The Outcomes of Logic-Based Argumentation Systems under Preferred Semantics

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Abstract. Logic-based argumentation systems are developed for reasoning with inconsistent information. They consist of a set of arguments, attacks among them and a semantics for the evaluation of arguments. *Preferred* semantics is favored in the literature since it ensures the existence of *extensions* (i.e., acceptable sets of arguments), and it guarantees a kind of maximality, accepting thus arguments whenever possible.

This paper proposes the first study on the outcomes under preferred semantics of logic-based argumentation systems that satisfy basic rationality postulates. It focuses on systems that are grounded on Tarskian logics, and delimits the number of preferred extensions they may have. It also characterizes both their extensions and their sets of conclusions that are drawn from knowledge bases. The results are disappointing since they show that in the best case, the preferred extensions of a system are computed from the maximal consistent subbases of the knowledge base under study. In this case, the system is coherent, that is preferred extensions are stable ones. Moreover, we show that both semantics are useless in thic case since they ensure exactly the same result as naive semantics. Apart from this case, the outcomes of argumentation systems are counter-intuitive.

1 Introduction

An important problem in the management of knowledge-based systems is the handling of inconsistency. Inconsistency may be present because the knowledge base includes default rules (e.g. [16]) or because the knowledge comes from several sources of information (e.g. [9]).

Argumentation theory is an alternative approach for reasoning with inconsistent information. It is based on the key notion of *argument* which explains why a conclusion may be drawn from a given knowledge base. In fact, an argumentation system is a set of arguments, an attack relation and a semantics for evaluating the arguments (see [5,13,14,17] for some examples of such systems). Surprisingly enough, in most existing systems, there is no characterization of the kind of outputs that are drawn from a knowledge base. To say it differently, the properties of those outputs are unknown. These properties should broadly depend on the chosen semantics. It is worth mentioning that in all existing systems, Dung's semantics [11] or variants of them are used. The so-called *Preferred* semantics is the most favored one. It enjoys a kind of maximality which leads to the acceptance of arguments whenever possible. This semantics was mainly proposed as an alternative for stable semantics which does not guarantee the existence of stable extensions. While we may find some works that investigate the

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outcomes of particular systems under stable semantics [8], there is no such work under preferred semantics. Thus, the outcomes under this semantics are still completely mysterious and unexplored.

This paper investigates for the first time the outcomes under preferred semantics of argumentation systems that are grounded on Tarskian logics [20] and that satisfy the basic rationality postulates proposed in [1]. We identify for the first time the maximum number of preferred extensions those systems may have, and characterize both their extensions and their sets of conclusions that are drawn from knowledge bases. The study completely abstracts from the logic and the attack relation. The results are disappointing. They show that in the best case, the preferred extensions of a system are computed from all the maximal consistent subbases of the knowledge base under study. In this case, the argumentation system is coherent, i.e., its preferred extensions coincide with its stable ones. In a companion paper [2], we have shown that stable semantics does not play any role in this case since the output of a system under this semantics is exactly what is returned by the same system under naive semantics (i.e., the maximal conflict-free sets of arguments). Consequently, preferred semantics is also useless in this case. In all the remaining cases we identified, the argumentation systems return counter-intuitive results. To sum up, preferred semantics is not commended for instantiations of Dung's framework with Tarskian logics.

The paper is organized as follows: we start by defining the logic-based argumentation systems we are interested in and by recalling the three basic postulates that such systems should obey. In a subsequent section, we investigate the properties of the preferred extensions of those systems. Next, we study the inferences that are drawn from a knowledge base by argumentation systems under preferred semantics. The last section is devoted to some concluding remarks.

2 Logic-Based Argumentation Systems and Rationality Postulates

In this paper, we consider abstract logic-based argumentation systems; that is systems that are grounded on *any* Tarskian logic [20] and that use *any* attack relation. Such abstraction makes our study very general.

According to Alfred Tarski, an abstract logic is a pair $(\mathcal{L}, \mathtt{CN})$ where $\mathcal{L}$ is a set of well-formed formulas. Note that there is no particular requirement on the kind of connectors that may be used. $\mathtt{CN}$ is a consequence operator that returns the set of formulas that are logical consequences of another set of formulas according to the logic in question. It should satisfy the following basic properties:

1. $X \subseteq \mathtt{CN}(X)$ (Expansion)
2. $\mathtt{CN}(\mathtt{CN}(X)) = \mathtt{CN}(X)$ (Idempotence)
3. $\mathtt{CN}(X) = \bigcup_{Y \subseteq_f X} \mathtt{CN}(Y)$[1] (Finiteness)
4. $\mathtt{CN}(\{x\}) = \mathcal{L}$ for some $x \in \mathcal{L}$ (Absurdity)
5. $\mathtt{CN}(\emptyset) \neq \mathcal{L}$ (Coherence)

The associated notion of *consistency* is defined as follows: A set $X \subseteq \mathcal{L}$ is *consistent* wrt a logic $(\mathcal{L}, \mathtt{CN})$ iff $\mathtt{CN}(X) \neq \mathcal{L}$. It is *inconsistent* otherwise. Besides, arguments are built from a *knowledge base* $\Sigma \subseteq \mathcal{L}$ as follows:

[1] $Y \subseteq_f X$ means that Y is a finite subset of X.

Definition 1 (Argument). *Let Σ be a knowledge base. An* argument *is a pair (X, x) s.t. $X \subseteq \Sigma$, X is consistent, and $x \in \mathtt{CN}(X)$*[2]*. An argument (X, x) is a* sub-argument *of another argument (X', x') iff $X \subseteq X'$.*

Notations: Supp and Conc denote respectively the *support* X and the *conclusion* x of an argument (X, x). For all $\mathcal{S} \subseteq \Sigma$, $\mathtt{Arg}(\mathcal{S})$ denotes the set of all arguments that can be built from $\mathcal{S}$ by means of Definition 1. Sub is a function that returns all the sub-arguments of a given argument. For all $\mathcal{E} \subseteq \mathtt{Arg}(\Sigma)$, $\mathtt{Concs}(\mathcal{E}) = \{\mathtt{Conc}(a) \mid a \in \mathcal{E}\}$ and $\mathtt{Base}(\mathcal{E}) = \bigcup_{a \in \mathcal{E}} \mathtt{Supp}(a)$. $\mathtt{Max}(\Sigma)$ is the set of all maximal (for set inclusion) consistent subbases of Σ. $\mathtt{Free}(\Sigma) = \bigcap \mathcal{S}_i$ where $\mathcal{S}_i \in \mathtt{Max}(\Sigma)$, and $\mathtt{Inc}(\Sigma) = \Sigma \setminus \mathtt{Free}(\Sigma)$. Finally, $\mathcal{C}_\Sigma$ denote the set of all minimal conflicts[3] of Σ.

