# On the Behaviour of Asymmetric Cylindrical Gears in Gear Transmissions

Alfonso Fuentes, Ignacio Gonzalez-Perez, Francisco T. Sanchez-Marin and Kenichi Hayasaka

Abstract Asymmetric gears have been proposed more than twenty years ago as the ultimate solution to increase the load capacity of gear drives while reducing their weight and dimensions. However, there are apparently contradictive statements in the literature regarding whether the higher pressure angle should be applied to the driving or coast side of the gear tooth surfaces. In this work, modern technologies of design and analysis of enhanced gear drives will be applied in order to validate the advantages of application of asymmetric gears and to determine what the right configuration of the asymmetric gear drive should be in terms of application of the higher pressure angle to the driving or coast side of the gear teeth. The evolution of contact and bending stresses as well as contact pressure for the whole cycle of meshing is investigated and compared for symmetric and asymmetric gears. Two configurations of asymmetric gears will be considered, taking into account both, the higher and the lower pressure angle for the driving side. In this way, the advantages of application of asymmetric cylindrical gears as well as the right configuration to get them are established.

Keywords Gear transmissions - Asymmetric gears - Finite element analysis -Stress analysis

F2012-C02-010

A. Fuentes (&) - I. Gonzalez-Perez Polytechnic University of Cartagena (UPCT), Cartagena, Spain e-mail: alfonso.fuentes@upct.es

F. T. Sanchez-Marin Universitat Jaume I, Castellón de la Plana, Spain

K. Hayasaka Space Creation Co. Ltd, Wan Chai, Japan

SAE-China and FISITA (eds.), Proceedings of the FISITA 2012 World Automotive Congress, Lecture Notes in Electrical Engineering 193, DOI: 10.1007/978-3-642-33744-4\_13, © Springer-Verlag Berlin Heidelberg 2013

## 1 Introduction

Asymmetric gears have been proposed more than twenty years ago as the ultimate solution to increase the load capacity of gear drives while reducing their weight and dimensions [\[1](#page-7-0)]. The term asymmetric means application of different pressure angles for both sides of gear tooth surfaces. Detailed procedures of design and computation of asymmetric spur gear geometry can be found in [[1,](#page-7-0) [2\]](#page-7-0). Other works [\[6–12](#page-7-0)] are focused in different aspects of design, analysis and optimization of asymmetric gear drives.

A common subject of investigation regarding applicability of asymmetric gears has been the determination whether the higher pressure angle should be applied for the driving or coast sides. One of the first works referring to the application of asymmetric gears and proposing using higher pressure angles for the coast side of the gear tooth surfaces in order to achieve the advantages enumerated above according to [\[2](#page-7-0)] was [\[3](#page-7-0)]. However, shortly after, many works applied the higher pressure angle for the driving side of the gear tooth surfaces in order to get an increased load capacity or reduced weight and dimensions of the gear set. Recently, in [\[4](#page-7-0)] the application of higher pressure angles for the coast side of the gear tooth surfaces is again proposed, showing the advantages by numerical and experimental tests of prototypes. These apparently contradictive statements have motivated this work, with the main goal of validation whether application of asymmetric gears yield indeed advantages and if so, determine what the right configuration should be in terms of application of the higher pressure angle to the driving or coast side of the gear teeth.

### 2 Methodology

In this work, modern technologies of design and analysis of enhanced gear drives will be applied based on the computerized generation of gears, application of tooth contact analysis and application of FEA enhanced models for stress analysis [[5\]](#page-7-0). Application of modern gear technologies of design and analysis allows the time of development of new gear drives to be drastically reduced, lowering production costs and offering high quality final products wherein gears are applied. In this case, those modern gear technologies will be applied by using the Integrated Gear Design (IGD) software developed by the Enhanced Gear Drives Research Group (GITAE) in order to investigate the advantages of application of asymmetric cylindrical gears with respect to traditional symmetric cylindrical gears.

The bending stresses as well as contact stresses and contact pressure will be determined by using enhanced finite element models that allows investigation of the formation of the contact patterns on the gear drive along the whole cycle of meshing. We will be considering 21 different positions of meshing along a cycle of meshing. The evolution of contact and bending stresses as well as contact

Pinion	Wheel
24	34
2.0	
1.0	
1.25	
20	
210.0	

Table 1 Common basic design data of all cases of design of a symmetric and asymmetric spur gear drive

pressure will be compared for symmetric and asymmetric gears in order to find and quantify the real advantages of application of asymmetric gears. Two configurations of asymmetric gears will be considered, taking into account both, the higher and the lower pressure angle for the driving side. In this way, this paper will definitely establish what the advantages of application of asymmetric cylindrical gears are and what configuration has to be used in order to get those advantages.

#### 3 Numerical Example and Discussions of Results

In order to establish the baseline design, a spur gear transmission  $20 \times 20$ (symmetric gear set with  $20^{\circ}$  for the pressure angle of the driving side and  $20^{\circ}$  for the coast side) has to be optimized. The common basic design data for the baseline design is shown in Table 1.

