The Influence of Torque and Speed Sensitive Differential Characteristics in a Front Wheel Drive Vehicle During On-Limit Manoeuvres

Anthony Tremlett, David Purdy, Nick Vaughan, Francis Assadian, Adrian Moore and Martin Halley

Abstract Passive Limited Slip Differentials (PLSD) are a well-established means of improving the traction limitation imposed by the open differential, and achieve this by transmitting a bias torque from the faster to slower rotating driven wheel. This torque bias is typically proportional to the differential input torque (torque sensing) or the speed difference between driven wheels (speed sensing). In the motorsport environment however, there exist devices which are able to bias torque through both methods simultaneously, but to date, remain unexplored with respect to their influence on handling and the differential models required to study them. Plate (Salisbury) type and Viscous Coupling (VC) differential models are formulated, then combined into a Viscous Combined Plate (VCP) model and used as a basis for handling characterisations through typical cornering scenarios. An 8 degree of freedom (DOF) vehicle model created in the MATLAB/Simulink environment is parameterised around a front wheel drive (FWD) saloon racing vehicle. Path preview steering control is used to give a robust means of comparing the necessary driver inputs to maintain a particular racing line, whilst reaching the lateral acceleration limit of the vehicle. At low lateral accelerations (<5 m/s^2) VC, VCP and locked differentials were shown to increase levels of understeer most, as the torque bias of a plate differential is proportionally less due to the minimal throttle input required to maintain vehicle speed. At these low speeds, any torque bias acts to reduce initial turn-in yaw rate response. At higher lateral accelerations (>5 m/s^2), inner wheel tyre saturation ultimately limits the maximum speed through a corner. The degree of

F2012-C02-003

A. Tremlett (✉) · D. Purdy · N. Vaughan · F. Assadian
Cranfield University, Cranfield, Bedfordshire, UK
e-mail: a.j.tremlett@cranfield.ac.uk

A. Tremlett · A. Moore · M. Halley
Xtrac Limited, Thatcham, Berkshire, UK

SAE-China and FISITA (eds.), *Proceedings of the FISITA 2012 World Automotive Congress*, Lecture Notes in Electrical Engineering 193,
DOI: 10.1007/978-3-642-33744-4_8,

torque bias that a PLSD can provide to the outer wheel will delay the onset of inner tyre saturation, and increase the maximum lateral acceleration limit. In raw performance terms, this means a locked differential is up to 0.05 s quicker through a constant 50 m radius corner when compared to an open differential equivalent.

Keywords Vehicle dynamics · Limited slip differential · Viscous coupling · Torque sensitive · Speed sensitive

Nomenclature

A_p	Empirical differential friction constant
$C_{\mu 1,2,3}$	Empirical friction surface constant
F_p	Differential preload force (N)
K_g	VC disc geometry correction factor
R_r	Mean contact radius between ramp and crosspin (m)
$R_{o,i}$	Outer and inner clutch surface/shear surface radius (m)
s	Distance between VC shear surfaces (m)
T_d	Differential input torque (Nm)
$T_{dd,dc}$	Drive and coast critical input torque (Nm)
$T_{p,vc,vcp}$	Plate, VC and VCP differential locking torque (Nm)
T_{max}	Normalisation locking torque (Nm)
Z_f	Number of friction/shear faces
θ_r	Drive/coast ramp angle (degrees)
$\mu_{p,r}$	Clutch plate and ramp-crosspin surface friction coefficient
μ_{max}	Normalisation friction coefficient
υ	VC fluid kinematic viscosity (mm^2/s)
ρ	VC fluid density (kg/m^3)
ω_d	Driven wheel speed difference (rpm)

1 Introduction

Automotive differentials are well-established as a means of transferring torque to wheels rotating at different speeds. Provided there is a sufficient level of grip at each wheel, the open differential is a perfectly satisfactory device. However, in extreme instances where traction at one wheel is compromised, Limited Slip Differentials (LSD) have been shown to offer distinct improvements in traction and vehicle stability [1–3]. LSDs achieve these performance benefits through the transfer of torque from the faster to slower driving wheel. In the majority of racing formulae, regulations dictate that this torque bias is controlled passively, either through differential input torque (torque sensing) or driven wheel speed difference (speed sensing) [4]. However, there exist devices which are able to bias torque

