Definition of Slippage Parameters of Friction Clutches in Gearboxes with Fixed Shaft Axles

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Abstract The paper is about a calculation method for slipping parameters of friction clutches (of the slipping work and time) in tractor gearboxes at various gear overlaps and at bench tests of full-sized prototypes.

Keywords Gearbox · Slippage · Clutch · Friction · Fixed shaft axles

Modern wheeled and track vehicles broadly use the gearboxes, where gearing is made both with and without interruption of power flow from the engine by means of friction clutches (FCs) with hydraulic constriction and different gear overlapping. However, until today, the gear shifting process executed inside the gearbox by means of FCs has not been completely studied.

As an example, let us consider the gear shifting process in a tractor gearbox. The gear shifting under load can be made both with and without a short-term interruption of power flow from the engine. The continuous (non-break) gearing ensured by simultaneous operation of two gears within a short period of time t_p (gear overlap time) has some peculiarities. The process of continuous gear shifting depends on the parameters of the FCs, which ensure shifting, the overlap duration, specifications of the tractor unit (or machine-tractor unit–MTU) and its operation conditions.

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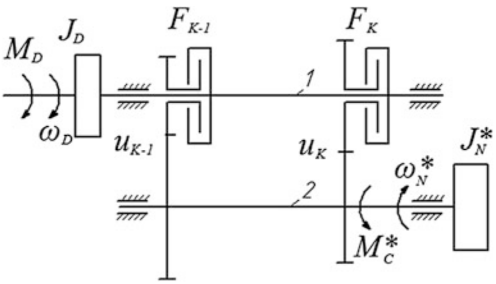

Fig. 1 Double-mass vehicle dynamic system with elementary gearbox. *1*—driving shaft, *2*—driven shaft; F_{K-1} and F_K—FCs of the K − 1 and K gears respectively; M_D—torque on the engine shaft, brought to the shaft of the engaged FC, ω_D—angular velocity of the engine shaft, brought to the shaft of the engaged FC; J_D—the inertia moment of moving parts of the engine and parts connected thereto, brought to the shaft of the engaged FC; M_C^*—the resistance moment to motion, of the tractor unit brought to the driven shaft of the gearbox, J_N^*—the inertia moment of the tractor unit brought to the driven shaft of the gearbox; ω_N^*—the angular velocity of the driven shaft of the gearbox; and u_{k-1} and u_k—the transmission ratios of the gearbox at the K − 1 and K gears respectively

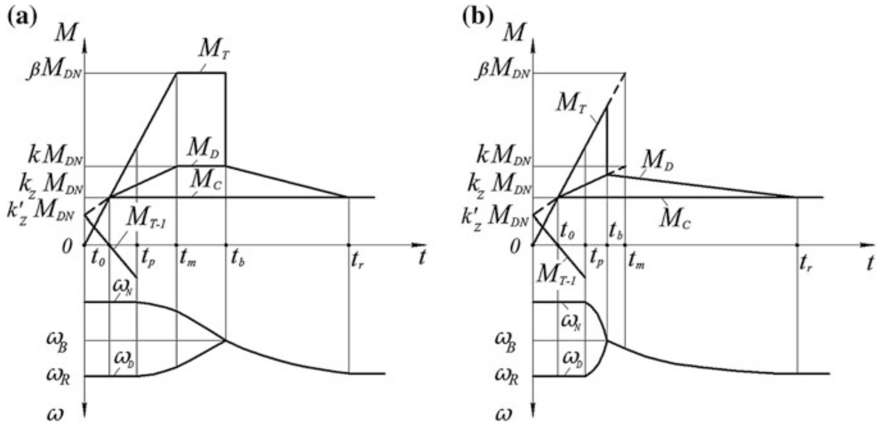

Fig. 2 Acceleration diagram of the tractor unit at gear shifting without interruption of the power flow from the engine. **a** with a flat section ($t_b > t_m$), **b** shape of "triangle" ($t_b \leq t_m$)

Let us consider the gear shifting process with overlap at shifting from lower to higher gear on the example of an elementary gearbox with two parallel shafts (Fig. 1).

