

Cooperation Policy Selection for Energy-Constrained Ad Hoc Networks Using Correlated Equilibrium

Dan Wu¹, Jianchao Zheng¹, Yueming Cai^{1,2}, Limin Yang¹, and Weiwei Yang¹

¹ Institute of Communications Engineering, PLA University of Science and
Technology, Nanjing, China

² National Mobile Communications Research Laboratory, Southeast University,
Nanjing, China

Abstract. Energy efficiency is crucial for energy-constrained ad hoc networks. Cooperative communication can be applied to significantly reduce energy consumption. Due to the selfishness and the self-organization of nodes, the relay requests can not always be accepted by potential relay nodes with only local information, and the network overall performance can not always be improved in a distributed way. In this work, we present a distributed cooperation policy selection scheme which allows nodes to autonomously make their own cooperation decisions to achieve the global max-min fairness in terms of energy efficiency. Specifically, since the correlated equilibrium can achieve better performance by helping the noncooperative players coordinate their strategies, we model a correlated equilibrium-based cooperation policy selection game, where the individual utility function is designed from the global energy efficiency perspective. We derive the condition under which the correlated equilibrium is Pareto optimal, and propose a distributed algorithm based on the regret matching procedure that converges to the correlated equilibrium. Simulation results are provided to demonstrate the effectiveness of the proposed scheme.

Keywords: ad hoc networks, cooperative communication, energy efficiency, outage probability, game theory, correlated equilibrium.

1 Introduction

Energy efficiency is of great importance to energy-constrained ad hoc networks, and the cooperative transmission technique is now widely considered as a promising approach to achieve energy efficiency [1]. The choice of cooperation policies is essential to exploit this energy saving potential of cooperation, however, existing methods are almost based on maximizing the throughput [2], minimizing the symbol error rate (SER) [3], etc. Since the nodes in ad hoc networks are distributed and selfish, how to select proper cooperation policies from a distributed perspective has been an important issue to be solved. Game theory provides a highly appealing mathematical tool for addressing the issue of node cooperation

in ad hoc networks, such as [4, 5]. Note that most of these works focus on the concept of Nash equilibrium in specific resource allocation games. However, the Nash equilibrium does not always lead to the best performance for the nodes which are distributed, competitive, and equipped with a low-level awareness of neighboring environments.

In this work, we aim at achieving a global objective (the max-min fairness in terms of energy efficiency with outage performance constraint) using a distributed scheme (cooperation policy selection by individual nodes) in energy-constrained cooperative ad hoc networks. Firstly, we transform a global objective into local objective function according to the relationship between them. Then, the resulting individual objective function triggers a cooperation policy selection game. Particularly, we focus on the correlated equilibrium to analyze the outcome of the proposed game. Since the correlated equilibrium directly considers the ability of nodes to coordinate actions, it is a better solution compared to the non-cooperative Nash equilibrium, and is naturally attractive for distributed adaptive algorithms to solve discrete problems. Recently, several wireless networking problems have been characterized by using the correlated equilibrium concept [6–8]. Furthermore, we prove that the correlated equilibrium of the proposed game is Pareto optimal in some specific cases. Also, we propose an algorithm based on the regret matching procedure to obtain the correlated equilibrium in a distributed manner. The resulting correlated equilibrium can help us select proper cooperation policies.

2 System Model and Problem Formulation

2.1 System Model

We consider an energy-constrained ad hoc network consisting of N nodes, where each node is endowed with a single antenna and a half-duplex transceiver. For a cooperative network model, a node plays the role of the source node (s) to send a number of data to a destination node (d), and the remaining $N - 1$ nodes form the set of potential relay nodes, denoted by $\mathcal{R}_p = \{r_j\}$. In general, the communication between the source and destination nodes is divided into two phases, i.e., a local broadcasting transmission and a long-haul cooperative transmission. In terms of both phases, the channels are modeled by a path loss exponent δ and frequency flat Rayleigh fading (i.e., $h_{s,r_j} \sim \mathcal{CN}(0, 1)$, $h_{s,d} \sim \mathcal{CN}(0, 1)$ and $h_{r_j,d} \sim \mathcal{CN}(0, 1)$ are unitary power, Rayleigh fading coefficients).