An argumentation system for reasoning over a knowledge base Σ is defined as follows.

Definition 2 (Argumentation system). *An* argumentation system *(AS) over a knowledge base Σ is a pair $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ such that $\mathcal{R} \subseteq \mathtt{Arg}(\Sigma) \times \mathtt{Arg}(\Sigma)$ is an* attack *relation. For $a, b \in \mathtt{Arg}(\Sigma)$, $(a, b) \in \mathcal{R}$ (or $a\mathcal{R}b$) means that a attacks b.*

Throughout the paper, the attack relation is left *unspecified*.

Arguments are evaluated using preferred semantics [11]. For the purpose of this paper, we also need to recall the definition of stable semantics. Preferred semantics is based on two requirements: *conflict-freeness* and *defence*. Recall that a set $\mathcal{E}$ of arguments is *conflict-free* iff $\nexists a, b \in \mathcal{E}$ such that $a\mathcal{R}b$. It *defends* an argument a iff $\forall b \in \mathtt{Arg}(\Sigma)$, if $b\mathcal{R}a$, then $\exists c \in \mathcal{E}$ such that $c\mathcal{R}b$.

Definition 3. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an AS and $\mathcal{E} \subseteq \mathtt{Arg}(\Sigma)$.*

- *$\mathcal{E}$ is an* admissible *extension iff $\mathcal{E}$ is conflict-free and defends all its elements.*
- *$\mathcal{E}$ is a* preferred *extension iff it is a maximal (for set inclusion) admissible extension.*
- *$\mathcal{E}$ is a* stable *extension iff $\mathcal{E}$ is conflict-free and attacks any argument in $\mathtt{Arg}(\Sigma) \setminus \mathcal{E}$.*

It is worth recalling that each stable extension is a preferred one but the converse is not true. Let $\mathtt{Ext}_x(\mathcal{T})$ denote the set of all extensions of $\mathcal{T}$ under semantics x where p and s stand respectively for preferred and stable. When we do not need to specify the semantics, we use the notation $\mathtt{Ext}(\mathcal{T})$ for short.

The set of conclusions drawn from a knowledge base Σ using an argumentation system $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ contains only the common conclusions of the extensions.

Definition 4 (Output). *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an AS over a knowledge base Σ. $\mathtt{Output}(\mathcal{T}) = \{x \in \mathcal{L} \mid \forall \mathcal{E} \in \mathtt{Ext}(\mathcal{T}), \exists a \in \mathcal{E}$ s.t. $\mathtt{Conc}(a) = x\}$.*

In [7], it was shown that not any instantiation of Dung's abstract argumentation framework is acceptable. Some instantiations like [15,18] may lead in some cases to undesirable outputs. Consequently, some rationality postulates that any system should obey

[2] Generally, the support X is minimal (for set inclusion). In this paper, we do not need to make this assumption.

[3] A set $C \subseteq \Sigma$ is a *minimal conflict* iff C is inconsistent and $\forall x \in C$, $C \setminus \{x\}$ is consistent.

were proposed. Postulates are desirable properties that any reasoning system should enjoy. In [1], those postulates were revisited and extended to any Tarskian logic. The first postulate concerns the closure of the system's output under the consequence operator $\mathtt{CN}$. The idea is that the formalism should not forget conclusions.

Postulate 1 (Closure under $\mathtt{CN}$). *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an AS over a knowledge base. $\mathcal{T}$ satisfies* closure *iff for all $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathtt{Concs}(\mathcal{E}) = \mathtt{CN}(\mathtt{Concs}(\mathcal{E}))$.*

The second rationality postulate ensures that the acceptance of an argument implies also the acceptance of all its sub-parts.

Postulate 2 (Closure under sub-arguments). *Let $\mathcal{T} = (\mathtt{Args}(\Sigma), \mathcal{R})$ be an AS. $\mathcal{T}$ is* closed under sub-arguments *iff for all $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, if $a \in \mathcal{E}$, then $\mathtt{Sub}(a) \subseteq \mathcal{E}$.*

The third postulate ensures that the set of conclusions supported by each extension is consistent.

Postulate 3 (Consistency). *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an AS over a knowledge base Σ. $\mathcal{T}$ satisfies* consistency *iff for all $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathtt{Concs}(\mathcal{E})$ is consistent.*

The following interesting result is shown in [1] under any acceptability semantics.

Proposition 1. *[1] Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an AS over a knowledge base Σ. If $\mathcal{T}$ satisfies consistency and closure under sub-arguments, then for all $\mathcal{E} \in \mathtt{Ext}(\mathcal{T})$, $\mathtt{Base}(\mathcal{E})$ is consistent.*

It was shown in [1] that in order to satisfy these postulates, the attack relation should capture inconsistency. This is an obvious requirement especially for reasoning about inconsistent information. Note also that *all* existing attack relations verify this property (see [14] for an overview of those relations defined under propositional logic).

Definition 5 (Conflict-dependent). *An attack relation $\mathcal{R}$ is* conflict-dependent *iff $\forall a, b \in \mathtt{Arg}(\Sigma)$, if $a\mathcal{R}b$ then $\mathtt{Supp}(a) \cup \mathtt{Supp}(b)$ is inconsistent.*

3 Properties of Preferred Extensions

Throughout the paper, we assume argumentation systems $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ that are built over a knowledge base Σ. These systems are assumed to be sound in the sense that they enjoy the three rationality postulates described in the previous section. It is worth recalling that the attack relation is a crucial parameter in a system since the satisfaction of the postulates depends broadly on it. For instance, it was shown in [1] that argumentation systems that use symmetric relations may violate the consistency postulate. This is particularly the case when the knowledge base contains a ternary or a n-ary minimal conflict (with $n > 2$). Thus, such symmetric systems [10] should be avoided and are not concerned by our study.

Our aim in this section is to investigate the properties of preferred extensions of sound argumentation systems. We will answer the following interesting questions.

1. What is the number of preferred extensions an AS may have?
2. What is the link between each preferred extension and the knowledge base Σ?
3. Is the set of formulas underlying a preferred extension consistent?
4. What is the real added value of preferred semantics compared to stable semantics? To put it differently, does preferred semantics solve any problem encountered by stable one?

