A Traffic Cellular Automaton with Time to Collision Incorporated

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Abstract. We present a new traffic CA model focused on an estimated time for a following vehicle to catch up with the one ahead (Time-to-Collision: TTC) , and investigate characteristics of the model with the simulation. We also analyze analytically the possibility of a collision between two cars in this model. The model is simulated under open boundary conditions and each car is parallel-updated. We draw some fundamental diagrams of the traffic flow with the simulation. In this figure, we find two distinct phases: a free flow and wide moving jam. And between the two phases, the region where the dots spread sparsely is seen clearly. In addition to this, by using different values of an important parameter, we can see the several patterns of the trajectory of vehicles. Based on these findings, we believe it is possible for this model to reproduce synchronized flow.

Keywords: TTC, Time-to-Collision, CA, cellular automaton, synchronized flow.

1 Introduction

Many researchers have studied the dynamics of traffic scientifically and proposed number of statistical physics models. Traffic flow is a complex system that can show asymptotic behaviors such as self-organized criticality and a phase transition from free flow phase to congestion phase. Due to its flexibility for modifying capability of capturing its self-organized criticality, the cellular automaton (CA) has been used successfully in modeling real behavior of traffic flow, and has become a popular tool for management of traffic [1,2,3].

The NaSch (Nagel-Schreckenberg) model [1] and the VDR (Velocity-Dependent-Randomization) model [2] were proposed in 1992 and 1998 respectively. The NaSch model is known as the model which is able to reproduce both a free flow and congestion state by increasing the possible sets of velocity value that vehicles can take. The VDR model can display a meta-stable state by introducing the slow-to-start effect. In addition to this, S-NFS model [4] proposed in 2006 shows a meta-stable state more clearly by forecasting a possible movement

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of a car ahead. And of course, there are many other mathematical models that reproduce such a behavior in a traffic flow.

In 2002, Kerner [5,6,7] proposed "Three-phase theory", which states that traffic flow has three phases; a free flow, synchronized flow and wide moving jams. He observed the existence of these three phases, and also proposed the models which can show all the phases [8].

Synchronized flow can be characterized as a sparsely clustered dots in the fundamental diagram of density and flow rate, while other phases, such as a free flow and wide moving jams are shown as dense packed clusters. Therefore, in synchronized flow, the average values of density and flow rate largely vary in time, and the average velocity can be spread from 20 km/h to 60 km/h. In the synchronized flow state, there are both the section where some small sized traffic jam occur and the section of cars moving smoothly. Synchronized flow is visualized as a phenomenon that these sections run waving through backward.

A concept of Time-to-Collision (TTC) was suggested by Lee [9] in 1976. The formula of TTC is described by the distance between a vehicle and the vehicle in front divided by the relative velocity of the two vehicles. It has been often used as the reference index for the risk of crash in a traffic model. However, TTC is rarely used to study a phenomenon of traffic jams but instead used to evaluate a safety of the traffic system as a whole [10,11]. In this paper, we regard TTC as a time allowance until the driver catches up to the car in front. Since TTC has a simpler formulation, it is easier to model behaviors of human drivers in real driving situation.

In this paper, we explain how vehicles run in the model considering the idea of TTC and also explain what is the condition of the parameters of the model to avoid any collision of vehicles in section 2. In section 3, we discuss the reproducibility of synchronized flow with figures of fundamental diagrams and time-space diagrams obtained by a few numerical simulations. Then, we conclude the findings and list what need to work in the section 4.

2 The Model with Time to Collision Incorporated

2.1 The Algorithm of the Model

We construct a model in which TTC is considered. The procedure is written as follows.

R0 random braking parameter: $@p = \begin{cases} p_0 \ (d_n^{t-1} = 0) \\ p_d \ (d_n^{t-1} > 0 \text{ and } v_n^t < v_{\max}) \\ p_s \text{ otherwise} \end{cases}$

R1 acceleration: $v_n^{t+1} \leftarrow \begin{cases} v_n^t + 1 \ (v_n^t < v_{\max} \text{ and } v_n^t < d_n^t) \\ v_n^t \quad \text{otherwise} \end{cases}$

R2 aim velocity : $\tilde{v}_n^{t+1} \leftarrow \begin{cases} v_{n+1}^t + \lfloor \frac{d_n^t}{c} \rfloor + 1 \ (v_n^t < d_n^t \text{ and } rand() < \frac{d_n^t}{c} - \lfloor \frac{d_n^t}{c} \rfloor) \\ v_{n+1}^t + \lfloor \frac{d_n^t}{c} \rfloor - 1 \ (v_n^t \geq d_n^t \text{ and } rand() > \frac{d_n^t}{c} - \lfloor \frac{d_n^t}{c} \rfloor) \\ v_{n+1}^t + \lfloor \frac{d_n^t}{c} \rfloor \quad \text{otherwise} \end{cases}$

