Design and Performance Prediction of a Tri-Mode Power-Split Transmission

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Abstract The tri-mode power-split transmissions, compared with the single-mode and dual-mode ones, can further optimize the operating point of the engine, make full use of the characteristics of the motor/generator and improve the driving performance and the fuel economy of the vehicles. As the structure of tri-mode systems is more complex, it is difficult to apply the design method of single-mode or dual-mode systems in the tri-mode ones. Therefore, it is quite necessary to further study the method of design and performance prediction of tri-mode power-split transmissions. The design method of power-split transmissions based on ideal driving performance and the operating characteristics of the components was put forward, and a tri-mode power-split transmission was designed. The optimal parameter matching method was developed by taking the driving performance, fuel economy and cost as optimization objective, the feasible schemes as research object and the system parameters and control parameters as optimized variables. Then, based on MATLAB and Simulink, a performance prediction method was provided and the steady-state performance and dynamic performance of the power-split transmission was predicted.

Keywords Tri-mode · Power-split · Design · Parameter · Prediction

The rapid development of automobile industry has not only given great convenience to people's life, but also brought up with serious energy and environment

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problems. Due to the shortage of oil resources, the oil price is increasingly rising; at the same time, the emission standard for automobiles is becoming more and more stringent. In order to solve these problems, almost all the leading automobile companies are developing new energy vehicles. As Toyota's single-mode Prius and GM's dual-mode SUV comes into the market, the power-split hybrid vehicle based on planetary gears becomes a research focus [[1\]](#page-12-0). The characteristics of single-mode, dual-mode and tri-mode power-split transmissions were analysed, and a tri-mode power-split transmission was designed in the paper. On this basis, the parameter matching and the performance prediction was done for the feasible schemes. It would be helpful for the design and performance prediction of various types of transmissions, especially the multi-mode power-split ones.

1 Design of Power-Split Transmission

1.1 Compare of Different Power-Split Transmissions

The power-split hybrid vehicle is the current research focus. Its basic structure is shown in Fig. 1. It is consisted with one engine, two motors/generators (MGs), one power battery pack and one power-split transmission. MGs can be used as both motor and generator. The operating principle is as follows: firstly the engine's power is split by the power-split transmission, and then one part of it drives the generator to generate electricity, which could be used to charge the battery or supply power to the motor; the other part is transmitted as the form of mechanical energy, which will be output after converged with the motor's power.

A power-split transmission is generally consisted with several planetary and clutches/brakes. Its performance will vary if the connecting form of the components is different. According to the number of its speed ranges, the power-split transmission can be divided into single-mode (Fig. [2\)](#page-2-0), dual-mode (Fig. [3](#page-2-0)) and multi-mode (Fig. [4](#page-2-0)) power-split transmission.

The single-mode power-split transmission, which has only one speed range, has become the research focus for a long time because of its simple structure. However,

Fig. 2 Single-mode power-split transmission of PRIUS and its speed characteristic

Fig. 3 Dual-mode power-split transmission of GM and its speed characteristic

Fig. 4 Multi-mode power-split transmission's speed characteristic

with a narrow speed range, it has a high demand to the speed and power of the MGs. At present, this system is mainly applied in small vehicles on city roads.

With two speed ranges, the dual-mode power-split transmission has a more complex structure. As it can expand the speed range and reduce the power demand of the MGs and the power storage, this structure was applied in SUVs which have a high demand to driving performances.

The multi-mode (including tri-mode) power-split transmission, consisted with three or more planetary and several clutches/brakes, has various connecting forms and performances. With a contrast with the single-mode, dual-mode and tri-mode power-split transmission, it can tell that on the condition of a constant speed range, the increase of modes can reduce the power demand of the MGs and the batteries. Thus, with the current limited condition of MGs and batteries, the performance demand of the hybrid transmission can be satisfied by increasing the modes.

1.2 Design Method of Power-Split Transmission

With only one planetary, the single-mode power-split transmission can be connected with other components through the sun, ring and carrier in six ways. Besides, by increasing the clutches/brakes, the performance of the system can be improved further. Due to the simple structure of the system, all the possible design schemes can be listed manually, and the optimal one can be got by analysis of each scheme's characteristics.