The basic parameters defining the gear shifting process are the work L and the time t_b of FC slippage F_K when shifting to higher gear. To define these parameters, let us make use of a theoretical diagram for acceleration of the tractor unit (Fig. 2), which assumes that the friction torque M_T of the engaged FC changes linearly. This assumption was confirmed by experimental acceleration studies of various tractor units, which established that at the regular rate of FC engagement F_K (see

Fig. 1), the friction torque M_T phases up under the law, which is close to linear [1–6].

When defining the law of changing the engine torque M_D, when the tractor unit is accelerating (see Fig. 2), let us assume that within the time interval (from 0 to t_0), it varies in proportion to the current time t of slippage from the value of $k'_z M_{DN}$ to the value of $k_z M_{DN}$. Here, k'_z and k_z are the load factors of the engine at the low and high gears, respectively; M_{DN} is the nominal torque of the engine reduced to the shaft of the engaged FC. We also assume that at the time point t_0 we have $M_D = M_T = M_C$. We further assume that within the time interval $(t_0 - t_m)$, by the end of which the FC engagement is over, that M_D also varies in proportion to the current time t of slippage and reaches the value of $k\,M_{DN}$, where k is the engine adjustability factor; and t_m is the FC engagement time. Within the time interval $(t_m - t_b)$, by the end of which the FC slippage is over (Fig. 2a), we assume that $M_D = k\,M_{DN} = \text{const}$. The torque of resistance to the tractor unit motion, which is brought to the FC shaft, is $M_C = \text{const}$ [1–6]. When drafting our calculation formulas, we neglect the yield effect and damping in the tractor transmission elements, the tangential yield and the slippage of the mover, and the gaps in toothed gears of the transmission and in the towing unit [1–6].

Based on the aforesaid, when a vehicle accelerates from rest, we assume for our calculations that it is to be a double-mass dynamic system of the tractor unit with one frictional bond. A similar dynamic system is used for calculating the slippage of automobile FCs [7].

In our case, the double-mass dynamic system of the tractor unit with two frictional bonds (see Fig. 1) is adopted for calculations, since at gear shifting such operation mode is possible, when the engine power is delivered at the same time by two FCs in the gearbox. The difference of our assumptions from those presented in the works [3–6] lies in the fact that we accelerate the driven parts of the engaged FC F_K not from the rest but starting from the angular velocity ω_N, which corresponds to the end of the acceleration of the tractor unit at the previous gear.

Let us conventionally split the gear shifting process into two phases: overlap and acceleration. We can split the overlap phase into two periods: optimal and excessive overlaps. The period of the optimal overlap corresponds to the period of time from 0 to t_0, where t_0 is the time, when the friction torque M_T of the engaged FC F_K reaches the value of the resistance moment to motion M_C of the tractor unit, brought to the shaft of the engaged FC at the higher gear (see Fig. 2). We assume that at the initial time $(t = 0)$ the FC F_{K-1} is engaged, and the power flow from the engine is delivered only through the gear $K - 1$ (Fig. 3a).

The gear shifting process starts with the engagement of the FC F_K and simultaneous disengagement of the FC F_{K-1}. We assume in this case that the FC F_{K-1} does not slip; therefore, the angular velocity of the driven shaft *2* does not change and corresponds to the gear $K - 1$, i.e. $\omega_2 = \omega^*_N = \omega_D / u_{k-1}$ (Fig. 3b). This is because the friction torque of the FC F_{K-1} is defined by the value of the static friction coefficient in contact of fixed plates, which is by its value significantly higher than the dynamic friction coefficient of the moving plates of the FC