We start by showing that the argumentation systems that satisfy consistency and closure under sub-arguments, satisfy also the strong version of consistency. Indeed, the union of the supports of all arguments of each preferred extension is a consistent subbase of Σ. This result is interesting since it is in accordance with the idea that an extension represents a *coherent position/point of view*.

Proposition 2. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an AS s.t. $\mathcal{R}$ is conflict-dependent and $\mathcal{T}$ satisfies consistency and closure under sub-arguments. For all $\mathcal{E} \in \mathtt{Ext}_p(\mathcal{T})$, $\mathtt{Base}(\mathcal{E})$ is consistent.*

Proof. Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an AS s.t. $\mathcal{R}$ is conflict-dependent and $\mathcal{T}$ satisfies consistency and closure under sub-arguments. From Proposition 1, it follows immediately that for all $\mathcal{E} \in \mathtt{Ext}_p(\mathcal{T})$, $\mathtt{Base}(\mathcal{E})$ is consistent.

In addition to the fact that the subbase computed from a preferred extension is consistent, we show next that it is unique. Indeed, the subbase computed from one extension can never be a subset of a subbase computed from another extension. Thus, the preferred extensions of an argumentation system return completely different subbases of Σ.

Proposition 3. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an AS s.t. $\mathcal{R}$ is conflict-dependent and $\mathcal{T}$ satisfies Postulates 2 and 3. For all $\mathcal{E}_i, \mathcal{E}_j \in \mathtt{Ext}_p(\mathcal{T})$, if $\mathtt{Base}(\mathcal{E}_i) \subseteq \mathtt{Base}(\mathcal{E}_j)$ then $\mathcal{E}_i = \mathcal{E}_j$.*

Proof. Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an AS s.t. $\mathcal{R}$ is conflict-dependent and $\mathcal{T}$ satisfies consistency and closure under sub-arguments. Assume that $\mathcal{E}_i, \mathcal{E}_j \in \mathtt{Ext}_p(\mathcal{T})$ and $\mathtt{Base}(\mathcal{E}_i) \subseteq \mathtt{Base}(\mathcal{E}_j)$. We first show that $\forall a \in \mathtt{Arg}(\mathtt{Base}(\mathcal{E}_i))$, $\mathcal{E}_j \cup \{a\}$ is conflict-free. Let $a \in \mathtt{Arg}(\mathtt{Base}(\mathcal{E}_i))$. Assume that $\mathcal{E}_j \cup \{a\}$ is not conflict-free. Thus, $\exists b \in \mathcal{E}_j$ such that $a\mathcal{R}b$ or $b\mathcal{R}a$. Since $\mathcal{R}$ is conflict-dependent, then $\mathtt{Supp}(a) \cup \mathtt{Supp}(b)$ is inconsistent. Besides, $\mathtt{Supp}(a) \subseteq \mathtt{Base}(\mathcal{E}_j)$. Thus, $\mathtt{Base}(\mathcal{E}_j)$ is inconsistent. This contradicts Proposition 2.

Let $\mathcal{E} = \mathcal{E}_j \cup (\mathcal{E}_i \setminus \mathcal{E}_j)$. From above, it follows that $\mathcal{E}$ is conflict-free. Moreover, $\mathcal{E}$ defends any element in $\mathcal{E}_j$ (since $\mathcal{E}_j \in \mathtt{Ext}_p(\mathcal{T})$) and any element in $\mathcal{E}_i \setminus \mathcal{E}_j$ (since $\mathcal{E}_i \in \mathtt{Ext}_p(\mathcal{T})$). Thus, $\mathcal{E}$ is an admissible set. This contradicts the fact that $\mathcal{E}_j \in \mathtt{Ext}_p(\mathcal{T})$.

In [2], we have shown that the subbases computed from the stable extensions of any argumentation system that satisfies the postulates are maximal (for set inclusion) consistent subbases of Σ. In what follows, we show that this is not necessarily the case for preferred extensions. Note that this does not mean that a preferred extension can never return a maximal consistent subbase. The previous result guarantees that the maximal consistent subbases containing a non-maximal subbase computed from a given extension will never be returned by any other extension of the same system.

Proposition 4. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an AS s.t. $\mathcal{R}$ is conflict-dependent and $\mathcal{T}$ satisfies consistency and closure under sub-arguments. Let $\mathcal{E} \in \mathtt{Ext}_p(\mathcal{T})$. If $\mathtt{Base}(\mathcal{E}) \notin \mathtt{Max}(\Sigma)$, then $\forall \mathcal{S} \in \mathtt{Max}(\Sigma)$ s.t. $\mathtt{Base}(\mathcal{E}) \subset \mathcal{S}$, $\nexists \mathcal{E}' \in \mathtt{Ext}_p(\mathcal{T})$ s.t. $\mathtt{Base}(\mathcal{E}') = \mathcal{S}$.*

Proof. Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an AS s.t. $\mathcal{R}$ is conflict-dependent and $\mathcal{T}$ satisfies consistency and closure under sub-arguments. Let $\mathcal{E} \in \mathtt{Ext}_p(\mathcal{T})$. Assume that $\mathtt{Base}(\mathcal{E}) \notin \mathtt{Max}(\Sigma)$ and that $\exists \mathcal{S} \in \mathtt{Max}(\Sigma)$ s.t. $\mathtt{Base}(\mathcal{E}) \subset \mathcal{S}$ and $\exists \mathcal{E}' \in \mathtt{Ext}_p(\mathcal{T})$ s.t. $\mathtt{Base}(\mathcal{E}') = \mathcal{S}$. Thus, $\exists x \in \mathcal{S} \setminus \mathtt{Base}(\mathcal{E}')$. Moreover, $\exists a \in \mathcal{E}'$ such that $x \in \mathtt{Supp}(a)$ and $a \notin \mathcal{E}$. Besides, from Proposition 2, it holds that $\mathcal{E} = \mathcal{E}'$. Contradiction.

The non-maximality of the subbases that are computed from preferred extensions is due to the existence of *undecided arguments* under preferred semantics. Indeed, in [6] another way of defining Dung's semantics was provided. It consists of labeling the nodes of the graph corresponding to the argumentation system with three possibles values: $\{$in, out, undec$\}$. The value *in* means that the argument is accepted, the value *out* means that the argument is attacked by an accepted arguments, and finally the value *undec* means that the argument is neither accepted nor attacked by an accepted argument. It is thus possible that some formulas appear only in undecided arguments.

