing procedure which allows the evaluation of the vehicle's transient response as well as its steady-state behaviour. In this manoeuvre the vehicle first runs in a straight line and then enters into a turn when the steering wheel is quickly rotated from the straight ahead position to a new one and held constant. This step change in the steer angle results into a steady-state cornering after the settling time is over. Figure 8 shows the outputs recorded during the simulation in which the 7 DOF vehicle model is subjected to step steer input of 3^{o} when it was moving with the speed of 65 km/h. Due to this sudden input the vertical loading on tyres is disturbed and the vehicle's load shifts from the left side towards the right. This can be observed in the second sub-figure of Fig. 8. The yaw-rate curves show that the steer-by-wire vehicle model without

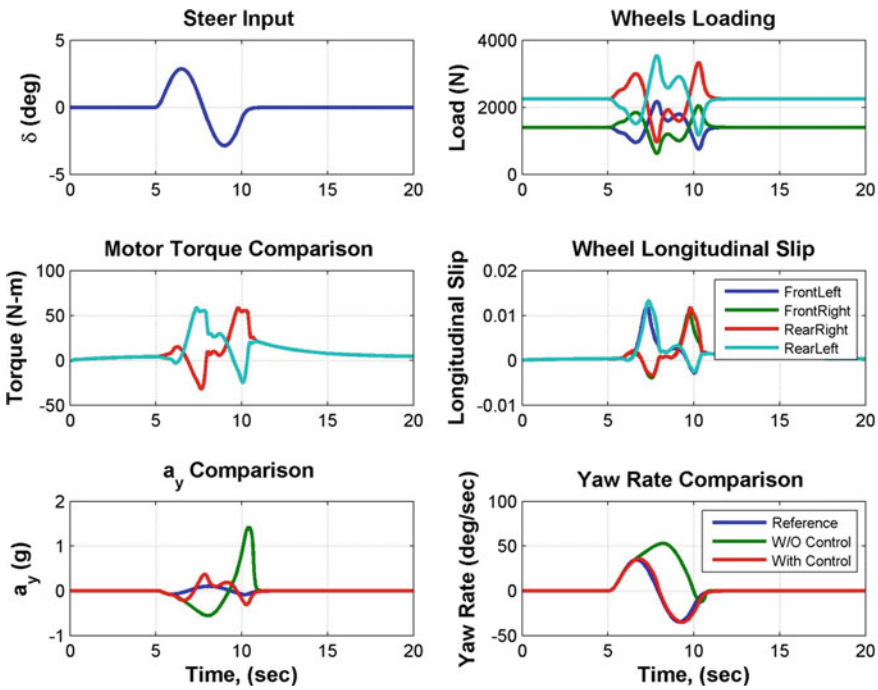


Fig. 9 Output recorded for a sine steer manoeuvre at the speed of 70 km/h

controller becomes unstable once subjected to this manoeuvre. This happens due to the lag induced by the actuators in vehicle's response. On the contrary the vehicle model with the controller is able to complete the manoeuvre like the reference vehicle model. The control action is applied by varying the torque values for each wheel. This is shown in the third sub-figure, which shows the correcting torque induced by the controller for each wheel. The fourth sub-figure is showing the longitudinal slip for the all four wheels during this manoeuvre.

4.5 Sine Steer Manoeuvre

The sine steer manoeuvre is used to evaluate the vehicle's transient lateral handling response. In this test a single sine wave has been used as a steer input for the vehicle. The amplitude and frequency of the sine steer command has been varied to test the control system's effectiveness to keep the vehicle stable at different values of lateral acceleration. Figure 9 shows information about various parameters and states recorded during a sine steer manoeuvre when a vehicle moving with a velocity of 70 km/h was subjected to a sine steer input of amplitude 3^{o} and frequency 0.2 Hz. During this manoeuvre the lateral acceleration value of the uncontrolled vehicle shows that it enters into highly non-linear region where it is difficult for an ordinary driver to control. On the contrary the vehicle equipped with the designed control system follows the reference signal and remains stable. The root mean square error values for lateral acceleration and yaw-rate are 0.06 g and 1.8 deg/s respectively.

5 Conclusion

This paper introduces the Simulink model of a steer-by-wire system, validated by experimental results. This steering model, in-conjunction with a seven degrees of freedom vehicle model, is used to show that this low performance steering system influences the handling response of the vehicle. Further a yaw controller based on LQR theory is developed, which improves the vehicle response by introducing the torque vectoring. Thus based on these simulation results it can be suggested that there is a possibility for the future vehicles having in-wheel electric motors to use low cost actuators for steering purpose.

References

1. Dick R (2005) Mercedes and auto racing in the belle epoque 1895–1915. McFarland and Company, Inc. Publishers, Jefferson
2. Ackermann J, Bünte T, Odenthal D (1999) Advantages of active steering for vehicle dynamics control. In: In Proceedings of 32nd international symposium on automotive technology and automation, pp 263–270
3. Kasselmann JT, Keranen TW (1969) Adaptive steering. Bendix Tech J 2:26–35
4. Esmailzadeh E, Vossoughi GR, Goodarzi A (2001) Dynamic modeling and analysis of a four motorized wheels electric vehicle. Veh Sys Dyn 35(3):163–194
5. Guntur R, Sankar S (1980) A friction circle concept for Dugoff's tyre friction model. Int J Veh Des 1(4):373–377
6. Dutton K, Thompson S, Barraclough B (1998) The art of control engineering. Addison-Wesley Longman Publishing Co., Inc. Boston, MA
7. Abe M (2009) Vehicle handling dynamics: theory and application. Elsevier, Oxford