Realistic Dynamic Posture Prediction of Humanoid Robot: Manual Lifting Task Simulation

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Abstract. A well known question mooted in biomechanics is how the central nerves system manages the body posture during various tasks. A 5DOF biomechatronical model of human body subjected to simulate the manual lifting task of humanoid robot. Simulation process is based on optimization approach named predictive dynamics using inverse dynamics. An objective function in term of ankle torques during lifting time, subjected to be minimized. It assumed that CNS considered this function to perform lifting motion balanced. In the other optimization-based simulations, balancing motion was guaranteed by a nonlinear inequality constraint which restricts the total moment arm of the links to an upper and lower boundary. In this method there is no need to use this constraint. Result shows that the motion is performed balanced. According to the comparison the results with the experimental data, the body posture of humanoid robots, predicted as similar as actual human posture.

Keywords: Human body simulation, Biomechatronical model of human body, Lifting motion modeling, ankle torques.

1 Introduction

Multibody dynamics of human body, subjected to an extensive area of researches such as robotics, biomechatronics, biomedical engineering etc, Because it can provide an approach to find some variables that are not possible to measure like: torques and internal forces of joints and stress exerted to joint's soft tissues. These mechanical parameters are so important to understand joint disease initiation and progression, like osteoarthritis [1], [2]. In addition to pathological aspects, simulation purposes are one of the major causes of human body modeling.

In order to know how the body postures varies during different movements to construct motion animation of human body, a model of whole human body dynamics applied to movement simulation process. Simulation and analysis of human movements commonly used for athletics in order to improve performance of the motion and so prevent injuries in cause of incorrect movements [3]. Some abnormalities accrued in

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parts of musculoskeletal system resulted in inaccurately function to muscle activation and control [4]-[6].

Therefore it is so important to know that how the central nervous system (CNS) controls the body posture varies during different tasks.

Biomechatronical model with large number of degree of freedom needed to done the human motion simulation more exactly and accurately. The multiplicity of joint space variables (DOFs) causes model maneuverable but creates redundancy problem. We face with the redundancy problem when the number of DOFs is more than needed to perform a task. This problem is also mooted in the robotic researches in kinematics, dynamics and control aspects [7]-[10]. Human body models usually contained large number of DOFs. For applying these models to motion simulation, optimization-based approaches are good methods to overcome with the redundancy problem. Some of these techniques are applied to robotic manipulator models with redundant DOFs [9] and [11]-[13]. Optimization-based solutions are suitable ways to solve problem with large number of variables, because this method uses a few amount of data as inputs to result a large number of variables as output set. The input contains two set of constraints impose to motion simulation process: 1. Constraints obtained from motion dynamics and 2. Variety limitation of variables to be optimized. The second type used as inequality constraints and the first one contain some algebraic and differential equations.

CNS manages the task with the balanced movements. Walking, running, sitting and lifting are good examples of tasks related to daily living activities performed completely balanced involuntarily. CNS uses an unknown algorithm to manage tasks unconsciously. Optimization-based simulation methods have performance analogous with CNS function caused balanced movements. These approaches used objective function description subjected to minimizing which is duality of CNS algorithm manner. On the other hand to simulate a movement as like as shape that biological system does, it assumed that optimization approach minimized the objective function considered that CNS try to minimize it too. Description of stability can be found in medicine and engineering as different meanings but these meanings follows joint goal. In optimization approach motion stability caused by constraints which restrict total moment arm of body segments (TMA) in each configuration, between horizontal position of heel and toe (base of support) [14],[15]. In fact this constraint prevent of figurate postures will caused to falling to forward and backward. In this research we use an optimization-based algorithm named predictive dynamics [16]. With objective function consist of ankle torque summation during lifting time. In this novel inverse dynamics, joints torque limitation and joint ranges of motion are used as constraints to shape the motion as lifting movement. In this algorithm by and large two kinds of constraints are used. 1. The constraints which shape the simulated motion as lifting movement consist of two type constraints. 1-1. Kinematical constraints which formed motion like ones which determine initial and final position of box, body collision avoidance and constraints which guarantee moving up motion of box. These kinds named "*kinematical governing constraints*". 1-2. Inverse dynamics of system is a differential equation implemented as equality constraint to govern the dynamics of motion to simulation process, named "*dynamic governing constrain*". 2. The 2nd type named "*bounder constraint*" which limits the range of variation of variables to be optimized. This classification illustrated in figure1.