Another particular property of preferred extensions is the fact that they may not be closed in terms of arguments. Indeed, they may not contain all the arguments that may be built from their bases. Indeed, it is possible that $\mathcal{E}$ is a preferred extension of a system and $\mathcal{E} \neq \mathtt{Arg}(\mathtt{Base}(\mathcal{E}))$. Surprisingly enough, the supports and conclusions of the missed arguments are conclusions of the extensions. Thus, even if an argument of $\mathtt{Arg}(\mathtt{Base}(\mathcal{E}))$ does not belong to the extension $\mathcal{E}$, all the formulas of its supports are conclusions of arguments in the extension, and the same holds for its conclusion.

Proposition 5. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an AS s.t. $\mathcal{T}$ is closed under $\mathtt{CN}$ and under sub-arguments. Let $\mathcal{E} \in \mathtt{Ext}_p(\mathcal{T})$. For all $a \in \mathtt{Arg}(\mathtt{Base}(\mathcal{E}))$, $\mathtt{Supp}(a) \subseteq \mathtt{Concs}(\mathcal{E})$ and $\mathtt{Conc}(a) \in \mathtt{Concs}(\mathcal{E})$.*

Proof. Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an AS s.t. $\mathcal{T}$ is closed under $\mathtt{CN}$ and under sub-arguments. Let $\mathcal{E} \in \mathtt{Ext}_p(\mathcal{T})$ and $a \in \mathtt{Arg}(\mathtt{Base}(\mathcal{E}))$. Thus, $\mathtt{Supp}(a) \subseteq \mathtt{Base}(\mathcal{E})$. Since $\mathcal{T}$ is closed under sub-arguments, then $\mathtt{Base}(\mathcal{E}) \subseteq \mathtt{Concs}(\mathcal{E})$ (proved in [1]). Thus, $\mathtt{Supp}(a) \subseteq \mathtt{Concs}(\mathcal{E})$. Besides, by monotonicity of $\mathtt{CN}$, $\mathtt{CN}(\mathtt{Supp}(a)) \subseteq \mathtt{CN}(\mathtt{Base}(\mathcal{E}))$. Since $\mathcal{T}$ is also closed under $\mathtt{CN}$, then $\mathtt{Concs}(\mathcal{E}) = \mathtt{CN}(\mathtt{Base}(\mathcal{E}))$ (proved in [1]). Thus, $\mathtt{CN}(\mathtt{Supp}(a)) \subseteq \mathtt{Concs}(\mathcal{E})$ and $\mathtt{Conc}(a) \in \mathtt{Concs}(\mathcal{E})$.

We show next that the free part of Σ (i.e., the formulas that are not involved in any conflict) is inferred by *any* argumentation system under preferred semantics. The reason is that the set of arguments built from $\mathtt{Free}(\Sigma)$ is an admissible extension of any argumentation systems whose attack relations are conflict-dependent. Thus, this is true even for systems that do not satisfy the postulates.

Proposition 6. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be s.t. $\mathcal{R}$ is conflict-dependent. The set $\mathtt{Arg}(\mathtt{Free}(\Sigma))$ is an admissible extension of $\mathcal{T}$.*

Proof. Let $(\mathtt{Arg}(\Sigma), \mathcal{R})$ be s.t. $\mathcal{R}$ is conflict-dependent. Let $a \in \mathtt{Arg}(\mathtt{Free}(\Sigma))$. Assume that $\exists b \in \mathtt{Arg}(\Sigma)$ s.t. $a\mathcal{R}b$ or $b\mathcal{R}a$. Since $\mathcal{R}$ is conflict-dependent, then $\exists C \in$

$\mathcal{C}_\Sigma$ such that $C \subseteq \mathtt{Supp}(a) \cup \mathtt{Supp}(b)$. By definition of an argument, both $\mathtt{Supp}(a)$ and $\mathtt{Supp}(b)$ are consistent. Then, $C \cap \mathtt{Supp}(a) \neq \emptyset$. This contradicts the fact that $\mathtt{Supp}(a) \subseteq \mathtt{Free}(\Sigma)$. Thus, $\mathtt{Arg}(\mathtt{Free}(\Sigma))$ is *conflict-free and can never be attacked.*

We show next that the set $\mathtt{Arg}(\mathtt{Free}(\Sigma))$ is contained in every preferred extension. This is true for any argumentation system that uses a conflict-dependent attack relation. That is, it is always true.

Proposition 7. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be s.t. $\mathcal{R}$ is conflict-dependent. For all $\mathcal{E} \in \mathtt{Ext}_p(\mathcal{T})$, $\mathtt{Arg}(\mathtt{Free}(\Sigma)) \subseteq \mathcal{E}$.*

Proof. Let $(\mathtt{Arg}(\Sigma), \mathcal{R})$ be s.t. $\mathcal{R}$ is conflict-dependent. Assume that $\exists \mathcal{E} \in \mathtt{Ext}_p(\mathcal{T})$ such that $\mathtt{Arg}(\mathtt{Free}(\Sigma)) \nsubseteq \mathcal{E}$. Thus, either $\mathcal{E} \cup \mathtt{Arg}(\mathtt{Free}(\Sigma))$ is conflicting or $\mathcal{E}$ does not defend elements of $\mathtt{Arg}(\mathtt{Free}(\Sigma))$. Both cases are impossible since arguments of $\mathtt{Arg}(\mathtt{Free}(\Sigma))$ neither attack nor are attacked by any argument.

The next result shows that formulas of $\mathtt{Free}(\Sigma)$ are always drawn from Σ under preferred semantics.

Proposition 8. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be s.t. $\mathcal{R}$ is conflict-dependent. It holds that $\mathtt{Free}(\Sigma) \subseteq \mathtt{Output}(\mathcal{T})$.*

Proof. Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be s.t. $\mathcal{R}$ is conflict-dependent. From Proposition 7, $\forall \mathcal{E} \in \mathtt{Ext}_p(\mathcal{T})$, $\mathtt{Arg}(\mathtt{Free}(\Sigma)) \subseteq \mathcal{E}$. Besides, $\forall x \in \mathtt{Free}(\Sigma)$, $(\{x\}, x) \in \mathtt{Arg}(\mathtt{Free}(\Sigma))$, thus $(\{x\}, x) \in \mathcal{E}$. Consequently, $\forall \mathcal{E} \in \mathtt{Ext}_p(\mathcal{T})$, $\mathtt{Free}(\Sigma) \subseteq \mathtt{Concs}(\mathcal{E})$. It follows that $\mathtt{Free}(\Sigma) \subseteq \bigcap \mathtt{Concs}(\mathcal{E})$ where $\mathcal{E} \in \mathtt{Ext}_p(\mathcal{T})$.