However, the dual-mode power-split transmission generally has several planetary and clutches/brakes, as well as tens of thousands connecting ways. Thus, it is very hard to find the optimal scheme in a manual way. The scheme design work has to be completed with computer. Different design methods of the hybrid system were adopted in references [[2,](#page-12-0) [3](#page-12-0)]. Based on the ideal driving performance and the characteristics of the power components, a design method of the multi-mode power-split transmission was put forward and a tri-mode power-split transmission was designed in the paper. The design procedure is shown as follows (Fig. [5\)](#page-4-0):

1.2.1 Ideal Driving Performance

The Ideal Driving Performance, including the vehicle speed range, the maximum driving torque and power, are the optimal driving performances of the vehicle. Therefore, the demand of characteristics of the power-split transmission, including the output speed range, the maximum driving torque and power, can be calculated through the ideal driving performance.

As shown in Fig. [6,](#page-4-0) the vehicle's ideal driving performance can be divided into two ranges: the output torque is constant in the low-speed stage, while the output power is constant in the high-speed stage. The maximum output speed of the power-split transmission can be calculated through the maximum vehicle speed,

$$
n_{\text{Max}} = f_1(v_{\text{Max}}) \tag{1}
$$

where, f_1 is related to the wheels' effective radius and the rear gear ratio. The maximum output speed determines the power-split transmission's output speed range. As it is easy to analyze the transmission speed, the output speed calculation should be firstly taken into consideration in the process of scheme design. If it does not match the speed condition, the scheme should be directly given up, so that much time will be saved in the later calculation.

$$
T_{\text{Max}} = f_2(D) \tag{2}
$$

where, D represents the dynamic factor. The system's maximum value of torque can be calculated through the vehicle's dynamic factor D. Function f_2 is related to the weight of the vehicle, the effective radius of the wheels and the rear gear ratio. The maximum output torque determines the power-split transmission's output

Fig. 5 Design procedure of power-split transmission

torque range. Therefore, in the process of scheme design, in order to satisfy the demand of the vehicle's driving performance, the maximum torque of the designed scheme should not lower than that.

$$
P = f_3(v, D) \tag{3}
$$

where, P stands for the vehicle's power requirement, and it is the function of the vehicle speed and the dynamic factor. Function f_3 is a binary function, which is only determined by the vehicle's weight. Generally, the main power source of hybrid vehicle is the engine. Thus, in order to meet the vehicle's driving power demand, the power-split transmission should be able to transmit the engine's power at any speed ratio.

Fig. 7 Engine characteristics Torque (Nm)

Fig. 8 MG characteristics Torque (Nm)

Here power components include the engine, MGs and power battery. For powersplit transmission, its inputs are determined by the characteristics of the power components. Therefore, for different power components, the best scheme of the system is also different. As the technical level of the power components is gradually improved, the current bad or even infeasible scheme may become the optimum one someday in future (Figs. 7 and 8).

It has been mentioned above that the characteristics of the power components have a great influence in the power-split transmission design. At the early stage of the scheme design, the characteristics of the power components should be determined according to the technical level of the current and near future, as well as some comprehensive factors, such as the space design inside the vehicle and the cost. The inputs of the power-split transmission designed should not go beyond the given characteristic range. In this way, the inputs condition of the power-split transmission could be determined.

1.2.3 Computer Aided Calculation

After the input and output conditions of the power-split transmission are got from the ideal driving performance and the characteristics of the power components, the structure scheme which satisfies the condition can be searched automatically by computer program calculation. The computer aided calculation process mainly includes the following parts (Fig. [9\)](#page-6-0):

Firstly, choose the components' model from the Model Library, which includes ordinary gear, planetary, clutch and brake at present, and some new transmission

 Ω

Motor speed (r·min⁻¹)

structures may be added in future. The current power-split transmissions are mostly using ordinary planetary, so the possible connecting ways are extremely limited. We add up the double and duplex planet multi-member planetary into the Model Library. It is useful for us to find the suitable scheme.