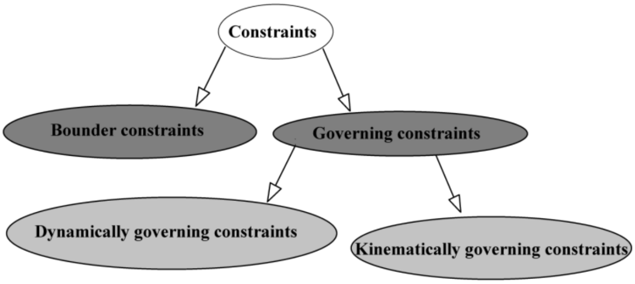

Fig. 1. Constraints classification

Ankle torque amplitude considered as stability index and the optimization algorithm tries to minimize integral of ankle torque squares during lifting time. A five DOF biomechatronical model of whole human body represented in part 2 obtained from kinematical modeling based on D-H method [17], [18]. Based on Lagrangian method dynamics of motion be formulated and results equation of motion named inverse dynamics. In part 3 simulation process is described and parts 4 and 5 present simulation results and conclusion respectively.

2 Modeling

A planar model with 5DOF in sagittal plane utilized to represent coordination system of human body (Fig. 2). All the limbs as shank, thigh, spine, arm and forearm subjected to modeling and considered as rigid bars with mass points at center of mass of each link which named: l_1, l_2, l_3, l_4, l_5 respectively. For human major joints as ankle, knee, hip, shoulder and elbow had considered joint angles in modeling to figurate human body posture and represented by the names:q_1, q_2, q_3, q_4, q_5 respectively.

The box assumed jointed to human body at the wrist with a horizontal orientation. Biomechatronical models of human body with coordination systems illustrated by fig. 2. Human body dynamics commonly model as open kinematics chain like robot manipulators as mentioned before [19], [20]. So the method used to modeling the dynamics of motion of this kinematical chain is like ones used for robotic manipulators. In this approach it's needed to calculate systems' kinetic and potential energies, and finally by minimizing the integral of system's lagrangian, the equations which govern the dynamics of motion will be obtained.

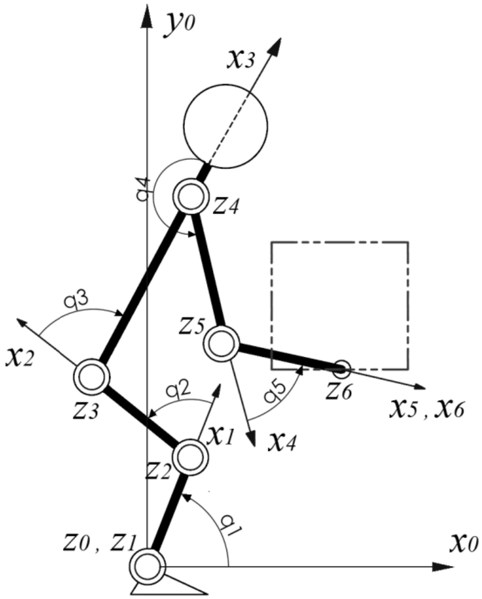

Fig. 2. 5DOF model of human body with coordination systems attached to each link

The kinetic energy of the model presented before define as bellow equation:

$$K = \frac{1}{2}\dot{q}^T D(q)\dot{q} \tag{1}$$

$$D(q) = \sum_{i=1}^{5}(m_i J_{Vc_i}{}^T J_{Vc_i} + J_{\omega_i}{}^T R_i^0 I_i R_i^0) \tag{2}$$