In [2], we have shown that formulas of the set $\mathtt{Free}(\Sigma)$ may be missed by argumentation systems under stable semantics. This is particularly the case when the systems do not have stable extensions. We have also seen that this problem is due to the use of skewed attack relations. Even if those relations ensure the rationality postulates, the corresponding systems do not return satisfactory results since they may miss intuitive conclusions like $\mathtt{Free}(\Sigma)$. The previous results show that since preferred semantics guarantees the existence of preferred extensions, then it guarantees also the inference of elements of $\mathtt{Free}(\Sigma)$.

The previous results make it possible to delimit the maximum number of preferred extensions a system may have. It is the number of consistent subbases of Σ that contain the free part of Σ and which are pairwise different. Note that this number is less than the number of consistent subbases of Σ.

Proposition 9. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an AS s.t. $\mathcal{R}$ is conflict-dependent and $\mathcal{T}$ satisfies consistency and closure under sub-arguments. It holds that $1 \leq |\mathtt{Ext}_p(\mathcal{T})| \leq |\mathtt{Cons}(\Sigma)|$ where $\mathtt{Cons}(\Sigma) = \{\mathcal{S} \mid \mathcal{S} \subseteq \Sigma,\ \mathcal{S}$ is consistent and $\mathtt{Free}(\Sigma) \subseteq \mathcal{S}\}$.*

Proof. From Proposition 2, each preferred extension returns a consistent subbase of Σ. From Proposition 3, it is not possible to have the same subbase several times. Finally, from Proposition 7, each preferred extension contains $\mathtt{Arg}(\mathtt{Free}(\Sigma))$.

Until now, we showed that preferred extensions reflect coherent points of view since they rely on consistent subbases of Σ. We also showed that when Σ is finite, each argumentation system that enjoy the rationality postulates has a *finite number of preferred extensions*. Proposition 9 provides the maximum number of such extensions. We thus answered all our questions.

4 Inferences under Preferred Semantics

In this section, we investigate the characteristics of the set $\mathtt{Output}(\mathcal{T})$ of any argumentation system $\mathcal{T}$ that satisfies the postulates. Indeed, we study the kind of inferences that are made by an argumentation system under preferred semantics. From the results of the previous section, this set is defined as follows.

Proposition 10. *Let* $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ *be an AS s.t.* $\mathcal{R}$ *is conflict-dependent and* $\mathcal{T}$ *satisfies the three postulates. It holds that* $\mathtt{Output}(\mathcal{T}) = \bigcap \mathtt{CN}(\mathcal{S}_i)$ *s.t.* $\mathcal{S}_i \in \mathtt{Cons}(\Sigma)$ *and* $\mathcal{S}_i = \mathtt{Base}(\mathcal{E}_i)$ *where* $\mathcal{E}_i \in \mathtt{Ext}_p(\mathcal{T})$.

Proof. Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an AS s.t. $\mathcal{R}$ is conflict-dependent and $\mathcal{T}$ satisfies the three postulates. Let $\mathcal{E} \in \mathtt{Ext}_p(\mathcal{T})$. Since $\mathcal{T}$ is closed both under CN and under sub-arguments, then $\mathtt{Concs}(\mathcal{E}) = \mathtt{CN}(\mathtt{Base}(\mathcal{E}))$ (result shown in [1]). From Proposition 7, $\mathtt{Free}(\Sigma) \subseteq \mathtt{Base}(\mathcal{E})$. Moreover, from Proposition 2, $\mathtt{Base}(\mathcal{E})$ is consistent. Thus, $\mathtt{Base}(\mathcal{E}) \in \mathtt{Cons}(\Sigma)$. From Definition 4, $\mathtt{Output}(\mathcal{T}) = \bigcap \mathtt{Concs}(\mathcal{E}_i)$, $\mathcal{E}_i \in \mathtt{Ext}_p(\mathcal{T})$. Thus, $\mathtt{Output}(\mathcal{T}) = \bigcap \mathtt{Base}(\mathcal{E}_i)$, $\mathcal{E}_i \in \mathtt{Ext}_p(\mathcal{T})$.

It is worth noticing that preferred semantics is more powerful than stable semantics *only* in case stable extensions do not exist and $\mathtt{Free}(\Sigma) \neq \emptyset$. Indeed, in this case the output set of any argumentation system is empty ($(\mathtt{Output}(\mathcal{T})) = \emptyset$) under stable semantics. Thus, the free formulas of Σ will not be inferred while they are guaranteed under preferred semantics. However, this does not mean that outputs under preferred semantics are "complete" and "intuitive". We show that some argumentation systems may miss in some cases some interesting conclusions. Worse yet, they may even return counter-intuitive ones. Let us illustrate our ideas on the following example.

Example 1. *Let us consider the following propositional knowledge base* $\Sigma = \{x, \neg x \wedge y\}$. *The two formulas are equally preferred. From Proposition 10, it follows that any reasonable argumentation that may be built over* Σ *will have one of the three following outputs:*

- $\mathtt{Output}_1(\mathcal{T}) = \emptyset$. *This is the case of systems that have a unique and empty extension, or those which have two extensions* $\mathcal{E}_1$ *and* $\mathcal{E}_2$ *where* $\mathtt{Base}(\mathcal{E}_1) = \{x\}$ *and* $\mathtt{Base}(\mathcal{E}_2) = \{\neg x \wedge y\}$.
- $\mathtt{Output}_2(\mathcal{T}) = \mathtt{CN}(\{x\})$. *This is the case of systems that have* $\mathcal{E}_1$ *as their unique extension.*
- $\mathtt{Output}_3(\mathcal{T}) = \mathtt{CN}(\{\neg x \wedge y\})$. *This is the case of systems that have* $\mathcal{E}_2$ *as their unique extension.*

Let us analyze the three cases. In the first one, the result is not satisfactory. Indeed, one may expect to have y *as a conclusion since it is not part of the conflict in* Σ. *Assume that*

x stands for "sunny day" and y for "My dog is sick". It is clear that the two information x and y are independent. This shows that argumentation systems are syntax-dependent. *The two other outputs ($\texttt{Output}_2(\mathcal{T})$ and $\texttt{Output}_3(\mathcal{T})$) are not satisfactory neither. The reason in these cases is different. For instance, in $\texttt{Output}_2(\mathcal{T})$, the formula x is inferred from Σ while $\neg x$ is not deduced. This discrimination between the two formulas is not justified since the two formulas of Σ are assumed to be equally preferred.*

Let us now analyze in detail all the possible situations that *may* occur. Throughout this sub-section, $\Re_p$ denotes the set of all attack relations that ensure the three postulates for any argumentation system, that is for any Σ. Indeed, $\Re_p = \{\mathcal{R} \subseteq \texttt{Arg}(\Sigma) \times \texttt{Arg}(\Sigma) \mid$ $\mathcal{R}$ is conflict-dependent and $(\texttt{Arg}(\Sigma), \mathcal{R})$ satisfies Postulates 1, 2 and 3 under preferred semantics$\}$ for all Σ. We distinguish two categories of attack relations: those that always lead to a unique extension ($\Re_u$) and those that may lead to multiple extensions ($\Re_m$), where $\Re_p = \Re_u \cup \Re_m$.