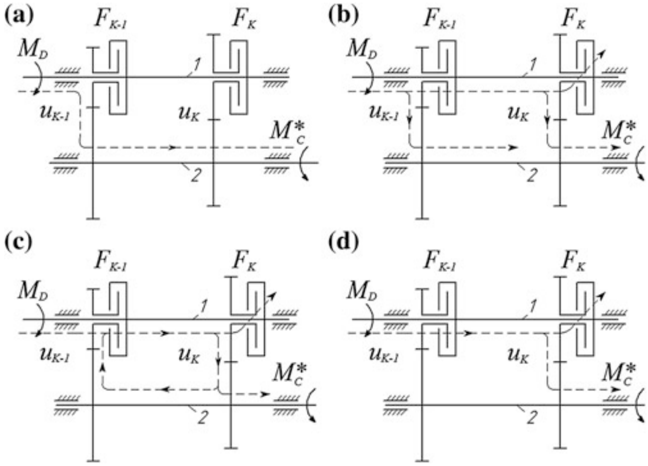

Fig. 3 Power flows in the gearbox. **a** at the engaged K − 1 gear, **b** at the optimal gear overlap, **c** at the excessive gear overlap, **d** at acceleration of the tractor unit at the *K* gear after the end of gear overlap

F_K. The angular velocity ω_N of the driven parts of the FC F_K is $\omega_N = \omega_D u_k / u_{k-1}$, and $u_k / u_{k-1} < 1$. Therefore, at the start of the gear shifting process, the driving plates of the FC F_K rotate faster than the driven ones. Consequently, the engaged FC F_K starts, while sliding, delivering the power from the driving shaft *1* to the driven shaft *2* of the gearbox. However, the value of the friction torque M_T of the FC F_K at the start of shifting (at $t < t_0$) is not enough to overcome the resistance moment to motion M_C of the tractor unit brought to the shaft of the engaged FC. Therefore, to overcome the resistance moment M_C, the FC F_{K-1} is also delivering the torque M_{T-1}, which value, at other equal conditions, depends on the value of friction torque M_T of the engaged FC F_K at the gear K. Thus, at this phase of gear shifting, the power is delivered from shaft *1* to shaft *2* by two parallel flows through the gears K − 1 and K (Fig. 3b).

As we show below, with increase of the torque M_T, the power flow delivered through the gear K increases, while the one delivered through the gear K − 1 decreases. The phase of the continuous gear shifting at the optimal overlap is over, when the power flow through the gear K − 1 becomes zero, although the FC F_{K-1}, and, therefore, the gear K − 1 may still be engaged. This happens at the time t_0.

The dynamic equations for the driving and driven parts of the engaged FC F_K look as follows (see Fig. 1):

$$M_D - M_T - M_{T-1} = J_D \frac{d\omega_D}{dt}; \tag{1}$$

$$M_{T-1} \frac{u_{k-1}}{u_k} + M_T - M_C = J_N \frac{d\omega_N}{dt}, \tag{2}$$

Where J_D is the inertia moment of the engine rotating elements and other parts linked thereto, brought to the shaft of the FC F_K; and J_N is the inertia moment of the tractor unit brought to the shaft of the FC F_K. It follows from Eq. (2) that at the first phase of gear shifting, the torque M_D is delivered from shaft *1–2* of the gearbox via both FCs at the same time by two parallel flows. In this case, each of the gears transforms the part of the torque M_D delivered to it according to its transmission ratio.

Let us assume that there is no change in angular velocity ω_D of the engine shaft during the overlap, i.e., $d\omega_D/dt = 0$.

Therefore, Eqs. (1 and 2) take the following forms:

$$M_D - M_T - M_{T-1} = 0; \tag{3}$$

$$M_{T-1}\frac{u_{k-1}}{u_k} + M_T - M_C = 0. \tag{4}$$

At the phase of optimal overlap in the time interval from 0 to t_0 (see Fig. 2), we have:

$$M_T = M_{DN}\,\frac{k_z\,t}{t_0}, \tag{5}$$

Where $t_0 = t_m\,k_z/\beta$.