In equation (2) $\dot{q}$ is 5×1 vector of angular velocities of joints, and $\dot{q}^T$is transpose matrix of$\dot{q}$, $D(q)$is 5×5 matrix related to mass and inertial properties of the model [24]. J_{Vc_i} , J_{ω_i} are 3×5 Jacobin matrix which translate linear and angular velocities of COM of i'th link to universal coordinate system respectively. $R_i^0$is rotational transformation matrix which interpret the orientation of i′th links from its coordinate to ground coordinate. $m_i$is mass of i'th link. By considering g as gravitational force vector, and r_{c_i} as height of i'th link's COM from ankle position, System's potential energy describes as bellow:

$$V = \sum_{i=1}^{5}\left(m_i g^T r_{c_i}\right) \tag{3}$$

A function of systems' energy which called Lagrangian calculated as bellow:

$$L = K - V = \frac{1}{2}\dot{q}^T D(q)\dot{q} - \sum_{i=1}^{5}\left(m_i g^T r_{c_i}\right) \tag{4}$$

A functional in term of systems' energy S defines as integral of system's lagrangian during lifting time interval$[0\ T]$, as bellow equation:

$$S = \int_{t=0}^{T} L(q, \dot{q}, t) dt \tag{5}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = \Gamma \tag{6}$$

Based on Hamiltonian principle extremizing integral S resulted in motion equation. Euler-Lagrange formulation (6) subjected to extremizing S[24]. In (6)Γ is generalized joints torque vector inserted. Finally general form of motion equations will be obtains as(7) [24].

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + V(q) = \Gamma \tag{7}$$

In (7)$C(q, \dot{q})$ is a term related to centrifugal and coriolis forces and $V(q)$ is gravitational forces vector, this term calculates as bellow:

$$C(q, \dot{q}) = \dot{D}(q) - \frac{1}{2}\dot{q}^T \left(\frac{\partial D(q)}{\partial q}\right) \tag{8}$$

$$V(q) = \left(\frac{\partial V}{\partial q}\right)^T \tag{9}$$

Generalized joint torque represented in (7) divided in two parts: 1. torques resulted in muscle forces and 2. Torques due to the box load exerted on wrist. These kinds obtain as (10):

$$\Gamma = \tau_{muscle} - \tau_{box} \quad ; \quad \tau_{box} = J^T(\ m_{box} g^T) \tag{10}$$

In (12) J^T is transpose of Jacobean matrix which project box load to joints m_{box} is box mass and g^T is transpose of gravity force vector.

3 Simulation Process

In this paper lifting movement simulation considered as optimization problem which CNS do either. In this problem an objective function subjected to be optimized with some constraints which limit the motions boundary to a feasible range to construct motion naturally. In other words it's being assumed that CNS try to minimize a particular function value to perform each task, and musculoskeletal system impose some constraints to the motion too. Predictive dynamics is a novel approach used to motion simulation [16], [23]; it implements inverse dynamics as a major constraint to modeling the dynamics of the motion in the simulation process. The joints torques and angles are the optimization process variables, so by using this method we can obtain joint angles and torques as output according to task constraints used as inputs. Simulation elements are described in bellow sections.

A. Objective function

Considering the lifting task as a simple inverted pendulum motion, can represents an insight to motion stability analysis. In other hand If lifting motion modeled as a inverted pendulum (fig. 3) [21] we can claim that magnitude of pendulum joints torque has relation to amount of deviation from stability position ($\theta = 0^{\circ}$) directly. Therefore we can use of a particular function which constructed in term of ankle torque as motion stability index. We propose this function as integral of ankle torque squares in each time sequence (11).

$$F(q,\tau,t) = \int_{t=0}^{T} \tau_{ankle}{}^{2}\, dt \tag{11}$$

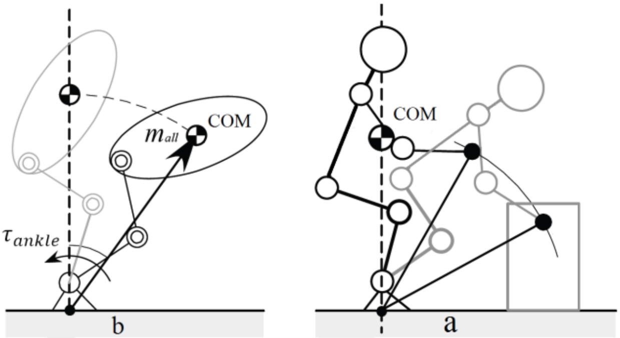

Fig. 3. a) 5DOF model of human body for lifting motion task, b) Inverted pendulum equivalent model for lifting task.