A torque of 210.0 N-m will be applied to the pinion for all cases of design. When investigating the evolution of maximum contact and bending stress, and contact pressure for the baseline design with no surface modifications, high contact stresses and contact pressures are obtained for the pinion and wheel tooth surfaces contacting on the top-edge. In order to get an optimized design, a top relief have been provided to the pinion and wheel members of the gear set, by application of a rack-cutter with modified generating profile as shown in Fig. [1.](#page-3-0)

As shown in Fig. [1](#page-3-0), a top-relief height  $h_{tr}$  of 0.7 mm has been used for the baseline design, in order to provide gear tooth surface modifications outside of the single tooth contact area, and not introduce, in this way, transmission errors into the gear drive design. Figure [2](#page-3-0) shows the modified top-edge area with a maximum profile modification at the top edge of 14.3  $\mu$ m for the pinion and 9.1  $\mu$ m for the gear.

Figure [3](#page-4-0) shows the evolution of contact and bending stresses (Figs. [3](#page-4-0)a, b) as well as contact pressure (Fig. [3c](#page-4-0)) for the pinion, along 21 contact points of a cycle of meshing. Several cases of design, obtained by application of gradually increased value of parabola coefficient of top-relief area of the generating rack-cutter profile (see Fig. [1\)](#page-3-0), have been investigated in order to establish the optimum value of top relief. The values in parenthesis in the legends of Fig. [3](#page-4-0) represent the maximum top relieves applied to the pinion. As represented in Fig. [3](#page-4-0), a value of 14.3  $\mu$ m of

<span id="page-3-0"></span>

Fig. 1 a Rack cutter with modified generating profile for top-relief generation. b Threedimensional model of the generating rack-cutter



Fig. 2 Comparison of top-reliefs modifications for optimized design of a pinion and b wheel members of the baseline design of the gear set

top relief yields a smooth evolution of contact stresses and contact pressure, and can be considered the optimum value of the top-relief for the base-line design. It has to be mentioned that higher or lower values than the optimal ones of top-relief yield high contact stresses and contact pressure, so that optimization of the toprelief is very important in order to get a smooth evolution of contact stress and contact pressure all over the cycle of meshing.

Figure [4](#page-4-0)a shows the contact stresses for the pinion at contact point number 7. As shown in Fig. [4](#page-4-0)a, an area of high contact stresses appears all over the top edge of the pinion tooth surfaces. In Fig. [4](#page-4-0)b, contact stresses for the optimized design with top-relief modification of  $14.3 \mu m$  are represented, reducing in this way substantially the maximum contact stress at the pinion tooth surfaces, allowing a smooth load transfer between consecutive gear teeth.

<span id="page-4-0"></span>

Fig. 3 Evolution of a contact stress, **b** bending stress, and c contact pressure for several cases of design



Fig. 4 Contact stresses for the pinion with a no top-relief, and **b** an optimized top-relief of 14.3 um

Figure [5](#page-5-0) shows three cases of design to be investigated and compared. A rotation counter clockwise will be considered to determine the driving side of the gear tooth surfaces. In Figure [5a](#page-5-0), a symmetric gear drive with  $20^{\circ}$  of pressure angle for both the driving and coast sides is shown. In Fig. [5b](#page-5-0) an asymmetric gear drive with pressure angle of  $30^{\circ}$  for the driving side and  $20^{\circ}$  for the coast side is represented, and finally in Fig. [5c](#page-5-0) an asymmetric gear drive with pressure angle of  $20^{\circ}$  for the driving side and  $30^{\circ}$  for the coast side is represented.

Figure [6](#page-5-0) shows the evolution of contact stress (Fig. [6a](#page-5-0)), bending stress (Fig. [6](#page-5-0)b), and contact pressure (Fig. [6](#page-5-0)c) for the pinion of cases of design obtained by considering increased pressure angle for the coast side with respect of baseline design. The legend in Fig. [6](#page-5-0) is self-explanatory. As shown in Fig. [6](#page-5-0)a, maximum contact stress on the pinion only depends on the pressure angle of the driving side. No reduction of contact stresses or contact pressure is obtained by considering different pressure angles for the coast side. In the same way, Fig. [6c](#page-5-0) shows that maximum contact pressure only depends as well on the pressure angle of the driving side. On the contrary, a reduction of bending stresses is obtained for higher pressure angles on the coast side. This result is in total agreement with statements in [\[3](#page-7-0), [4](#page-7-0)] and according to experimental tests presented in [\[4](#page-7-0)].