During the first phase, the source node chooses $n_t - 1$ nodes to form the set of relay nodes \mathcal{R} , and broadcasts its data to them. Due to the broadcasting nature of the wireless channel and the goal of guaranteeing the nodes in \mathcal{R} decode correctly, the capacity region of the local broadcasting transmission is constrained to:

$$\min_{r_j \in \mathcal{R}} \left\{ \frac{1}{2} \log_2 \left(1 + \frac{p_s^{\text{co},1}}{\sigma^2} \kappa d_j^{-\delta} |h_{s,r_j}|^2 \right) \right\} \geq \mathcal{C}_{out}, \quad (1)$$

$$\Rightarrow p_s^{\text{co},1} \geq \frac{(2^{2C_{\text{out}}} - 1) \sigma^2 \kappa^{-1}}{\min_{r_j \in \mathcal{R}} \left\{ d_j^{-\delta} |h_{s,r_j}|^2 \right\}}, \quad (2)$$

where C_{out} is the outage capacity, $p_s^{\text{co},1}$ is the power needed for broadcasting, σ^2 is the Gaussian noise variance, d_j is the local distance between the source node and r_j , and κ is a constant which depends on the propagation environment.

During the second phase, the relay nodes form the virtual MISO with the source node based on distributed space time codes (DSTC), and jointly transmit the data to the destination node with the transmission power $p^{\text{co},2}$. Each transmitting member has the same transmission power, i.e., $p_s^{\text{co},2} = p_{r_j}^{\text{co},2} = p^{\text{co},2}/n_t$, $\forall r_j \in \mathcal{R}$, $|\mathcal{R}| = n_t - 1$. Furthermore, the outage probability is

$$P_{\text{out}} = \Gamma \left(n_t, \frac{(2^{2C_{\text{out}}} - 1) \sigma^2 \kappa^{-1} d^\delta}{p^{\text{co},2}/n_t} \right), \quad (3)$$

where d is the long-haul transmission distance between the transmitting members and the destination node, and $\Gamma(n_t, b) = \frac{1}{(n_t-1)!} \int_0^b x^{n_t-1} e^{-x} dx$.

2.2 Problem Formulation

In order to determine globally optimal cooperation policies in terms of energy efficiency, it is necessary to deal with two tricky problems: i) the CSI should be acquired for all links and for all time; ii) improving energy efficiency focuses on not only reducing the energy consumption of the whole network, but also making balanced use of each nodes energy. To this end, we denote outage performance as the target, which depends on large scale channel effects and models small scale fading via using its statistical description. Moreover, we should formulate the optimal cooperation policy selection problem from a global and max-min fairness perspective. Hence, we describe the optimization problem as maximizing the residual energy of worst-off node with a outage performance constraint, i.e,

$$\max_{\mathcal{R}, \mathbf{p}} \left\{ u(\mathcal{R}, \mathbf{p}) = \min \left\{ E_s - \frac{1}{2} (p_s^{\text{co},1} + p_s^{\text{co},2}), E_{r_j} - \frac{1}{2} p_{r_j}^{\text{co},2}, \forall r_j \in \mathcal{R} \right\} \right\}, \quad (4)$$

subject to

$$p_s^{\text{co},1} \geq \frac{(2^{2C_{\text{out}}} - 1) \sigma^2 \kappa^{-1}}{\min_{r_j \in \mathcal{R}} \left\{ d_j^{-\delta} |h_{s,r_j}|^2 \right\}}, \quad (4.1)$$

$$P_{\text{out}} \leq P_{\text{out}}^{\text{thr}}, \quad (4.2)$$

where $u(\mathcal{R}, \mathbf{p})$ represents a global objective function, E_s and E_{r_j} are the residual energy of the source and potential relay nodes, respectively, $P_{\text{out}}^{\text{thr}}$ is the threshold value of the outage probability, and $\mathbf{p} = [p_s^{\text{co},1}, p_s^{\text{co},2}, p_{r_1}^{\text{co},2}, \dots, p_{r_{N-1}}^{\text{co},2}]$ is the

transmission power vector. The solution of (4)-(4.2) guarantees the participation of a proper number of nodes and the proper power allocation among these nodes. Actually, the energy-constrained nodes prefer to consume as little energy as possible, subject to the constraint on the desired outage performance, i.e., constraints (4.1) and (4.2). Hence, the transmission power vector is dependent on the potential relay nodes' decisions, i.e., $\mathbf{p}(\mathcal{R})$. Once \mathcal{R} is determined, \mathbf{p} can be obtained by transforming (4.1) and (4.2) into equality constraints. That is, we can rewrite $u(\mathcal{R}, \mathbf{p})$ as $u(\mathcal{R})$, following this convention below. However, due to the distributed and selfish features of the nodes, $u(\mathcal{R})$ can not be easily evaluated by any one node, and hence an appropriate substitute must be found.