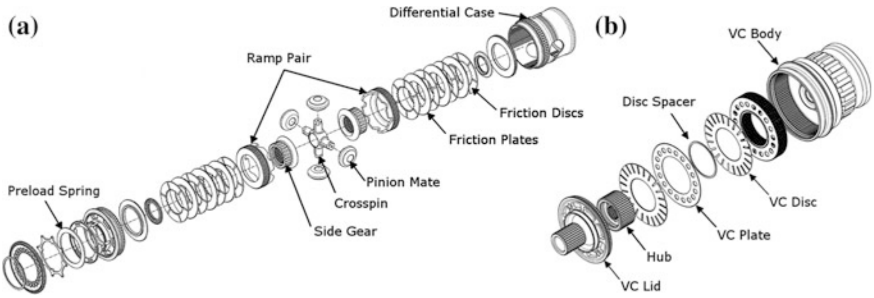

Fig. 1 **a** Exploded view of Xtrac [4] plate differential and **b** viscous coupling

through both methods simultaneously, but to date, remain unexplored with respect to their influence on handling and the differential models required to study them.

This paper aims to formulate a more comprehensive plate differential model and combine with a Viscous Coupling (VC) unit to produce a Viscous Combined Plate (VCP) simulation model. This is primarily to allow the influence of its locking characteristics on vehicle handling to be studied, but also for future optimisation of the torque bias itself. Figure 1a shows an exploded view of an Xtrac [5] plate type, or 'Salisbury' differential [3], in which a torque bias is generated through the thrust loads of a ramp pair acting against two wet clutch packs. Conversely, the speed sensitive VC (see Fig. 1b) relies on the torque generated from a number of plates shearing through a viscous silicon fluid to generate its torque bias [3].

2 Differential Simulation Models

2.1 Torque Sensitive: Plate Type

The most basic method to calculate the locking torque generated by a clutch plate assembly uses uniform pressure clutch theory [6]. This analysis can be extended and applied to the particular case of a plate differential, by incorporating the effects of ramp angle (θ_r) and the number of friction faces (Z_f) [7]. Furthermore, it is common to allow a residual clamp force (F_p) or 'preload' to clamp the plate pack. This is both to improve lockup response and to allow a degree of torque bias to take place when little differential input torque (T_d) can be distributed to each driven wheel (e.g. when a wheel is in mid-air due to a curb strike). The resulting locking torque (T_p) can be defined for both on-throttle (drive) and off-throttle (coast) conditions. Below critical drive and coast input torques (T_{dd}, T_{dc}), T_p is equal to a preload locking torque (T_{lp}). Above these critical input torques, T_p is governed by a more significant ramp locking torque (T_{lr}). These regions are defined in Eqs. 1 and 2 and typical locking characteristics are shown in Fig. 2a.

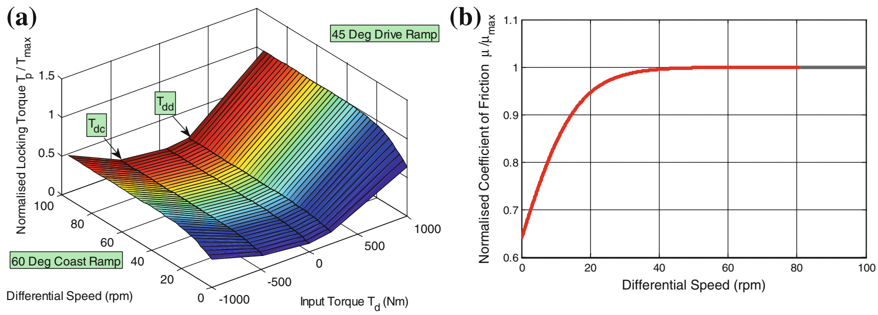

Fig. 2 **a** Normalised locking characteristics of an Xtrac [4] plate differential, **b** normalised coefficient of friction approximation with differential speed

$$Ramp\ Locking\ Torque\ \ (T_{lr})\ \frac{2}{3}\left[\frac{T_d}{R_r}\left(\frac{\cos\theta_r - \mu_r \sin\theta_r}{\sin\theta_r + \mu_r \cos\theta_r}\right)\right]\mu_p Z_f\left(\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right) \tag{1}$$

$$Preload\ Locking\ Torque\ \ (T_{lp})\ \frac{2}{3}\left(\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right)\mu_p F_p Z_f + A_p T_d \tag{2}$$