Then, from Eqs. (3 and 4), with account of (5), we have:

$$M_D = M_C\frac{u_k}{u_{k-1}} + M_T\left(1 - \frac{u_k}{u_{k-1}}\right). \tag{6}$$

Since resistance moment to motion M_C of the tractor unit at gear K brought to the shaft FC F_K is $M_C = k_z\,M_{DN}$, and $u_k/u_{k-1} = k'_z/k_z$,

then, Eq. (6) takes the following form:

$$M_D = k'_z\,M_{DN} + M_T\left(1 - \frac{u_k}{u_{k-1}}\right) = k'_z\,M_{DN} + M_{DN}\,\frac{k_z\,t}{t_0}\left(1 - \frac{u_k}{u_{k-1}}\right). \tag{7}$$

From Eq. (3) with account of (7) we have:

$$M_{T-1} = k'_z\,M_{DN} - M_T\,\frac{u_k}{u_{k-1}} = k'_z\,M_{DN} - M_{DN}\,\frac{k_z\,t}{t_0}\times\frac{u_k}{u_{k-1}}. \tag{8}$$

The work of the slippage of the FC F_K within the time interval $(0 - t_0)$ is defined by the following expression (see Fig. 2):

$$L_0 = \int_0^{t_0} M_T(\omega_D - \omega_N)dt.$$

By replacing the values of M_T, ω_D and ω_N by their values for the first phase, we have:

$$L_0 = \frac{M_{DN}\,\omega_R\,k_z^2 t_m}{2\,\beta}\left(1 - \frac{u_k}{u_{k-1}}\right),$$

where $\omega_R = \omega_{DH} - k_z'(\omega_{DH} - \omega_{DN})$. Here, ω_R and ω_{DH} are the angular velocities of the engine shaft respectively at operating load at gear $K - 1$ and at idle speed brought to the shaft of the engaged FC.

If upon the end of the first phase, the FC F_{K-1} is not disengaged, then the second phase of gear shifting (with the excessive overlap) starts. It starts at the moment of time t_0, when the torque M_{T-1} delivered by the FC F_{K-1} becomes equal to zero (see Fig. 2). We should note here that at the moment of time $t = t_0$, the torque M_{T-1} on the shaft of the FC F_{K-1} becomes equal to zero even at the completely disengaged FC. This is because at the moment of time $t = t_0$, the value of friction torque M_T on the shaft of the FC F_K becomes sufficient for overcoming the resistance to motion of the tractor unit, and the FC F_{K-1} does not slip.

At $t > t_0$, the friction torque M_T at the engaged FC $F_K$keeps going up. As a result, the gear to be engaged starts delivering to the driven shaft greater power than it is necessary to ensure the motion of the tractor unit at the speed V_{K-1}, which corresponds to gear $K - 1$; this is supposed to cause acceleration of the tractor unit from speed V_{K-1} up to V_K. However, if the friction torque M_{T-1} of the FC F_{K-1} of the gear to be disengaged is still high enough, then, the gear $K - 1$ prevents acceleration of the tractor unit by putting out the excessive power, which is delivered by gear K, back from driven shaft 2 to driving shaft 1 of the gearbox (Fig. 3c). This results in circulation of power in the circuit of the unit of the gearbox formed by shafts 1 and 2, gears, $K - 1$, and K. Therefore, at the first phase, the possibility of accelerating the tractor unit at slippage of the FC F_K with a constant angular velocity is excluded (see Fig. 2). However, the circulation causes no interruption of the power flow from the engine to the driving wheels of the tractor, since only an excessive part of the power delivered to the driven shaft by gear Kis circulating. At the full disengagement of the FC F_{K-1}, the phase of excessive overlap is over.