B. Constrants

The constraints used in this research are: joints torques and angles limitations, initial and final position of box, elevating constraint, inverse dynamics, and body collision avoidance constraint used for prevent of collision box with body. Vertical position of wrist (12) is a function of joint angles $q(t)$ calculated from forward kinematics (22):

$$Y_{wrist}(t) = y\big(q(t)\big) \tag{12}$$

In each time sequence $Y_{wrist}(t)$should be higher than previous sequence:

$$Y_{wrist}(i) > Y_{wrist}(i-1) \tag{13}$$

So the elevating constraint defined by:

$$y\big(q(t-1)\big) - y\big(q(t)\big) < 0 \tag{14}$$

In fact elevating constraint guaranteed moving up motion of box during time sequences. The equality constraints which impose initial and final position of box to optimization process are defined by (15).

$$\begin{cases} x(q(0)) - x_{initial} = 0 \\ y(q(0)) - y_{initial} = 0 \\ x(q(T)) - x_{final} = 0 \\ y(q(T)) - y_{final} = 0 \end{cases} \tag{15}$$

Which $X_{wrist}(t)$ and $Y_{wrist}(t)$ are respectively horizontal and vertical position of wrist considered fixed to box. According to fourfold constraints set (15) wrist should be placed at initial position $x_{initial}$,$y_{initial}$ and final position x_{final} , y_{final} of box at $t = 0$ and $t = T$ respectively. Inverse dynamics constraint express as bellow:

$$\tau - \tau_{invd} = 0 \ ; \ \tau_{invd} = f(q,t) \tag{16}$$

In equation (16) τ is joints torque vector should be predicted, and τ_{invd} is joints torque vector obtained from inverse dynamics. Body collision avoidance implemented in this simulation is a systematic method to check the penetration value of the box into the body in each iteration of optimization process to determine horizontal position of the box to collision avoidance adaptively. This process described as bellow briefly:

The collision avoidance considered in optimization process as a constraint to prevent penetration of box with the body. It's inequality constraint and defined as a term of sufficient horizontal distance dx which wrist should move to prevent collision box with the body.

$$X_{bounndary} - X_{wrist_d} < 0 \tag{17}$$

$$X_{bounndary} = X_{wrist_{pr}} + d_x \tag{18}$$

Distance which wrist should move to arrive to horizontal boundary position $X_{bounndary}$ represent by d_x, and boundary position is a horizontal position of wrist which box edge touch the body. $X_{wrist_d}$is desire horizontal position of wrist which should be greater than $X_{bounndary}$. $X_{wrist_{pr}}$is horizontal position of wrist obtained from optimization algorithm in current iteration.

According to Figure 5 penetration value of the box in the body d would be obtained through equation (19), X_{body} and X_{edge} obtain from body line and box line respectively.