Where Z_f is the number of friction faces, μ_p the coefficient of friction between friction faces and A_p an empirical constant representing the inherent friction of rotating elements within the differential housing (bearings, side gear, pinion mates). Crucial to the accuracy of this model, is the calculation of plate friction. The friction coefficient changes with many factors including clamp load, oil temperature, differential speed and friction material [8, 9]. One of the most relevant of these factors is sensitivity of the friction coefficient with differential speed. Most simply, this can be represented by an approximation of empirical data defined in Eq. 3 and depicted in Fig. 2b.

$$\mu_p = C_{\mu 1} \tanh\left(C_{\mu 2}\omega_d\right) + C_{\mu 3} \tag{3}$$

where $C_{\mu 1}$, $C_{\mu 2}$, $C_{\mu 3}$ are empirically determined constants and ω_d is the difference in rotational speed of the friction faces. The resulting locking torque gradient shown in Fig. 2a is governed mainly by input torque but also relative wheel speed at lower speeds. The locking torque gradient is an important aspect, since it will control the rate of differential torque bias and the resulting yaw moment imparted to the vehicle through longitudinal tyre forces. As the input torque and therefore torque bias varies substantially during typical driving conditions, this has the potential to influence vehicle stability.

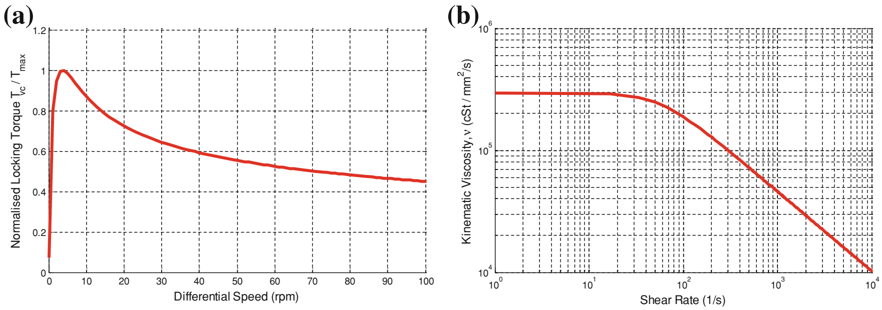

Fig. 3 **a** Normalised VC locking characteristics and **b** variation of fluid viscosity with shear rate

2.2 Speed Sensitive: Viscous Coupling

In contrast to torque sensing differentials, speed sensing differentials are not reliant on an input torque to generate locking. Of the current types in use, VCs used in conjunction with a conventional bevel gear open differential have proved popular since they provide a reliable, cost efficient solution, which can be easily tuned to give a range of locking characteristics. Of crucial importance is the fluid viscosity relationship with temperature and shear rate [10, 11]. Assuming the device is maintained at a constant operating temperature an isothermal representation can be used to calculate the locking torque generated from viscous shearing of the silicon fluid. This is defined in Eq. 4 [10] where, ρ and υ are the density and kinematic viscosity of the fluid, s the distance between shear plates, and K_g a correction factor used to account for the reduction in shear area due to plate geometry (e.g. slots and perforations).

$$VC\ Locking\ Torque\ \ (T_{vc}) = Z_f \frac{K_g \rho \pi \upsilon \omega_d}{2s} \left(R_o^4 - R_i^4\right) \qquad (4)$$

Typical locking torque characteristics and the variation of fluid viscosity with shear rate are shown in Figs. 3a and b, at a nominal fluid temperature of 25 °C.

2.3 Viscous Combined Plate

The characteristics of both differentials can be utilised by coupling the VC unit between the outputs of a conventional plate differential, as shown in the schematic in Fig. 4a. This yields what is termed a VCP Differential. Consequently, locking torque is now governed both by input torque (T_d) and the difference in driven wheel speed (ω_d). Total locking torque (T_{vcp}) can be described by Eqs. 5 and 6 and is depicted in Fig. 4b.