The dynamic equations for the driving and driven parts of the engaged FC F_K for the time interval $\left(t_0 - t_p\right)$ differ from Eqs. (1 and 2) by the sign at torque M_{T-1} :

$$M_D - M_T + M_{T-1} = J_D\frac{d\omega_D}{dt}; \tag{9}$$

$$M_T - M_{T-1}\frac{u_{k-1}}{u_k} - M_C = J_N\frac{d\omega_N}{dt}. \tag{10}$$

Here, the behavior of the friction torque M_T of the engaged FC F_K is defined similarly to the previous phase (see Fig. 2) by expression (5).The overcoming of load of M_C, which is accompanied by circulation of power and slippage of the FC F_K, can result in overload of the engine. To define the dependence of the angular velocity of the engine shaft ω_D on the duration of power circulation (the time of the excessive overlap) and other parameters, let us make use of equations of the torques (9 and 10), as well as of the connection equation for driving and driven plates of the FC F_K :

$$\omega_N = \omega_D \frac{u_k}{u_{k-1}}. \tag{11}$$

As a result, we have that:

$$\omega_D = \omega_R + \frac{M_{DN}\beta(t_p - t_0)^2}{2t_m\left[J_D + J_N\left(\frac{u_k}{u_{k-1}}\right)^2\right](\beta - k_z)} \cdot \left[k - \beta\left(1 - \frac{u_k}{u_{k-1}}\right) - k_z'\right]. \tag{12}$$

It follows from the analysis of expression (12) that the decrease of angular velocity ω_D of the engine shaft at gear shifting with excessive overlap becomes more significant with the increase of the load factor k_z' of the engine at gear $K - 1$, safety factor β of the FC F_K, and value u_{k-1}/u_k. In this case, the higher is the adjustability factor k of the engine the smaller is the decrease of the angular velocity ω_D of its shaft. However, the practice shows that the angular velocity ω_D of the engine shaft at the phase of excessive overlap has little changes. Therefore, to schematize the law of ω_D change, in order to simplify the mathematical manipulations, let us assume that at the phase of excessive overlap, as well as at the phase of optimal overlap, the angular velocity of the engine shaft, and, therefore, of the driven parts of the transmission do not change.

In time interval $\left(t_0 - t_p\right)$ of gear shifting (see Fig. 2), the work of the slippage of the FC F_K is:

$$L_1 = \int_{t_0}^{t_P} M_T(\omega_D - \omega_N)\,dt.$$

Then, with account of expressions (5, 11) and at the condition that $d\omega_D/dt = 0$, we have:

$$L_1 = \frac{M_{DN}\omega_R k_z\left(t_p - t_0\right)^2}{2t_0}\left(1 - \frac{u_k}{u_{k-1}}\right).$$

Further, let us consider the phase of acceleration of the driven parts of the tractor unit, which starts after disengaging the FC F_{K-1}. At the start of this phase, the angular velocity of the driven plates of the FC $F_K \omega_N = \omega_D u_k/u_{k-1}$. Therefore, the driven plates of the FC F_K rotate at smaller angular velocity than the driving ones; the FC slips and the power from shaft 1–2 is delivered through gear K (Fig. 3d). As a consequence, the torque M_T causes an acceleration of the tractor unit and at the same time a decrease of the angular velocity of the engine shaft (see Fig. 2). We should note here that only at this phase, the speed ratio $u = \omega_D/\omega_N$ of the unit of the gearbox changes from u_{k-1} to u_k. Then, after the end of the slippage of the FC F_K, the tractor unit accelerates at gear K up to speed V_K of the steady motion. Here, the gear shifting process and the acceleration of the tractor unit is over.

This phase of shifting differs from the process of starting the tractor unit from rest by means of the main FC basically only in their initial conditions.