$$d = X_{body} - X_{edge} \tag{19}$$

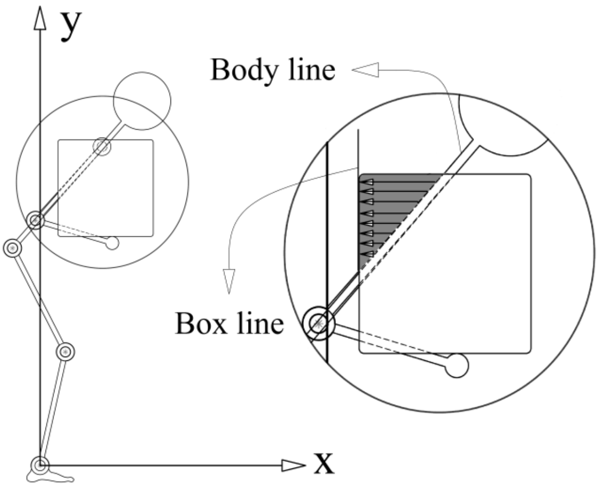

Fig. 4. Collision box with body, box line, body line and penetration area

Maximum penetration value determine the horizontal distance which wrist should move to arrive to boundary position, d is penetration index so d_x is maximum value of d.

$$d_x = MAX(d) \tag{20}$$

4 Results

The optimization process designed for 10 evenly distributed time sequences. By considering 5 angles of joints and 5 joint's torques for each time segment, we have

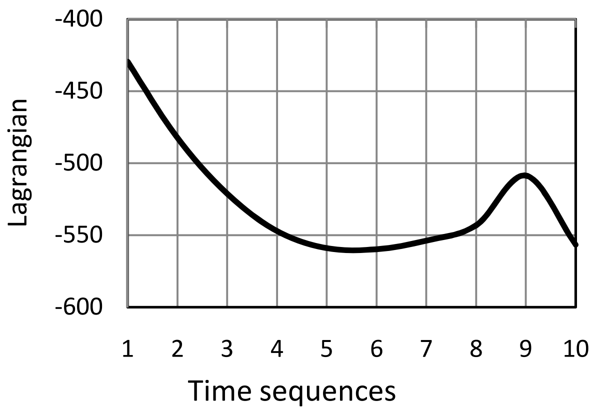

Fig. 5. Lagrangian values during time sequences

100 variables subjected to be optimized. Inertial properties considered as data used previously [22]. The experimental data used in [23] implemented to validate the simulation process. Lifting task parameters presented in table I.

Lagrangian value calculated from (4) plotted for time sequences and illustrate in figure6. It shows that lagrangian value is always minus, so according to (4) it have been concluded that the motion is so slowly or in other words it's static motion approximately. Respect to this statement, a statically index presented to evaluate the motion stability during all the time sequences.

Total moment arm (TMA) of all the links are calculated as (21) it's calculated from the moments respect to all of the links weight for each configuration related to time sequence.

$$TMA(t) = \frac{1}{m_{all}} \sum_{i=1}^{N} m_i x_i(t) \tag{21}$$

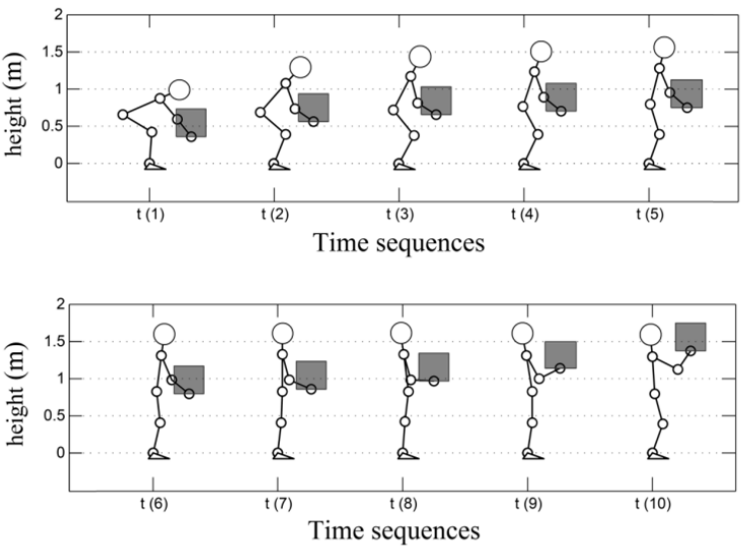

Fig. 6. Body postures of humanoid robot, during lifting task. The horizontal axis is time in term of second, by scaling 0.12.