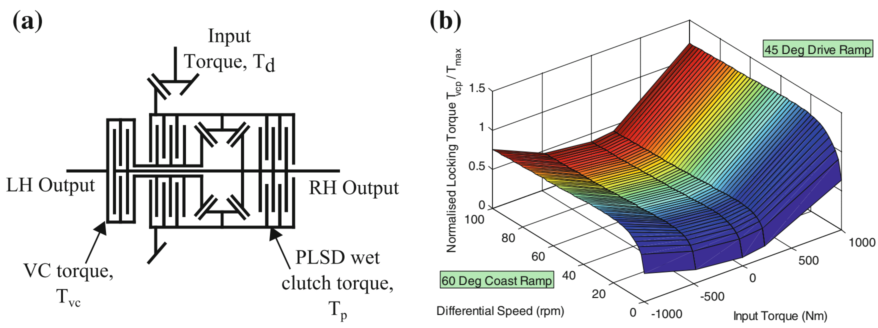

Fig. 4 **a** VCP schematic and **b** normalised locking characteristics of VCP differential

$$VCP\ Locking\ Torque\ \ (T_{rvcp}) = T_{lr} + T_{vc}\ \text{(ramp region)} \tag{5}$$

$$VCP\ Locking\ Torque\ \ (T_{pvcp}) = T_{lp} + T_{vc}\ \text{(preload region)} \tag{6}$$

3 Vehicle Model

To evaluate the influence of plate, VC and VCP locking characteristics on vehicle handling, an 8 degree of freedom (DOF) model constructed in the Matlab/Simulink environment was used (see Fig. 5). The equations of motion for the system describe longitudinal, lateral, yaw and roll vehicle motions in addition to four wheel rotations [12]. This was combined with a non-linear Pacejka tyre model [13] and vehicle parameters from a FWD saloon racing vehicle. To replicate a degree of driver realism, a preview steer driver model was employed [14] which uses a pre-defined path to generate steering corrections based on vehicle position and yaw angle. Longitudinal velocity is controlled with a Proportional Integral (PI) throttle parameter (ranging from 0 to 1), used in conjunction with saturation functions generated from each tyre. This effectively limits the control input under conditions in which the tyres have exceeded their optimum longitudinal slip ratio (i.e. slip at which peak longitudinal force is generated). The differential models were included as look-up tables and considered perfect actuators, which did not account for any dynamic clutch engagement effects.

4 Simulation Results

4.1 Constant Radius Acceleration

To evaluate the performance of each differential at the traction limit, a 30 m radius skid pan test was conducted. The vehicle was first held at a steady state speed to provide initial starting conditions and then requested to accelerate at 0.2 g until the