At the start of this phase of gear shifting, the relative angular velocity ω_{rel} of the driven and driving plates of the FC F_K is defined from the following expression:

$$\omega_{rel} = \omega_D - \omega_N = \omega_R\left(1 - \frac{u_k}{u_{k-1}}\right).$$

The work of the slippage of the FC F_K for the case of accelerating the tractor unit according to the diagram with a flat section (see Fig. 2a) within the time interval $\left(t_p - t_m\right)$ is:

$$L_2 = \int_{t_p}^{t_m} M_T(\omega_D - \omega_N)\,dt. \tag{13}$$

Within the preset time interval $\left(t_p - t_m\right)$, according to the above assumptions, the torques M_T and M_D are defined from the following expressions:

$$M_T = \frac{M_{DN}(\beta - k_z)\,(t - t_0)}{(t_m - t_0)} + M_C; \tag{14}$$

$$M_D = \frac{M_{DN}(k - k_z)\,(t - t_0)}{(t_m - t_0)} + M_C, \tag{15}$$

and the changes of the angular velocities ω_D and ω_N—from the following expressions:

$$\omega_D = \omega_R - \frac{M_{DN}(\beta - k)(t - t_0)^2}{2J_D(t_m - t_0)}; \tag{16}$$

$$\omega_N = \omega_R\frac{u_k}{u_{k-1}} + \frac{M_{DN}(\beta - k_z)\,(t - t_0)^2}{2J_N(t_m - t_0)}. \tag{17}$$

By using dependences (14, 15, 16, and 17), after respective transformations, the integral of expression (13), which presents the work of the slippage within the time interval $\left(t_p - t_m\right)$, is written as follows:

$$L_2 = \frac{M_{DN}\beta}{t_m}\left[\frac{\omega_R}{2}\left(1-\frac{u_k}{u_{k-1}}\right)\left(t_m^2-t_p^2\right)-\frac{M_{DN}}{24}\left(t_m-t_p\right)^2\left(\frac{\beta-k}{J_D}+\frac{\beta-k_z}{J_N}\right)\right.$$
$$\left.\times\left((3t_m+t_p)-\frac{(t_p-t_0)(3t_p+t_m)}{t_m-t_0}\right)\right]. \tag{18}$$

For the case of accelerating the tractor unit according to the "triangular" diagram (see Fig. 2b), within the time interval $\left(t_p - t_b\right)$, the work of the slippage will be:

$$L_2 = \int_{t_p}^{t_b} M_T(\omega_D - \omega_N)dt. \tag{19}$$

Then, with account of expressions (14, 15, 16, and 17), the integral of expression (19), which presents the work of the slippage within the time interval $\left(t_p - t_m\right)$, is written as follows:

$$L_2 = \frac{M_{DN}\beta}{t_m}\left[\frac{\omega_R}{2}\left(1-\frac{u_k}{u_{k-1}}\right)\left(t_b^2-t_p^2\right)-\frac{M_{DN}}{24(t_m-t_p)}\left(\frac{\beta-k}{J_D}+\frac{\beta-k_z}{J_N}\right)\right.$$
$$\left.\times\left((t_b-t_p)^3(3t_b+t_p)-\frac{(t_p-t_0)}{(t_m-t_0)}\left\{(t_b-t_m)^3(3t_b+t_m)+(t_m-t_p)^3(3t_p+t_m)\right\}\right)\right]. \tag{20}$$

Here, the time of slippage t_b is defined from (16, 17) from condition $t = t_b$ and $\omega_D = \omega_N$:

$$t_b = \sqrt{(t_m-t_0)\left(t_p - t_0 + \frac{2\omega_R\left(1-\frac{u_k}{u_{k-1}}\right)}{M_{DN}\left(\frac{\beta-k}{J_D}+\frac{\beta-k_z}{J_N}\right)}\right)} + \frac{t_m k_z}{\beta}. \tag{21}$$