Unique Extension. Let us focus on argumentation systems $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ that satisfy the three postulates and that use attack relations of the set $\Re_u$. Thus, $\texttt{Ext}_p(\mathcal{T}) = \{\mathcal{E}\}$. Three possible situations may occur:

$\texttt{Ext}_p(\mathcal{T}) = \{\emptyset\}$**:** In this case, the output set is empty, and consequently, there is no free formula, i.e. all the formulas of Σ are involved in at least one conflict.

Property 1. *Let $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ be an AS s.t. $\mathcal{R} \in \Re_u$. If $\texttt{Ext}_p(\mathcal{T}) = \{\emptyset\}$, then $\texttt{Output}(\mathcal{T}) = \emptyset$ and $\texttt{Free}(\Sigma) = \emptyset$.*

Proof. Since $\texttt{Ext}_p(\mathcal{T}) = \{\emptyset\}$, then from Definition 4 $\texttt{Output}(\mathcal{T}) = \emptyset$. From Proposition 8, it follows that $\texttt{Free}(\Sigma) = \emptyset$.

Note that the fact that $\texttt{Free}(\Sigma) = \emptyset$ does not imply that $\texttt{Ext}_p(\mathcal{T}) = \{\emptyset\}$. At a first glance, the previous result may seem reasonable since all the formulas in Σ are conflicting and are all equally preferred. However, Example 1 shows that this is not the case since there are some interesting formulas that may be missed.

$\texttt{Ext}_p(\mathcal{T}) = \{\texttt{Arg}(\texttt{Free}(\Sigma))\}$**:** Argumentation systems that have the unique preferred extension $\texttt{Arg}(\texttt{Free}(\Sigma))$ return as conclusions all the formulas that follow under $\texttt{CN}$ from $\texttt{Free}(\Sigma)$.

Property 2. *Let $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ be an AS s.t. $\mathcal{R} \in \Re_u$. If $\texttt{Ext}_p(\mathcal{T}) = \{\texttt{Arg}(\texttt{Free}(\Sigma))\}$, then $\texttt{Output}(\mathcal{T}) = \texttt{CN}(\texttt{Free}(\Sigma))$.*

Proof. Let $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ be an AS s.t. $\mathcal{R} \in \Re_u$. Assume that $\texttt{Ext}_p(\mathcal{T}) = \{\texttt{Arg}(\texttt{Free}(\Sigma))\}$. Let $\mathcal{E} = \texttt{Arg}(\texttt{Free}(\Sigma))$. Since $\mathcal{T}$ is closed both under sub-arguments and $\texttt{CN}$, then $\texttt{Concs}(\mathcal{E}) = \texttt{CN}(\texttt{Base}(\mathcal{E}))$ ([1]). Thus, $\texttt{Concs}(\mathcal{E}) = \texttt{CN}(\texttt{Free}(\Sigma))$. Besides, $\texttt{Output}(\mathcal{T}) = \texttt{Concs}(\mathcal{E})$, thus $\texttt{Output}(\mathcal{T}) = \texttt{CN}(\texttt{Free}(\Sigma))$.

It is worth mentioning that such outputs correspond exactly to the so-called *free consequences* developed in [4] for handling inconsistency in propositional knowledge bases. The authors in [4] argue that this approach is very conservative. Indeed, if $\texttt{Free}(\Sigma)$ is empty, then nothing can be drawn from Σ. This may lead to miss intuitive formulas as shown in the next example.

Example 2. *Let us consider the following propositional knowledge base $\Sigma = \{x, \neg x \wedge y, z\}$. It can be checked that $\mathtt{Free}(\Sigma) = \{z\}$. Thus, any reasonable argumentation system that may be built over Σ and that uses an attack relation of category $\Re_u$ will have the set $\mathtt{CN}(\{z\})$ as output. However, y should also be inferred from Σ.*

$\mathtt{Ext}_p(\mathcal{T}) = \{\mathcal{E}\}$ where $\mathtt{Arg}(\mathtt{Free}(\Sigma)) \subset \mathcal{E}$: In this case, there is at least one argument in the extension $\mathcal{E}$ whose support contains at least one formula which is involved in at least one conflict in Σ. However, since $\mathtt{Base}(\mathcal{E})$ is consistent, then there are some formulas involved in the same conflict which are not considered. Then, there is a discrimination between elements of Σ which leads to ad hoc results as shown by the following result.

Proposition 11. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an AS s.t. $\mathcal{R} \in \Re_u$. If $\mathtt{Ext}_p(\mathcal{T}) = \{\mathcal{E}\}$ and $\mathtt{Arg}(\mathtt{Free}(\Sigma)) \subset \mathcal{E}$, then $\exists x \in \mathtt{Inc}(\Sigma)$ s.t. $x \in \mathtt{Output}(\mathcal{T})$ and $\exists x' \in \mathtt{Inc}(\Sigma)$ s.t. $x' \notin \mathtt{Output}(\mathcal{T})$.*

Proof. Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an AS s.t. $\mathcal{R} \in \Re_u$. Assume that $\mathtt{Ext}_p(\mathcal{T}) = \{\mathcal{E}\}$ and $\mathtt{Arg}(\mathtt{Free}(\Sigma)) \subset \mathcal{E}$. Thus, $\exists a \in \mathcal{E}$ and $a \notin \mathtt{Arg}(\mathtt{Free}(\Sigma))$. Consequently, $\mathtt{Supp}(a) \nsubseteq \mathtt{Free}(\Sigma)$. Thus, $\exists x \in \mathtt{Supp}(a)$ and $x \notin \mathtt{Free}(\Sigma)$. Thus, $x \in \mathtt{Inc}(\Sigma)$. Moreover, since $\mathcal{T}$ is closed under sub-arguments, then from [1], $\mathtt{Base}(\mathcal{E}) \subseteq \mathtt{Output}(\mathcal{T})$, then $x \in \mathtt{Output}(\mathcal{T})$. Besides, $\{x\}$ is consistent since $\mathtt{Supp}(a)$ is consistent. Thus, $\exists C \in \mathcal{C}_\Sigma$ such that $|C| > 1$ and $x \in C$. Since $\mathcal{T}$ satisfies consistency, then $C \nsubseteq \mathtt{Output}(\mathcal{T})$. Thus, $\exists x' \in \mathcal{C}$ such that $x' \notin \mathtt{Output}(\mathcal{T})$.