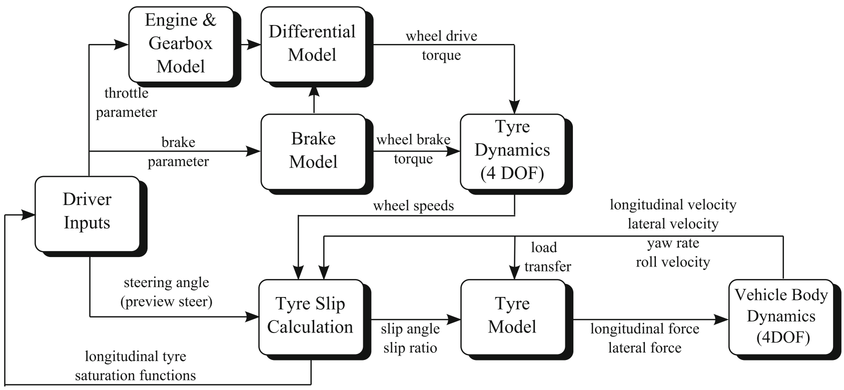

Fig. 5 8 DOF Vehicle Model Structure

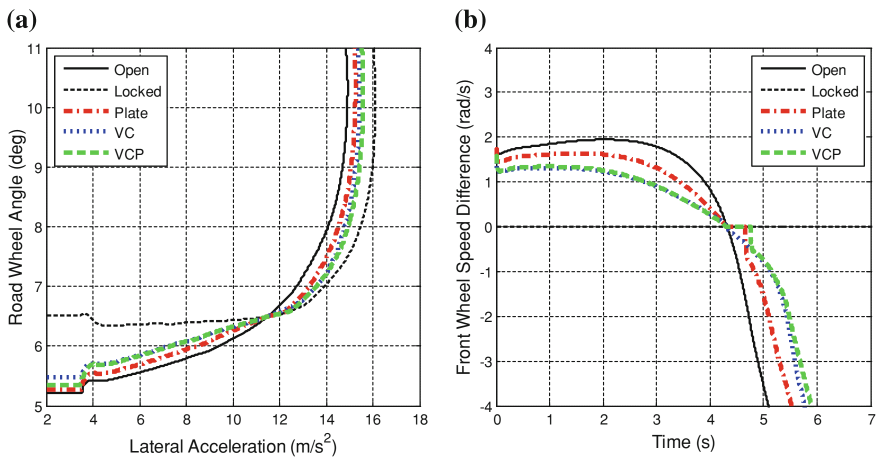

Fig. 6 **a** Understeer gradient comparison of plate, VC and VCP differentials, **b** front wheel speed difference, (30 m Skid pan radius, 0.2 g longitudinal acceleration)

lateral acceleration limit of the vehicle had been reached. Figure 6 shows the steering angle and resulting driven wheel speed difference required to maintain the 30 m radius. A maximum path error of ±0.2 m was permitted throughout the manoeuvre.

Figure 6a shows that at low lateral accelerations (<5 m/s^2), VC, VCP, plate and locked differentials increase the steering required to keep the vehicle on the 30 m radius. This steering increase is proportional to the torque bias generated by each device. As lateral acceleration increases to 12 m/s^2 the plate and VCP differentials force driven wheel speeds to synchronise and 'lock' (see Figs. 6b, 7b). This occurs when the differential can support a sufficient bias torque to overcome the difference in longitudinal tyre forces. Both differential types remain locked until

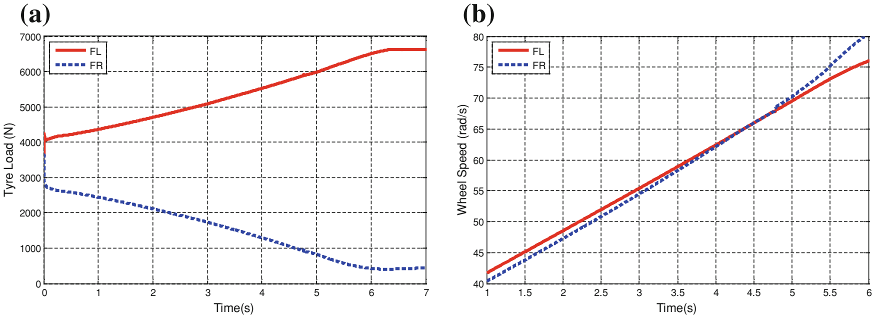

Fig. 7 **a** Front left (*FL*) and front right (*FR*) tyre loads, **b** Front wheel speeds, (VCP differential)

the difference in longitudinal tyre forces becomes more significant. In this case, this has occurred due to lateral load transfer (see Fig. 7a) compromising inner (front right) wheel traction. The locked differential is shown to increase the lateral acceleration limit from that of an open differential (14.9 m/s^2) up to 16.1 m/s^2.

4.2 Constant Velocity Corner

A further series of simulated test manoeuvres were conducted through a sequence of straight—constant radius bend (50 m)—straight, negotiated at three constant speeds (15, 27.5 and 28.0 m/s). These speeds were intended to start well below the vehicle lateral acceleration limit and to increase up to and beyond the lateral acceleration that the tyres could support. At the lowest speed of 15 m/s the road wheel angle trace (Fig. 8a) shows that the locked differential requires the highest steering angle, with the VC, VCP and plate all falling in between the locked and open differential steering traces. Figure 8c shows the yaw moment generated by each differential which is directly related to the magnitude of the differential locking torque. The locked differential yaw moment dominates due to the higher longitudinal slip induced at each driven wheel. However, due to the relatively low throttle input (Fig. 8b) required to maintain vehicle speed, the plate yaw moment is proportionally small. Importantly, the direction of the torque bias gives a negative moment, i.e. the differential acts to reduce the initial turn-in yaw moment and effectively increases the yaw damping of the car. In raw performance terms however, the manoeuvre time of each differential type is identical (see Table 1). The longitudinal and lateral tyre force saturation plots shown in Fig. 9 also confirm that the tyres are well below their adhesion limit.