Secondly, make limitation to the number of the components according to the space inside the vehicle, size of the components and cost of the system. Later, list all the connecting schemes with computer program, and generate the constraint equation automatically. The reference [[4\]](#page-12-0) completed the choosing process by matrix analysis and calculation. Here we use the method of graph theory, the specific introduction of which can be found in reference [\[5](#page-12-0)].

Lastly, based on the characteristics of the power components, analyze the speed, torque and power of all the possible schemes, so as to judge that whether they meet the demands generated from the ideal driving performance. Only all the performance demands are satisfied, the corresponding connecting scheme can be saved and further analyzed.

1.3 A Tri-mode Power-split Transmission

By using the method mentioned above, a tri-mode power-split transmission was designed in this paper. Its structure is shown as the following Fig. [10](#page-7-0):

The tri-mode power-split transmission is consisted with three planetary, two clutches, two brakes, and two MGs. Here 'i' stands for the input shaft which is connected with the engine, and 'o' represents the output shaft which is connected with the driving shaft. Remarkably, there is a dual-planet planetary in the transmission, the speed and torque characteristics of which are different from that of an ordinary planetary. It is particularly suitable to be used here, as it can make the power-split transmission meet the input and output performance demands well.

Fig. 10 A tri-mode powersplit transmission

Table 1 Operating mode and clutches/brakes' condition

The tri-mode power-split transmission has three EVT modes and three fixed gear ratio (FG). The relationship between the modes and the state of clutches and brakes is shown in Table 1. Methods of parameter matching and performance prediction of the power-split transmission will be introduced in detail by taking this structure as an example in the following parts.

2 Parameter Matching

There are many references introducing parameter matching $[6-8]$, and the related technics have already been mature. The optimization based parameter matching method was adopted here. The optimal parameter matching method was developed by taking the driving performance, fuel economy and cost as optimization objective, the feasible schemes as research object and the system parameters and control parameters as optimized variables.

Most of the existing parameter matching methods all took driving performance, fuel economy and emission as optimization objectives. The tri-mode power-split transmission, applied in heavy off-road vehicles, emphasizes on the cost rather than the emission performance, so the comprehensive cost of the power-split transmission was taken as an optimization objective,

$$
J = \text{Min}[-f_1(x), f_2(x), f_3(x)] \tag{4}
$$

where, $f_1(x)$ is the function of the driving performance. The better the driving performance is, the larger $f(x)$ is. Thus, its opposing value was taken as one of the

optimization objectives. $f_2(x)$ is the function of fuel consumption and $f_3(x)$ is the function of cost. Both of them are the smaller the better, so they are taken as another two optimization objectives.

The constraint conditions contained the operating range of power components, the speed, torque and power equations of the system and other performance constraints,

$$
\begin{cases} g(x) \le 0 \\ h(x) = 0 \end{cases}
$$
 (5)

With the optimization objectives and constraint conditions, the optimization model could be built by utilizing computer programming. Here the optimizing variables mainly conclude the characteristic parameters such as the power of the engine, electric motor, and battery. As it is related with the dynamic programming algorithm, the control variable needs to be chosen as the optimizing variables, which will be specifically introduced in the following part.

3 Performance Predictions

For the power-split transmission is a multiple-degree-of-freedom system, the components can work in different operating states, although the demands are the same. Thus the performance of the system largely depends on the quality of the control strategy. It can be known from the control theory that strategies based on optimization are better than those based on rules, and the performance of global optimization is better than that of instantaneous optimization [\[9](#page-12-0)]. The dynamic programming algorithm is a kind of global optimization algorithm which can achieve the optimum comprehensive performance of the system. As the calculated amount of dynamic programming algorithm is huge, it would be difficult to implement if the dynamic model is directly used. Therefore, the steady-state model was adopted at first, and the optimal scheme and system parameter was found through steady-state optimization by MATLAB. Then the dynamic model was built with Simulink, and the dynamic performance of the system was predicted through dynamic simulation.

3.1 Dynamic Programming Algorithm

The dynamic programming algorithm is a kind of multiple procedure decision optimization algorithm. According to the demands of control accuracy and calculating time, the driving circle needs to be divided into several segments. The calculation is made from back to front, and the optimum performance of the system can be realized within the whole time range.