<span id="page-5-0"></span>

Fig. 5 Cases of design of a symmetrical gear drive; b asymmetric gear drive with higher pressure angle for the driving side; and c asymmetric gear drive with higher pressure angle for the coast side



Fig. 6 Evolution of a contact stress, **b** bending stress, and c contact pressure for the pinion of symmetric and asymmetric gear drives for increasing angle of pressure angle for the coast side

Figure  $\overline{7}$  $\overline{7}$  $\overline{7}$  shows the evolution of contact stress (Fig.  $\overline{7}$ a), bending stress (Fig. [7](#page-6-0)b), and contact pressure (Fig. [7](#page-6-0)c) for the pinion of cases of design obtained by considering increased pressure angle for the driving side with respect of baseline design. As shown in Fig. [7a](#page-6-0), the minimum contact stress is obtained for the case of design with higher pressure angle for the driving side. Figure [7](#page-6-0)b shows also that bending stresses are reduced for case of design with higher pressure angle for the driving side. Same results are obtained when considering contact pressure. By observing Fig. [7](#page-6-0), we can conclude that application of higher pressure angle for the driving side, not only a reduction of bending stresses is obtained but also reduction of contact stresses and contact pressure is expected, increasing in this way life and endurance of the gear drive.

Finally, in order to evaluate what design is more favourable, the baseline design is compared with asymmetric designs considering the same increased angle of pressure angle for the coast and driving sides of the pinion and wheel tooth surfaces. Figure [8](#page-6-0)a shows that reduction of contact stresses is only achieved for design  $30 \times 20$ , wherein higher pressure angle is applied for the driving sides. Regarding bending stresses, Fig. [8](#page-6-0)b shows that reduction is achieved in almost similar values for both asymmetric designs, being this result in agreement with previously published works with apparently contradictive statements. Contact pressure is reduced significantly for the case of design  $30 \times 20$ .

<span id="page-6-0"></span>

Fig. 7 Evolution of a contact stress, **b** bending stress, and c contact pressure for the pinion of symmetric and asymmetric gear drives for increasing angle of pressure angle for the driving side



Fig. 8 Comparison of evolution of a contact stress, **b** bending stress, and c contact pressure for base line design and asymmetric gear drives for higher pressure angle applied to the driving side (case  $30 \times 20$ ) and to the coast side (20  $\times$  30)

# 4 Conclusions

The developed research allows the following conclusions to be drawn:

- 1. The application of asymmetric gears is directed indeed to the reduction of contact and bending stresses, and therefore contributes to the increase of the endurance and life of the gear drive.
- 2. Maximum contact stresses and contact pressure on the gear teeth depends only on the pressure angle of the contacting side (driving side) of the gear drive, no matter what the pressure angle of the coast side is.
- 3. Bending stresses are reduced when higher pressure angles are used, not only for the coast side as stated in  $[3, 4]$  $[3, 4]$  $[3, 4]$  but also for the driving side.
- 4. The driving side of the gear drive should have the highest pressure angle, when the goal is to reduce contact stresses and contact pressures, and not only reduce bending stresses.

Acknowledgments The authors express their deep gratitude to the Spanish Ministry of Economy and Competitiveness—MINECO (formerly Ministry of Science and Innovation) for the financial support of research projects ref. DPI2010-20388-C02-01 (financed jointly by FEDER) and DPI2010-20388-C02-02.

## <span id="page-7-0"></span>**References**

- 1. Kapelevich AL (2000) Geometry and design of involute spur gears with asymmetric teeth. Mech Mach Theor 35(117-130): 2000
- 2. Litvin FL, Lian Q, Kapelevich AL (2000) Asymmetric modified spur gear drives: reduction of noise, localization of contact, simulation of meshing and stress analysis. Comput Methods Appl Mech Eng 188(1–3):363–390
- 3. Hayer L (1986) Advanced transmission components investigation. Sikorski Aircraft Division, USAAVRADCOM TR-82-D-11
- 4. Sanders A, Houser DR, Kahraman A, Harianto J, Shon S (2011) An experimental investigation of the effect of tooth asymmetry and tooth root shape on root stresses and single tooth bending fatigue life of gear teeth. In: Proceedings of the ASME 2011 international design engineering technical conferences, IDETC/CIE 2011, Washington DC, USA
- 5. Litvin F, Fuentes A (2004) Gear geometry and applied theory. Cambridge University Press, New York
- 6. DiFrancesco G, Marini S (1997) Structural analysis of asymmetrical teeth: reduction of size and weight. Gear Technol 14(5):47–51
- 7. Karpat F, Cavdar K, Babalik FC (2005) Computer aided analysis of involute spur gear with asymmetric teeth. VDI Berichte 1904(I):145–163
- 8. Kapelevich AL (2009) Direct design of asymmetric gears: approach and application. In: International conference on motion and power transmissions
- 9. Kapelevich AL (2010) Measurement of directly designed gears with symmetric and asymmetric teeth. In: International conference on gears
- 10. Kapelevich AL (2011) Asymmetric gears: parameter selection approach. In: International conference on power transmissions of Xian
- 11. Kapelevich AL, Shekhtman YV (2009) Tooth fillet profile optimization for gears with symmetric and asymmetric teeth. Gear Technology, pp 73–79
- 12. Yang SC (2005) Mathematical model of a helical gear with asymmetric involute teeth and its analysis. International Journal of Advanced Manufacturing Technology 26(5):448–456