When the vehicle speed is increased to 27.5 m/s both the steering angle and throttle required to maintain the manoeuvre speed increase dramatically (see Fig. 10a, b). Due to lateral load transfer, an open differential promotes inner wheel saturation, and road wheel angle increases to almost 10° to allow the vehicle to negotiate the corner. The remaining devices which support a torque bias (plate,

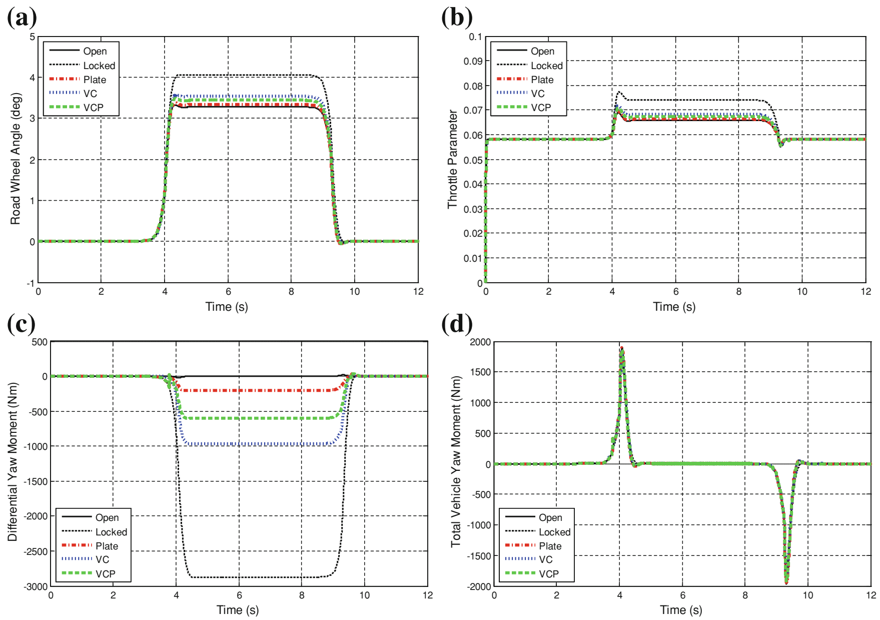

Fig. 8 **a** Road wheel angle, **b** throttle parameter, **c** differential yaw moment and **d** total vehicle yaw moment (sub-limit, constant vehicle speed 15 m/s, 50 m radius turn)

Table 1 Constant velocity corner manoeuvre time(s) and maximum lateral acceleration (m/s^2)

	Open	Locked	Plate	VC	VCP
Manoeuvre time (15.0 m/s)	13.240	13.240	13.240	13.240	13.240
Peak lateral acceleration	4.55	4.56	4.55	4.56	4.55
Manoeuvre time (27.5 m/s)	7.254	7.226	7.226	7.226	7.226
Peak lateral acceleration	15.29	15.78	15.62	15.73	15.70
Manoeuvre time (28.0 m/s)	7.169	7.097	7.111	7.109	7.103
Peak lateral acceleration	15.26	15.87	15.64	15.75	15.72

VC, VCP) show reduced levels of steering, but importantly the differential yaw moment now reverses its direction from being negative (understeer) to a positive, oversteer moment at 2.5 s (see Fig. 10c).

The magnitude of the moment generated by plate and VCP differentials has also increased due to the higher throttle and therefore higher differential input torque. However, at this higher lateral acceleration, all differential yaw moments are now much smaller than those generated by lateral tyre forces. This can be seen in Fig. 10d, where the total turn-in yaw moment has increased to over 4kNm, with the differential yaw moment only starting to approach—1kNm before switching directions at 2.5 s. This means that the closer the vehicle is to its lateral acceleration limit, the less dominant differential yaw moments become in influencing