Fig. 11 Operating states of components of the tri-mode power-split transmission

The inherent resistance model of the battery was adopted, the power of which is shown as follows,

$$
P_{batt} = V_{oc}I_{batt} - I_{batt}^2 R_{batt} \tag{6}
$$

where, V_{oc} is the open-circuit voltage, I_{batt} is the charge or discharge current and R_{batt} is the inherent resistance. SOC can be got through the following expression,

$$
\frac{d}{dt}(SOC) = \frac{-Ibatt}{3600Cbatt} \tag{7}
$$

where, C_{batt} is the capacity of the power battery. Then the following result can be derived through the express (6) and (7) :

$$
\frac{d}{dt}(SOC) = -\frac{V_{oc} - \sqrt{V_{oc}^2 - 4P_{batt}R_{batt}}}{7200C_{batt}R_{batt}}\tag{8}
$$

3.2 Steady-State Performance Prediction

The optimal structure scheme, system parameters and control parameters were obtained by solving the steady-state optimization model. Then the operating state of each component, and the output speed and torque of the power-split transmission was calculated, which was shown in Figs. 11 and [12.](#page-10-0)

Fig. 12 Driving performance of the tri-mode power-split transmission

The abscissa and the ordinate respectively stand for the vehicle speed and the operating condition of the components. The operating condition of the components at different vehicle speed is obtained through steady-state calculation, so that it can provide necessary condition for the prediction of the system's steady-state performance.

The abscissa stands for vehicle speed, the blue lines stand for the dynamic factors of three EVT modes, the red stands for the ideal dynamics factors and the black lines stand for the dynamic factors of three FG modes. The value of the vehicle's maximum dynamic factor is larger than that of the ideal dynamics characteristic at each speed. Therefore, this tri-mode power-split transmission is able to meet the vehicle's driving performance demand.

3.3 Dynamic Performance Prediction

After the scheme design, parameter matching and steady-state performance prediction of the power-split transmission had been made through steady-state calculation, the dynamics model of the chosen scheme would be built, and its dynamic performance would be predicted through dynamic simulation. The dynamic model of a complete vehicle was built by using the common models in Simulink, Simdriveline and SimpowerSystem. The control system model was built by using Stateflow and some common models. There are many references introducing the model building method of the hybrid system. Hence, the simulation result of the dynamic performance prediction is directly given in the paper, which is shown in Figs. [13](#page-11-0) and [14](#page-11-0).

In Fig. [13,](#page-11-0) the abscissa and the ordinate respectively represent the simulation time and the operating condition of the components. Due to the usage of the dynamic model, the operating condition of the components has a slight fluctuation with the time. But the fluctuation range is relatively small. The operating condition

Fig. 13 Operating states of power components in dynamic simulation process

Fig. 14 Engine's operating points distribution in process of dynamic simulation

of the components obtained through dynamic simulation will provide references for system's dynamic performance prediction.

It can be seen from Fig. 14 that most of the engine's operating points are near the lowest fuel consumption rate, so the whole vehicle's fuel economic performance is guaranteed. It can be known from the dynamic simulation result that the tri-mode power-split transmission designed in this paper can not only adapt the change of vehicle speed rapidly, but also optimize the operating zone of the engine, so that the system has good driving performance and fuel economic. Therefore, the method of combining the scheme design, parameter matching and performance prediction is effective and successful.

4 Conclusions

After comparing the current structures of power-split transmission, a design method of the power-split transmission based on ideal driving characteristic and component's output characteristic was put forward. An optimal parameter matching method was developed by taking the driving performance, fuel economy and cost as optimization objective, the feasible schemes as research object and the system parameters and control parameters as optimized variables. Besides, a performance prediction method based on MATLAB and Simulink was provided. The scheme design, parameter matching and performance prediction, respectively as the foundation, necessary condition and evaluation criterion, were inseparable. It can be known from the steady-state calculation and dynamic simulation results that the designed power-split transmission could achieve good driving performance and fuel economic, which proved that the research method in this paper was feasible.

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