Let us illustrate this result on the following critical example.

Example 2 (Cont): Assume again the propositional knowledge base $\Sigma = \{x, \neg x \wedge y, z\}$. Argumentation systems of the previous category may return either $\mathcal{E}_1$ or $\mathcal{E}_2$ (not both) such that $\mathtt{Base}(\mathcal{E}_1) = \{x, z\}$ and $\mathtt{Base}(\mathcal{E}_2) = \{\neg x \wedge y, z\}$. In the first case, $\mathtt{Output}(\mathcal{T}) = \mathtt{CN}(\{x, z\})$. Thus, $x \in \mathtt{Output}(\mathcal{T})$ while $\neg x \notin \mathtt{Output}(\mathcal{T})$. In the second case, $\mathtt{Output}(\mathcal{T}) = \mathtt{CN}(\{\neg x \wedge y, z\})$, thus $\neg x \in \mathtt{Output}(\mathcal{T})$ while $x \notin \mathtt{Output}(\mathcal{T})$. Both cases are undesirable since there is no reason to privilege x over $\neg x$ and vice versa. Remember the case where x stands for "sunny day" and y for "my dog is sick".

Multiple Extensions. Let us now tackle the second category of attack relations: the ones that may lead to multiple preferred semantics. Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an AS s.t. $\mathcal{R} \in \Re_m$ and let $\mathtt{Ext}_p(\mathcal{T}) = \{\mathcal{E}_1, \ldots, \mathcal{E}_n\}$ such that $n \geq 1$. We have seen previously that each preferred extension gives birth to a consistent subbase of Σ. This subbase may be either maximal (for set inclusion) or not. Moreover, the subbases of some extensions of the same system may be maximal while those of the remaining extensions not. In what follows, we study three possible cases.

Case 1: In this case, an argumentation system has at least one preferred extension whose corresponding base is not maximal (i.e. $\exists \mathcal{E} \in \mathtt{Ext}_p(\mathcal{T})$ such that $\mathtt{Base}(\mathcal{E}) \notin \mathtt{Max}(\Sigma)$). The output set may be counter-intuitive since some priority will be given to some formula. Let us consider the following example.

Example 3. *Let us consider the following propositional knowledge base that contains four equally preferred formulas: $\Sigma = \{x, x \to y, z, z \to \neg y\}$. It can be checked that* $\mathtt{Free}(\Sigma) = \emptyset$. *An argumentation system that fits in Case 1 would have for instance, two preferred extensions $\mathcal{E}_1$ and $\mathcal{E}_2$ such that* $\mathtt{Base}(\mathcal{E}_1) = \{x, x \to y, z\}$ *and* $\mathtt{Base}(\mathcal{E}_2) = \{z, z \to \neg y\}$. *Note that* $\mathtt{Base}(\mathcal{E}_1) \in \mathtt{Max}(\Sigma)$ *while* $\mathtt{Base}(\mathcal{E}_2) \notin \mathtt{Max}(\Sigma)$. *The output of this system is* $\mathtt{Output}(\mathcal{T}) = \mathtt{CN}(\{z\})$. *Thus, $z \in$* $\mathtt{Output}(\mathcal{T})$ *while the three other formulas of Σ are not elements of* $\mathtt{Output}(\mathcal{T})$. *This result is unjustified since all the formulas of Σ are involved in the conflict and are equally preferred.*

Case 2: The bases of all the preferred extensions of an argumentation system are maximal (for set inclusion). However, not all the maximal consistent subbases of Σ have a corresponding preferred extension (i.e. $\forall \mathcal{E} \in \mathtt{Ext}_p(\mathcal{T})$, $\mathtt{Base}(\mathcal{E}) \in \mathtt{Max}(\Sigma)$ and $|\mathtt{Ext}_p(\mathcal{T})| < |\mathtt{Max}(\Sigma)|$). The same problem described in Case 1 is encountered here. Indeed, some formulas are privileged over others in an ad hoc way. Let us consider the following example.

Example 3 (Cont): Let us consider again the knowledge base of Example 3. An argumentation system that fits in Case 2 would have for instance, two preferred extensions $\mathcal{E}_1$ and $\mathcal{E}_2$ such that $\mathtt{Base}(\mathcal{E}_1) = \{x, x \to y, z\}$ and $\mathtt{Base}(\mathcal{E}_2) = \{x, x \to y, z \to \neg y\}$. Note that both subbases are maximal. It is easy to check that $x, x \to y \in \mathtt{Output}(\mathcal{T})$ while $z, z \to \neg y \notin \mathtt{Output}(\mathcal{T})$. This result is again unjustified.

Case 3: The bases of all the preferred extensions of an argumentation system are maximal (for set inclusion). Moreover, any maximal consistent subbases of Σ has a corresponding preferred extension in the argumentation system (i.e. $\forall \mathcal{E} \in \mathtt{Ext}_p(\mathcal{T})$, $\mathtt{Base}(\mathcal{E}) \in \mathtt{Max}(\Sigma)$ and $|\mathtt{Ext}_p(\mathcal{T})| = |\mathtt{Max}(\Sigma)|$). The outputs of such systems are exactly the common conclusions that are drawn under CN from the maximal consistent subbases of Σ.

Property 3. *Let $\mathcal{T} = (\mathtt{Arg}(\Sigma), \mathcal{R})$ be an AS s.t. $\mathcal{R} \in \Re_m$. If $\forall \mathcal{E} \in \mathtt{Ext}_p(\mathcal{T})$, $\mathtt{Base}(\mathcal{E}) \in \mathtt{Max}(\Sigma)$ and $|\mathtt{Ext}_p(\mathcal{T})| = |\mathtt{Max}(\Sigma)|$, then $\mathtt{Output}(\mathcal{T}) = \bigcap \mathcal{S}_i$ where $\mathcal{S}_i \in \mathtt{Max}(\Sigma)$.*

Proof. This follows from the definition of $\mathtt{Output}(\mathcal{T})$ and the fact that for each preferred extension $\mathcal{E}$, $\mathtt{Concs}(\mathcal{E}) = \mathtt{CN}(\mathtt{Base}(\mathcal{E}))$.