$m_{all}$is total weight of body and $x_i(t)$ is horizontal position of $i'th$ links at time t, and N is number of links. Optimized joint angles show that how the body posture varies during lifting task, it illustrated in figure 6.

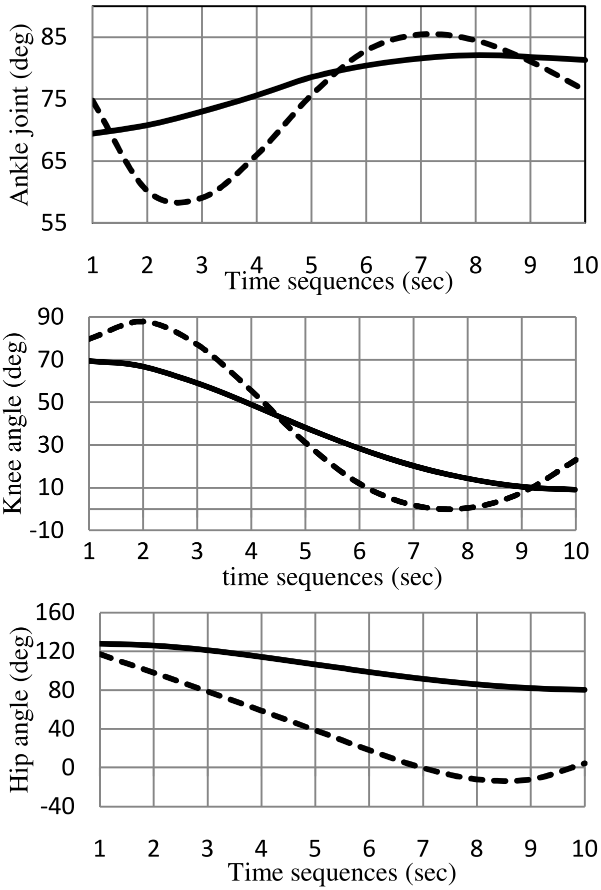

Fig. 7. Joints angles profiles resulted in optimization process in comparison with experimental results. The solid line is experimental and the dashed line is predicted profile. The vertical axis is joint angle and is in term of degree. The horizontal axis is time in term of second, by scaling 0.12.

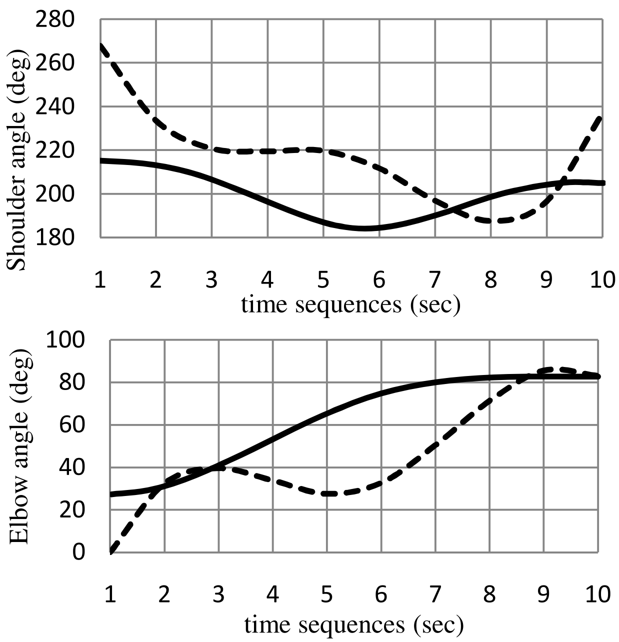

Fig. 7. (*continued*)

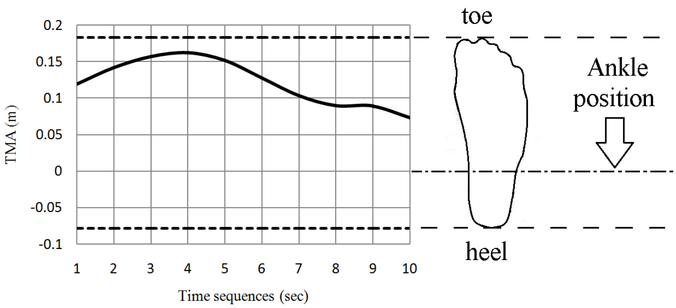

Fig. 8. TMA values during lifting time. To prevent falling forward or backward, TMA values should be restricted between base of support (distance between heel and toe).