3 Correlated Equilibrium-Based Cooperation Policy Selection Game

The nodes in ad hoc networks are distributed and selfish, hence, their actions are strictly determined by self interest. For the purpose of distributed operation, we transform the behavior of $u(\mathcal{R})$ into the individual objective functions u_{r_j} in the following sense. On the one hand, any potential relay node may be turned into the source node at the next time. Therefore, it would like to help the current source node, and expects a favor in return. The residual energy of source node can be viewed as the return on the kindness of relay nodes. On the other hand, we take into account the self-concern of each potential relay node, and employ the additional energy cost for cooperative transmission to reflect this. Then, u_{r_j} can lead us to a cooperation policy selection game which is modeled as

$$G = \left\langle \mathcal{R}_p, \{\mathcal{A}_{r_j}\}_{r_j \in \mathcal{R}_p}, \{u_{r_j}\}_{r_j \in \mathcal{R}_p} \right\rangle, \quad (5)$$

where the components of the game are given in the list:

1. The set of potential relay nodes \mathcal{R}_p is the set of players.
2. $\mathcal{A}_{r_j} = \{0, 1\}$ is the set of relay decision strategies for player r_j . Specifically, if r_j chooses to take part in the cooperative transmission, i.e., $r_j \in \mathcal{R}$, $A_{r_j} = 1$; otherwise, $A_{r_j} = 0$.
3. $u_{r_j} : \mathcal{A} \rightarrow \mathbb{R}$ is the individual utility that maps the joint strategy spaces $\mathcal{A} = \mathcal{A}_{r_1} \times \cdots \times \mathcal{A}_{r_{N-1}}$ to the set of real numbers. More precisely, the individual utility function should comply with the transformation rule mentioned above, that is

$$u_{r_j}(A_{r_j}, A_{-r_j}) = E_s - \frac{p_s^{\text{co},1} + p_s^{\text{co},2}}{2} - A_{r_j} \alpha_{r_j} \frac{p_{r_j}^{\text{co},2}}{2E_{r_j}}, \quad (6)$$

where A_{-r_j} represents the joint strategies of the other players, and α_{r_j} is the pricing parameter of r_j , which weighs its cost compared to its reward. Also, $A = (A_{r_j}, A_{-r_j})$ is called a strategy profile.

Formally, the proposed game G can be expressed as

$$\max_{A_{r_j} \in \mathcal{A}_{r_j}} u_{r_j}(A_{r_j}, A_{-r_j}), \quad \text{for all } r_j \in \mathcal{R}_p, \tag{7}$$

subject to constraints (4.1) and (4.2). Note that u_{r_j} is also a function of A_{-r_j} , because it depends on the number of the relay nodes and the worst local CSI among the relay nodes, which are related to A_{-r_j} .

In order to analyze the outcome of the proposed game G , we focus on an important generalization of the Nash equilibrium, known as the correlated equilibrium. For the distributed, competitive ad hoc network, the correlated equilibrium permits to coordinate the cooperation policy selection among potential relay nodes, hence, may lead to the most relevant noncooperative solution. There are several benefits for considering a correlated equilibrium, which are summarized in [6].

Definition 1. Let $\Delta\mathcal{A}$ be the set of probability distributions on \mathcal{A} . A correlated strategy $P = (P(A))_{A \in \mathcal{A}} \in \Delta\mathcal{A}$ is a correlated equilibrium if for every strategy $A_{r_j} \in \mathcal{A}_{r_j}$ such that $P(A_{r_j}) > 0$, and every alternative strategy $\tilde{A}_{r_j} \in \mathcal{A}_{r_j}$, it holds that

$$\sum_{A_{-r_j}} P(A_{r_j}, A_{-r_j}) u_{r_j}(A_{r_j}, A_{-r_j}) \geq \sum_{A_{-r_j}} P(A_{r_j}, A_{-r_j}) u_{r_j}(\tilde{A}_{r_j}, A_{-r_j}). \tag{8}$$

P provides each player r_j with a private “recommendation” $A_{r_j} \in \mathcal{A}_{r_j}$, so as to allow a weak form of cooperation.