It is worth noticing that this output corresponds to the *universal consequences* developed in [19] for handling inconsistency in propositional knowledge bases. Thus, argumentation systems of this category generalize the coherence-based approach to any logic. Consequently, they inherit its problems, namely the one described in Example 1 (missing intuitive conclusions). It is also worth recalling that there exist attack relations that lead to this result. Assumption attack developed in [12] is one of them. Indeed, any argumentation system that use this relation will have the output described in Property 3. Finally, we have shown in another paper that the stable extensions of any argumentation system (that satisfies consistency and closure under sub-arguments) return maximal consistent subbases of Σ. We show next that when all the maximal subbases of Σ have a corresponding stable extension in a system, then this latter is certainly coherent, i.e., its preferred extensions are stable ones.

Attack relation	Cases	Output	Problem
$\mathcal{R} \in \Re_u$	$\texttt{Ext}_p(\mathcal{T}) = \{\emptyset\}$ $\texttt{Ext}_p(\mathcal{T}) = \{\texttt{Arg}(\texttt{Free}(\Sigma))\}$ $\texttt{Ext}_p(\mathcal{T}) = \{\mathcal{E}\}$ and $\texttt{Arg}(\texttt{Free}(\Sigma)) \subseteq \mathcal{E}$	$\emptyset$ $\texttt{CN}(\texttt{Free}(\Sigma))$ $\texttt{CN}(\mathcal{S}), \mathcal{S} \in \texttt{Cons}(\Sigma)$	M M U
$\mathcal{R} \in \Re_m$	$\exists \mathcal{E}_i$ s.t. $\texttt{Base}(\mathcal{E}_i) \notin \texttt{Max}(\Sigma)$ $\forall \mathcal{E}_i, \texttt{Base}(\mathcal{E}_i) \in \texttt{Max}(\Sigma)$ and $\lvert\texttt{Ext}_p(\mathcal{T})\rvert < \lvert\texttt{Max}(\Sigma)\rvert$ $\forall \mathcal{E}_i, \texttt{Base}(\mathcal{E}_i) \in \texttt{Max}(\Sigma)$ and $\lvert\texttt{Ext}_p(\mathcal{T})\rvert = \lvert\texttt{Max}(\Sigma)\rvert$	$\bigcap_{i=1.k} \texttt{CN}(\mathcal{S}_i), \{\mathcal{S}_1, \ldots, \mathcal{S}_k\} \subseteq \texttt{Cons}(\Sigma)$ $\bigcap_{i=1.k} \texttt{CN}(\mathcal{S}_i), \{\mathcal{S}_1, \ldots, \mathcal{S}_k\} \subset \texttt{Max}(\Sigma)$ $\bigcap \texttt{CN}(\mathcal{S}_i), \mathcal{S}_i \in \texttt{Max}(\Sigma)$	U U M

Fig. 1. Outcomes under preferred semantics (M stands for missing conclusions and U for undesirable ones)

Proposition 12. *Let* $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ *be an AS over a knowledge base* Σ *s.t.* $\mathcal{R} \in \Re_p$. *If* $\lvert\texttt{Ext}_s(\mathcal{T})\rvert = \lvert\texttt{Max}(\Sigma)\rvert$, *then* $\mathcal{T}$ *is coherent.*

Proof. Let $\mathcal{T} = (\texttt{Arg}(\Sigma), \mathcal{R})$ be an AS over a knowledge base Σ such that $\mathcal{R} \in \Re_p$. By definition of stable semantics, $\texttt{Ext}_s(\mathcal{T}) \subseteq \texttt{Ext}_p(\mathcal{T})$. Assume now that $\mathcal{E} \in \texttt{Ext}_p(\mathcal{T})$. From Proposition 2, $\texttt{Base}(\mathcal{E})$ is consistent. Thus, $\exists \mathcal{S} \in \texttt{Max}(\Sigma)$ such that $\texttt{Base}(\mathcal{E}) \subseteq \mathcal{S}$. From Property 4 in [3], it holds that $\texttt{Arg}(\texttt{Base}(\mathcal{E})) \subseteq \texttt{Arg}(\mathcal{S})$. However, $\mathcal{E} \subseteq \texttt{Arg}(\texttt{Base}(\mathcal{E}))$ and $\texttt{Arg}(\mathcal{S}) \in \texttt{Ext}_s(\mathcal{T})$. Thus, $\mathcal{E} \subseteq \texttt{Arg}(\mathcal{S})$ where $\texttt{Arg}(\mathcal{S}) \in \texttt{Ext}_p(\mathcal{T})$. This contradicts the fact that $\mathcal{E}$ is a preferred extension, thus maximal.

The results of this section show that reasoning under preferred semantics is not recommended. Figure 1 summaries the different outputs that may be encountered under this semantics.

5 Conclusion

In this paper, we characterized for the first time the outcomes of argumentation systems under preferred semantics. To the best of our knowledge there is no work that tackled this issue. In [8], the author studied the outcomes of a very particular system under stable semantics. In [14], the authors focused on argumentation systems that are grounded on propositional logic and studied the properties of various systems using specified attack relations. The focus was mainly on the satisfaction of rationality postulates. In this paper, we assume abstract logic-based argumentations in which neither the underlying logic nor the attack relations are specified. This abstraction makes our results more general and powerful. Moreover, among all the possible instantiations of this setting, we considered those that satisfy some basic rationality postulates. Indeed, systems that violate those postulates should be avoided as they certainly lead to undesirable results. A first important result consists of delimiting the maximum number of preferred extensions a system may have. We have shown that if the knowledge base under study is finite, then any argumentation system buit over it has a finite number of preferred extensions. Then, we have shown that from each preferred extension, a (maybe maximal) consistent subbase of the knowledge base is computed. This subbase contains all the formulas that are not involved in any conflict. We have then shown that in the best case, reasoning under preferred semantics may lead to missing some interesting conclusions. This is mainly due to the fact that argumentation systems are syntax-dependent. Moreover, they coincide with the coherence-based approach [4,19], thus inherit all its weaknesses. In the worst case, preferred semantics will lead to undesirable conclusions. The main problem here is that the attack relation defines some "artificial" priorities between the formulas of the knowledge base leading to ad hoc outputs. The only good news is that preferred semantics performs better than stable one in the sense that it guarantees the inference of the free formulas (i.e., the formulas that are not involved in any conflict).

To sum up, we have shown (for the large class of argumentation systems we discussed) that preferred semantics should be avoided.

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