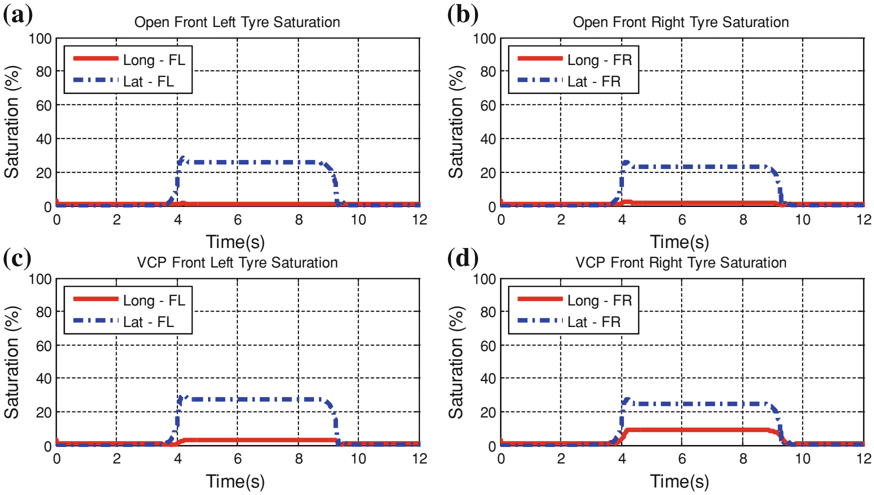

Fig. 9 **a** *Front left* and **b** *front right* tyre saturation for an open differential, **c** *front left* and **d** *front right* tyre saturation for a VCP differential

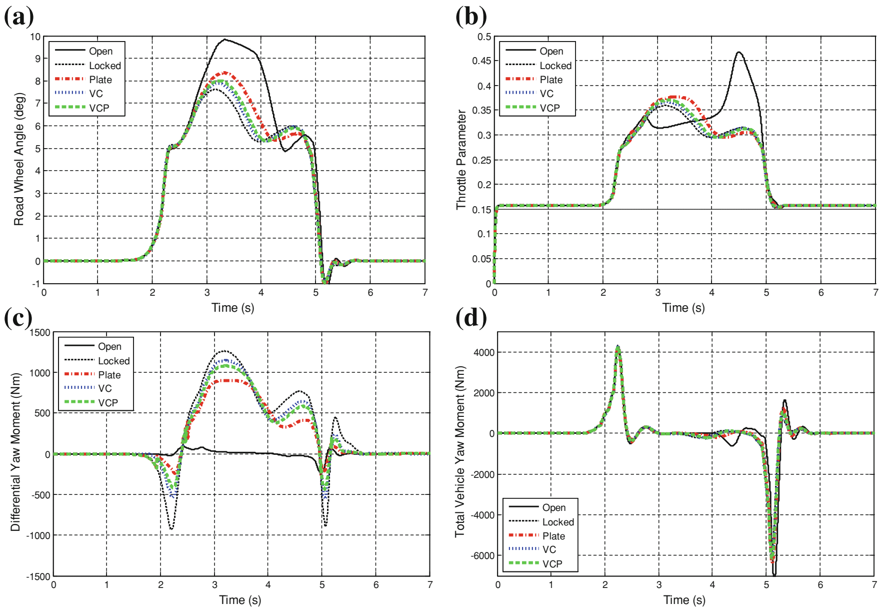

Fig. 10 **a** Road wheel angle, **b** throttle parameter, **c** differential yaw moment, **d** total vehicle yaw moment (on-limit, constant vehicle speed = 27.5 m/s, 50 m radius turn)

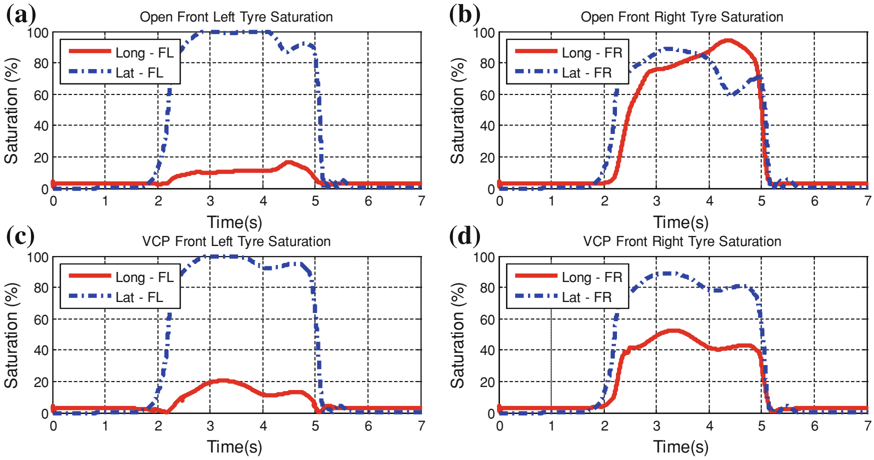

Fig. 11 **a** *Front left* and **b** *front right* tyre saturation for an open differential, **c** *front left* and **d** *front right* tyre saturation for a VCP differential

the fundamental understeer/oversteer balance of the car. Nevertheless, any differential torque bias helps to limit the longitudinal slip of the inside tyre and maintain a greater lateral tyre force utilisation. This is shown on the lateral force saturation curve of the inner tyre of the VCP (see Fig. 11d). Furthermore, this is reflected in the peak lateral acceleration and manoeuvre time (Table 1) which now shows a more marked performance improvement for locking differentials, both at 27.5 and 28 m/s. It must be borne in mind however, that although offering the largest performance benefit, a locked differential will also dramatically increase tyre wear.