Theorem 1. A correlated equilibrium always exists in the cooperation policy selection game G . □

Proof. The result from [9] shows that every finite game has a correlated equilibrium. Hence, Theorem 1 is justified, and enables the application of the proposed game. ■

Theorem 2. In the game G , if a CE achieves the highest social welfare, denoted as P^* , it is Pareto optimal. □

Proof. To find $P^*(\cdot)$, we first introduce an appropriate objective function, i.e.,

$$\max \sum_{A \in \mathcal{A}} P(A) \sum_{r_j \in \mathcal{R}_p} u_{r_j}(A), \tag{9}$$

subject to constraint (8), and

$$P(A) \geq 0, \tag{9.1}$$

$$\sum_{A \in \mathcal{A}} P(A) = 1. \tag{9.2}$$

where (9) means that P^* is the solution to the highest social welfare, and constraints (8), (9.1) and (9.2) guarantee that P^* is the CE. If the resulting correlated equilibrium P^* is not Pareto efficient, there exists a different probability distribution \tilde{P} such that $\sum_{A \in \mathcal{A}} \tilde{P}(A) u_{r_j}(A) \geq \sum_{A \in \mathcal{A}} P^*(A) u_{r_j}(A)$ for $\forall r_j \in \mathcal{R}$, and $\sum_{A \in \mathcal{A}} \tilde{P}(A) u_{r_j}(A) > \sum_{A \in \mathcal{A}} P^*(A) u_{r_j}(A)$ for some r_j , thus resulting in a higher value for the expected sum of utilities which contradicts the fact that P^* is the optimal solution to (9). This completes the proof. ■

P^* can be obtained by linear programming method. However, all information is required to be available for optimization. The requirement is not possible for distributed r_j .

4 Distributed Algorithm for Cooperative Policy Selection

4.1 Algorithm Description

In this section, we present a distributed algorithm based on the regret matching procedure of [10] to obtain the set of correlated equilibria. Suppose that the proposed game G is played repeatedly through time: $n = 1, 2, \dots$. At time $n + 1$, given a history of play $h^n = (A^\tau)_{\tau=1}^n \in \prod_{\tau=1}^n \mathcal{A}$, each potential relay node $r_j \in \mathcal{R}_p$ chooses $A_{r_j}^{n+1} \in \mathcal{A}_{r_j}$ according to the average regret at time n . Then, the cooperative policy selection algorithm is executed independently by each potential relay node and summarized as follows.

1. **Initialization:** At the initial time $n = 1$, the source node calculates the minimum transmission power $p^{\text{co},2}$ which satisfies constraint (4.2) and broadcasts the value to each potential relay node. Each potential relay node knows its CSI with the source node and initializes its strategy $A_{r_j}^1 \in \mathcal{A}_{r_j}$ arbitrarily.
2. **Iterative Update Process:** At the time n , each potential relay node r_j chooses a strategy $i \in \mathcal{A}_{r_j}$, and informs the source node of its choice. Then, the source node broadcasts the information which includes the number of relay nodes $n_t - 1$ and the worst-off CSI with relay nodes, i.e., $\omega^n = \min_{r_j \in \mathcal{R}} \left\{ d_j^{-\delta} |h_{s,r_j}|^2 \right\}$.
 - **Utility Update:** Each potential relay node r_j calculates its utility $u_{r_j}(A^n)$ according to (6), where $p_s^{\text{co},2} = p_{r_j}^{\text{co},2} = p^{\text{co},2}/n_t$, and $p_s^{\text{co},1}$ is the minimum value which satisfies constraint (4.1). Similarly, r_j calculates the utility for choosing the different strategy $k \in \mathcal{A}_{r_j}$.
 - **Average Regret Update:** If r_j replaces strategy i , every time that it was played in the past, with the different strategy $k \in \mathcal{A}_{r_j}$, the resulting difference in r_j 's average utility up to time n is

$$D_{r_j}^n(i, k) = \frac{1}{n} \sum_{\tau \leq n: A_{r_j}^\tau = i} \left[u_{r_j}(k, A_{-r_j}^\tau) - u_{r_j}(A^\tau) \right]. \quad (10)$$

$D_{r_j}^n(i, k)$ represents the average regret at time n for not having played, every time that i was played in the past, the different strategy k .