The work of the slippage L_3 for the case of the final phase of accelerating of the tractor unit under the diagram with the a flat section (see Fig. 2a) within the time interval $(t_m - t_b)$ is calculated by the following formula:

$$L_3 = \int_{t_m}^{t_b} M_T(\omega_D - \omega_N)dt. \tag{22}$$

When defining L_3 within the time interval $(t_m - t_b)$, by assuming that the friction torque of the engaged FC $M_T = M_{Tmax} = \beta M_{DN} = \text{const}, M_D = k M_{DN} = \text{const}$, from Eqs. (1 and 2), we find:

$$\omega_D = \omega_R - \frac{M_{DN}(\beta - k)}{J_D}\left[\frac{(\beta - k_z)\,t_m}{2\beta} + (t - t_m)\right]. \tag{23}$$

$$\omega_N = \omega_R \frac{u_k}{u_{k-1}} + \frac{M_{DN}(\beta - k_z)}{J_N}\left[\frac{(\beta - k_z)\,t_m}{2\beta} + (t - t_m)\right]. \tag{24}$$

By using expressions (23, 24) and values of the torques M_T and M_D for intervals $(t_m - t_b)$, we reduce the work of the slippage presented by the integral of dependence (22) to the following form:

$$L_3 = \frac{M_{DN}\beta\,(t_b - t_m)}{2}\left\{2\omega_R\left(1 - \frac{u_k}{u_{k-1}}\right) - M_{DN}\left[t_b - t_p\right]\left(\frac{\beta - k}{J_D} + \frac{\beta - k_z}{J_N}\right)\right\}.$$

Here, time of slippage t_b is defined from (23, 24) at the condition that $t = t_b$ and $\omega_D = \omega_N$.

Then:

$$t_b = \frac{\omega_R\left(1 - \frac{u_k}{u_{k-1}}\right)}{M_{DN}\left(\frac{\beta - k}{J_D} + \frac{\beta - k_z}{J_N}\right)} + \frac{t_m + t_p}{2}. \tag{25}$$

The total work of the slippage for the acceleration diagram of the tractor unit with a flat section (Fig. 2a) is defined from the following expression:

$$L = L_0 + L_1 + L_2 + L_3, \tag{26}$$

Where L_2 is defined from expression (18), and time t_b of the slippage—from expression (25).

For the case with the "triangular" acceleration diagram (Fig. 2b), the total work of the slippage will be:

$$L = L_0 + L_1 + L_2, \tag{27}$$

where L_2 is defined from expression (20), and time t_b of the slippage—from expression (21).

To calculate the work of the slippage L of the FC, we need to know, which of the cases the acceleration of the tractor unit should be referred to (see Fig. 2). With this aim, from expression (21 or 25), let us define time t'_m of engagement of the FC for a particular case of accelerating the tractor unit, at which $t_m = t_b$:

$$t'_m = \frac{2\omega_R\left(1 - \frac{u_k}{u_{k-1}}\right)}{M_{DN}\left(\frac{\beta - k}{J_D} + \frac{\beta - k_z}{J_N}\right)} + t_p. \tag{28}$$

If $t'_m > t_m$, then $t_b > t_m$ (Fig. 2a). Then work L and time t_b of the slippage of the FC of the gear to be engaged are defined respectively from expressions (26 and 25).

If $t'_m \le t_m$, then $t_b \le t_m$ (Fig. 2b). Then work L and time t_b of the slippage of the FC are defined respectively from expressions (27 and 21).

One of the most important parameters of the accelerating process of the MTU (machine-tractor unit) is a minimum angular velocity ω_b of the engine shaft at the end of the slippage of the FC. To define ω_b, we need to substitute $t = t_b$ for the case when $t_b > t_m$ (Fig. 2a) into expression (23 or 24), and for the case when $t_b \le t_m$ (Fig. 2b)—into expression (16 or 17). As a result, we have the general dependence, according to which:

$$\omega_b = \omega_R \frac{1 + \frac{u_k}{u_{k-1}} \frac{J_N(\beta - k)}{J_D(\beta - k_z)}}{1 + \frac{J_N(\beta - k)}{J_D(\beta - k_z)}}. \tag{29}$$

When defining the acceleration time of the MTU t_r at the gear to be engaged, let us consider the time interval $(t_b - t_r)$, when the acceleration is ensured by the torque backup of the engine. Let us write down the equation of the torques based on double-mass dynamic model relating to the elementary unit of the gearbox (Fig. 1) with the condition that the FC in this time interval has already no slippage:

$$M_D - M_C = (J_D + J_N)\frac{d\omega_D}{dt}. \tag{30}$$

At other conditions being equal, the acceleration time t_r of the MTU depends on the type of the acceleration diagram (Fig. 2). Therefore, let us consider both cases of accelerating the MTU at the preset gear.