5 Conclusions

This paper has described a plate differential model that is able to show the effect of ramp angle, friction faces and preload on locking characteristics. This is combined with a VC model to produce a novel VCP differential, capable of biasing torque through torque sensing and speed sensing means. An 8 DOF vehicle model was used to show how each device influences handling during on-limit manoeuvres. It is apparent that there are two modes of operation:

Low speed, low lateral accelerations (<5 m/s^2):

- VC and VCP differentials give higher levels of understeer than plate types, since torque bias is speed and therefore manoeuvre radius dependant. The relatively small throttle input required to maintain vehicle speed means the plate differential generates a proportionally smaller bias torque. However, this also depends on the particular plate differential configuration (ramp angle, friction faces and preload).

- All differential types, (excluding open) will generate an understeer moment during turn-in. This reduces vehicle agility but may also be used to help stability under heavy braking.

High speed, high lateral accelerations (>5 m/s^2):

- Greater amounts of differential locking increase the vehicle lateral acceleration limit by delaying the onset of inner tyre saturation. This limit was shown to increase from 14.9 m/s^2 for an open differential to 16.1 m/s^2 for a locked differential (8 % increase).
- At high lateral accelerations, due to lateral load transfer, the inner tyre becomes unladen and allows the direction of torque transfer to reverse for all but open differentials. This generates an oversteer yaw moment, but is small when compared to the yaw moment generated by lateral tyre forces.
- VC, VCP and plate locking differentials can be tuned to give equivalent levels of locking which will all approach the maximum lateral performance of a locked differential. The point at which this will occur depends on corner radius and throttle application.
- A locked differential was shown to be up to 0.05 s quicker through a constant 50 m radius corner when compared to an open differential.

References

1. Kopf P, Escher M, Gazyakan U, Oberhauser M (1990) Optimization of traction and driving stability in 2-wheel drive cars by means of electro-hydraulic limited-slip differentials, SAE paper 905108
2. Morselli R, Zanasi R, Sandoni G (2006) Detailed and reduced dynamic models of passive and active limited-slip car differentials. Math Comput Modell Dyn Syst 12(4):347–362
3. Milliken WF, Milliken DL (1995) Race car vehicle dynamics, SAE
4. FIA technical regulations,—http://www.fia.com/sport/regulations. Accessed 9 July 2012
5. Xtrac, Xtrac Limited, Kennet Park, Gables Way, Thatcham, Berkshire, RG19 4ZA
6. Rothbart HA, Brown TH (Jr.) (2006) Mechanical design handbook: measurement, analysis, and control of dynamic systems. McGraw Hill, New.York
7. Dickason I (2008) Development of a theoretical locking model for a motorsport mechanical plate differential, Master's thesis, School of engineering. Cranfield University
8. Maki R (2005) Wet clutch tribology, Doctoral thesis, Lulea University of technology, Department of applied physics and mechanical engineering
9. Ivanovic V, Deur J, Herold Z, Hancock M, Assadian F (2008) Experimental setups for active limited slip differential dynamics research, SAE world congress and exhibition. April 2008, (2008-01-0302)
10. Mohan SK, Ramarao BV (2003) A comprehensive study of self-induced torque amplification in rotary viscous couplings. J Tribol 125(1):110–120
11. Peschke W (1986) A viscous coupling in the drive train of an all-wheel-drive vehicle, SAE technical paper series 860386

12. Genta G (1997) Motor vehicle dynamics: modeling and simulation (series on advances in mathematics for applied sciences—vol. 43), World Scientific Publishing Co. Pte. Ltd
13. Pacejka HB, Besselink IJM (1997) Magic formula tyre model with transient properties. Veh Syst Dyn Suppl 27:234–249
14. Sharp RS, Casanova D, Symonds P (2000) Mathematical model for driver steering control, with design, tuning and performance results. Veh Syst Dyn 33(5):289–326