- **Strategy Update:** According to the resulting average regret, r_j updates its relay decision strategy at the time $n + 1$:

$$A_{r_j}^{n+1} = \begin{cases} i, & D_{r_j}^n(i, k) \leq 0 \\ k, & \text{others} \end{cases} \quad (11)$$

Notes and Comments.

1. The proposed algorithm has low complexity. At each iteration, each r_j performs one table lookup to calculate its utility, two additions and two multiplication to update its regret value, and one comparison to determine the next strategy.
2. The proposed algorithm does not need r_j to know the individual strategies and utilities of other nodes, the global network structure, etc. This accords with the distributed characteristics of ad hoc networks.
3. We expand the applications of correlated equilibrium to the cooperation policy selection of ad hoc networks. Compared to [6, 7], we integrate the transmit power strategy selection into the algorithm, and modify the strategy update process in accordance with the specific space of two strategies, which avoids the bad convergence of fewer strategies.

4.2 Convergence Analysis

Let $z^n \in \Delta\mathcal{A}$ be the empirical distribution of play to time n , which can be viewed as an average or moving average frequency of play and given by

$$z^{n+1} = z^n + \frac{1}{n+1} (\mathbf{e}_{A^{n+1}} - z^n). \quad (12)$$

where $\mathbf{e}_{A^{n+1}} = [0, 0, \dots, 1, 0, \dots, 0]$ is the $|\mathcal{A}|$ dimensional unit vector with the one in the position of A^{n+1} .

Theorem 3. *If every potential relay node follows the proposed algorithm, the empirical distributions of play z^n converge almost surely as $n \rightarrow \infty$ to the set of correlated equilibria of the cooperation policy selection game G . \square*

The proof that z^n converges to the set of correlated equilibria is presented in [6] and [10] respectively. Here, we only summarize and compare the two proofs.

1. In [6], the proof is based on a stochastic approximation convergence proof. A continuous time random process $z^n(t)$ is constructed by interpolating z^n . The tail behavior of the sequence $\{z^n\}$ is captured by the behavior of $z^n(t)$ for large t . Moreover, the trajectory of $z^n(t)$ converges almost surely to a trajectory whose dynamics are given by a different inclusion. Then, the asymptotically stable properties of the different inclusion tell us the tail behavior of $\{z^n\}$.
2. In [10], the proof relies on a recursive formula for the distance of the vector of regrets to the negative orthant. Particularly, in order to satisfy the conditions of Blackwell's approachability theorem, a multi-period recursion, where a large block of periods is combined together, substitutes for a one-period recursion.

5 Simulation Results and Analysis

In this section, we conduct simulations to study the performance of the proposed scheme over an energy-constrained cooperative ad hoc network. For both the local broadcasting and the long-haul cooperative channels, Rayleigh fading coefficients are modeled as unitary power, complex Gaussian random variables. The constant κ is set to 1, and the path loss exponent δ is set to 3. The Gaussian noise variance σ^2 is 10^{-12} W, and the outage capacity \mathcal{C}_{out} is 1.4bps/Hz. The threshold value of outage probability is 10^{-4} . Besides, the pricing parameters of all nodes are set as one, but they have their own different residual energy.

Fig. 1 plots the evolution of regret value of worst player, when there exist 5, 10, 15 and 20 potential relay nodes, respectively. No matter how many potential relay nodes are placed in the ad hoc network, the correlated equilibrium can be obtained via using the proposed algorithm. From Fig. 1, we can find that: i) the individual regret value depends on not only its own strategy, but also the strategies chosen by other potential relay nodes, hence it can reflect the global convergence performance; ii) the more the potential relay nodes, the slower the convergence speed.

Fig. 2 presents the max-min fairness (see equation (4) subject to (4.1) and (4.2)) with different long-haul transmission distances. We exploit three algorithms which are plotted as reference, i.e., exhaustive search among all possible relay node combinations (Algorithm 1), complete cooperation (Algorithm 2) and complete noncooperation (Algorithm 3) among all potential relay nodes. Although Algorithm 1 can achieve the best cooperation policies from a max-min fairness perspective, its complexity is exponential. Our algorithm can obtain the same max-min fairness as Algorithm 1 with small complexity. In Algorithm 2, all potential relay nodes contribute their residual energy. Indeed, energy consumption caused by increasing the distance is averaged among all the nodes. However, it is disadvantageous for the node which has little residual energy with increasing the distance, hence, the max-min fairness becomes poor. In Algorithm 3, the