5 Conclusion

Simulation process implemented 5DOF biomechatronical model of human body to simulate lifting motion by using predictive dynamics approach. The constraints which applied to this process, limit motion space to a feasible region that human limbs move through it. Major constraint named inverse dynamics, implement the dynamics of the motion in simulation process and finally the optimized postures shaped by objective function minimization. Figure 6 Shows that posture variation does in a natural shape. The box motion is extremely uprising, and it situate at initial and final position exactly and also it hasn't collision to the body in all of the postures. Figure 7 compare the simulation process results with experimental results due to the CNS, on the other hand figure 7 shows the operation of optimization approach in contrast with CNS action to manage the body posture. Although the curves of the predicted profiles for ankle, knee, hip and elbow are not match exactly with the experimental results, but the trend of predictive angle profiles of these joints has good compatibility with experimental results. But against these joints, the results for shoulder joint haven't good correlation with the experimental results. It caused because of complexity of shoulder structure which needs to modeling more exactly and also considering suitable and sufficient constraints.

Figure 8 illustrate the TMA values during lifting time and its boundaries. According to this figure, Lifting movement performed completely balanced because TMA have values between upper and lower boundaries. In other words minimizing ankle torque summation can guarantee motion balancing.

According to figure 6 it is concluded that: in the sequences 1 to 6, the box lifted by action of joints of lower limbs: ankle, knee and hip. In the remained sequences lifting motion continued by action of shoulder and elbow joints. In other hand this simulation approach is able to simulate leg lift (squat) motion accurately. It can be used to construct the skillful movement for humanoid robots as like as motion that human does. The movement planned based on properties of robot's bodies which presented in table 2 and lifting task parameters presented in table 1.

Table 1. Parameters of Task and related values

Parameters	Values
Box depth	0.370 (m)
Box height	0.365 (m)
Box weight	9 (kg)
Initial height	0.365 (m)
Final height	1.37 (m)
Initial horizontal position	0.490 (m)
Final horizontal position	0.460 (m)
Lifting time duration	1.2 (s)

Units are used in table; s = second, m = meter, kg = kilogram.

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Appendix

Body segments parameters of biomechatronical model presented in figure 1 is as follow:

Table 2. Parameters of Body

segments	Inertia (N.m^2)	COM (m)	Mass (kg)	Length (m)
l_1	0.183	0.147	9.68	0.50
l_2	0.505	0.209	33.48	0.46
l_3	0.325	0.360	41.65	0.66
l_4	0.070	0.144	6.36	0.32
l_5	0.035	0.130	4.80	0.30

Transformer matrixes used to calculate forward kinematics are as bellow matrixes:

$$ {}^{0}_{6}T = {}^{0}_{1}T \cdot {}^{1}_{2}T \cdot {}^{2}_{3}T \cdot {}^{3}_{4}T \cdot {}^{4}_{5}T \cdot {}^{5}_{6}T \tag{22} $$

$$ {}^{0}_{1}\boldsymbol{T} = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{1}_{2}\boldsymbol{T} = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & l_1 \\ S\theta_2 & C\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}\boldsymbol{T} = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & l_2 \\ -S\theta_2 & -C\theta_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{3}_{4}\boldsymbol{T} = \begin{bmatrix} C\theta_4 & -S\theta_4 & 0 & l_3 \\ -S\theta_4 & -C\theta_4 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{4}_{5}\boldsymbol{T} = \begin{bmatrix} C\theta_5 & -S\theta_5 & 0 & l_4 \\ S\theta_5 & C\theta_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{5}_{6}\boldsymbol{T} = \begin{bmatrix} 1 & 0 & 0 & l_5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$