By solving Eq. (30) for the case of accelerating the MTU under the diagram with a flat section (Fig. 2a) at $t = t_r$ и $\omega_D = \omega_R$ we find the acceleration time of the MTU at the engaged gear:

$$t_r = t_b + \frac{2(\omega_r - \omega_b)(J_D + J_N)}{M_{DN}(k - k_z)}. \tag{31}$$

By solving Eq. (30) for the case of accelerating MTU according to the "triangular" diagram (Fig. 2b) at $t = t_r$ and $\omega_D = \omega_R$, we find the acceleration time of the MTU at the gear to be engaged:

$$t_r = t_b + \frac{2(\omega_r - \omega_b)\,(J_D + J_N)}{M_{DN}\,(k - k_z)}\sqrt{\frac{t_m}{t'_m}}. \tag{32}$$

The obtained expressions for calculating the work and the time of FC slippage at gear shifting in the gearbox with different overlap degrees are universal, since they allow calculating the work and the time of the FC slippage and acceleration of the MTU both at gear shifting with various overlap degrees, and at tractor's acceleration and start from the rest. When the tractor starts from the rest and accelerates at the preset gear, we assume the overlap time $t_p = 0$, and the ratio $u_k/u_{k-1} = 0$.

Table 1 Comparison of experimental and calculated values of FC slippage at gear shifting in the gearbox

Source of information	Work type	t_m, s	t_p, s	β	J_D	J_N	M_{DN},N•m	ω_{DH}, rad/s	k	k_z	$\frac{u_k}{u_{k-1}}$	L, J		Error, %
					kg•m^2							Experiment	Calculation	
[2]	Tillage	1.5	0.71	2.0	1.67	2.8	195.2	201	1.10	0.88	0.85	3300	3704	+10.9
		1.5	0.78	2.0	1.67	2.8	195.2	201	1.10	0.88	0.85	3500	3800	+7.9
		1.5	0.84	2.0	1.67	2.8	195.2	201	1.10	0.88	0.85	3700	3857	+4.1
		1.5	0.59	2.0	1.67	2.3	195.2	201	1.10	0.72	0.68	8500	8850	+4.0
		1.5	0.66	2.0	1.67	2.3	195.2	201	1.10	0.72	0.68	8700	8910	+2.4
		1.5	0.72	2.0	1.67	2.3	195.2	201	1.10	0.72	0.68	8850	8986	+1.5
[5]	Tillage	1.2	0.54	1.9	2.55	1.03	274.4	220	1.05	0.85	0.90	8624	9648	+10.6
	Sowing	1.0	0.37	1.9	2.55	1.16	274.4	220	1.05	0.70	0.90	7742	7727	−0.2

To verify the reliability of the mathematical model of the FC slippage process in the gearbox, and the calculation methods of the slippage work at gearing under various overlaps, we made a comparison of the calculated and experimental values of the work L of the slippage. The experimental data for the case of the optimal overlap were taken from the work of Ananyin [5], where the optimal overlap at gear shifting at tillage and sowing was ensured by installing a free-travel clutch into the gearbox. The results of the experimental studies at gear shifting without interruption the power flow for various overlaps were borrowed by us from the work of Lvovskiy [2].

The comparison of calculated and experimental data is presented in Table 1.

It follows from the analysis of the results that the divergence of calculated and experimental results of the work of the FC slippage in the gearbox with various degrees of the overlap does not exceed 10.9 %.

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