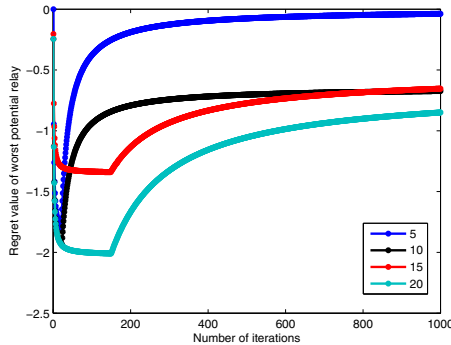


Fig. 1. Evolution of regret value of worst player for different number of potential relay nodes

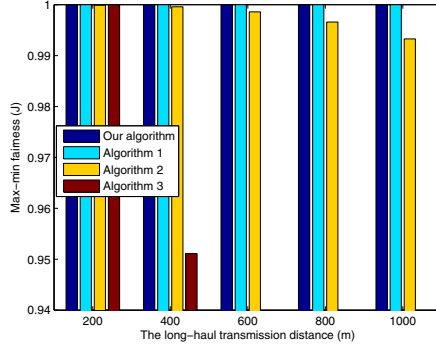


Fig. 2. Max-min fairness vs. the long-haul transmission distance in different algorithms

source node consumes much more energy than the other algorithms. Hence, it is more possible that the source node drains its energy firstly, while the other nodes have much retaining energy.

6 Conclusions

In this work, we design a cooperation policy selection scheme to achieve the global max-min fairness in terms of energy efficiency with outage performance constraint in energy-constrained cooperative ad hoc networks. Specifically, we model a cooperation policy selection game and focus on the CE to analyze the proposed game. Moreover, we develop an algorithm based on the regret matching procedure to obtain the correlated equilibrium. From the resulting correlated equilibrium, we can determine the proper cooperation policy. Both the theoretical analysis and simulation results demonstrate the efficiency of the proposed scheme.

Acknowledgement. This work is supported by the NSF of China (Grant No. 60972051, 61001107), the Major National Science & Technology Specific Projects (Grant No. 2010ZX03006-002-04), and the Open Research Fund of National Mobile Communications Research Laboratory, Southeast University (Grant No. 2010D09).

References

1. Wang, X., Vasilakos, A., Chen, M., Liu, Y., Kwon, T.: A survey of green mobile networks: Opportunities and challenges. *ACM/Springer Mobile Networks and Applications* (2011), doi:10.1007/s11036-011-0316-4
2. Dai, L., Chen, W., Cimini Jr., L.J., Letaief, K.B.: Fairness improves throughput in energy-constrained cooperative ad-hoc networks. *IEEE Transactions on Wireless Communications* 8(7), 3679–3691 (2009)
3. Qu, Q., Milstein, L.B., Vaman, D.R.: Cooperative and constrained MIMO communications in wireless ad hoc/sensor networks. *IEEE Transactions on Wireless Communications* 9(10), 3120–3129 (2010)

4. Sergi, S., Pancaldi, F., Vitetta, G.M.: A game theoretical approach to the management of transmission selection scheme in wireless ad hoc networks. *IEEE Transactions on Communications* 58(10), 2799–2804 (2010)
5. Sergi, S., Vitetta, G.M.: A game theoretical approach to distributed relay selection in randomized cooperation. *IEEE Transactions on Wireless Communications* 9(8), 2611–2621 (2010)
6. Krishnamurthy, V., Maskery, M., Yin, G.: Decentralized adaptive filtering algorithms for sensor activation in an unattended ground sensor network. *IEEE Transactions on Signal Processing* 56(12), 6086–6101 (2008)
7. Maskery, M., Krishnamurthy, V., Zhao, Q.: Decentralized dynamic spectrum access for cognitive radios: cooperative design of a non-cooperative game. *IEEE Transactions on Communications* 57(2), 459–469 (2009)
8. Han, Z., Pandana, C., Liu, K.: Distributive opportunistic spectrum access for cognitive radio using correlated equilibrium and no-regret learning. In: *WCNC*. IEEE (2007)
9. Hart, S., Schmeidler, D.: Existence of correlated wquilibria. *Mathematics of Operations Research* 14(1), 18–25 (1989)
10. Hart, S., Mas-Colell, A.: A simple adaptive procedure leading to correlated equilibrium. *Econometrica* 68(5), 1127–1150 (2000)