R3 deceleration: $v_n^{t+1} \leftarrow \min\left(v_n^{t+1}, \tilde{v}_n^{t+1}\right)$
R4 random braking $v_n^{t+1} \leftarrow \max\left(v_n^{t+1} - 1, 0\right)$ with probability p
R5 moving: $x_n^{t+1} \leftarrow x_n^t + v_n^{t+1}$

x_n^t and v_n^t are the location and velocity of the n th vehicle at time t respectively, and the $n+1$ th vehicle represents the vehicle in front of n th vehicle. d_n^t is the gap between the n th car and the $n+1$ th car, that is $d_n^t = x_{n+1}^t - x_n^t - 1$. Figure 1 shows a schema for the variables in the model. $\lfloor\cdot\rfloor$ is floor function.

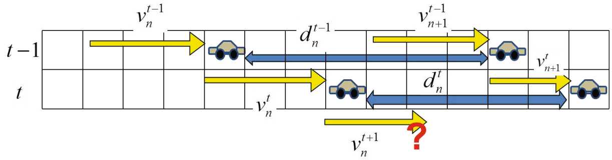

Fig. 1. schema for the variables. v_n^{t+1} is determined by x_n^t, v_n^t and d_n^t

In this model slow-to-start effect [2,4,13] is considered by making p to have three possible different values at R0. Cars accelerate gradually not to exceed a limiting speed at R1. In addition, we consider a drivers' mind that they tend to avoid a collision. When the distance to the car in front is short for their own velocity $v_n^t \geq d_n^t$, they don't accelerate. Likewise, when the distance between two cars is short at R2, they take an aim velocity that is 1 smaller value than it is not. We take the idea of TTC into R2. At R2, each driver adjusts the velocity to make sure that one has more than c seconds to catch up to a car in front. Suppose a time to catch up the car in front is t_{TTC}. Then using relative velocity $(v_n^t - v_{n+1}^t)$, we can write

$$\left(v_n^{t+1} - v_{n+1}^t\right) t_{TTC} = d_n^t. \tag{1}$$

From this equation, the range of v_n^{t+1} that makes t_{TTC} more than c (second) is

$$v_{n+1}^t \leq v_n^{t+1} \leq v_{n+1}^t + \frac{d_n^t}{c}. \tag{2}$$

Therefore, the velocity of each car is within this range. Each driver looks at the velocity of the car in front one time step (1 second) ahead. It is just the update time step of CA.

Drivers follow the optimal velocity $v_n^{t+1} = v_{n+1}^t + \frac{d_n^t}{c}$ at R2. This model only uses integer, so rational numbers like $\frac{d_n^t}{c}$ is linearly interpolated in this model. If a car applies brakes, then a velocity $\tilde{v}_n^{t+1}$ is again computed in R2, then used in R3. In this model the maximal acceleration is one cell per one step (7.5 m/s^2). Therefore, even if acceleration more than one cell per one step is required in order to reach the optimal velocity, cars can't accelerate up to the value. This is why we use the value $\tilde{v}_n^{t+1}$ only when $\tilde{v}_n^{t+1}$ becomes less than v_n^{t+1} at R3. Drivers apply brakes randomly at R4 and the finally, cars advanced at the step R5.

2.2 Risk of Collision

If the velocity is larger than the distance to a car in front, the danger of collision increases. In this situation, if the car ahead does not move at the next time ($v_{n+1}^{t+1} = 0$), a collision occurs. However, it is possible to have $v_n^{t+1} \geq d_n^t$ because there is no condition between $v_{n+1}^t + \frac{d_n^t}{c} + 1$ and d_n^t. Therefore, it is not clear if this model is a collision free or not.

For a car not to collide with a car in front at time t, it is sufficient if a condition $v_n^{t+1} \leq d_n^t$ is satisfied. Therefore, for the model to be collision free, we find the sufficient condition is "$v_n^{t+1} \leq d_n^t$ if $v_n^t \leq d_n^{t-1}$" at any possible values of n and t. In addition, we suppose initial conditions of all cars satisfy $v_n^1 \leq d_n^0$ for all n.

Before going further, we write down the relation between the velocity of the front car and that of the following car. We reduce R1, R2 and R3 into the form of

$$v_n^{t+1} = \begin{cases} \min\left(v_n^t + 1, v_{n+1}^t + \lfloor \frac{d_n^t}{c} \rfloor + 1\right) & (v_n^t < v_{\max} \text{ and } v_n^t < d_n^t) \\ \min\left(v_n^t, v_{n+1}^t + \lfloor \frac{d_n^t}{c} \rfloor + 1\right) & \text{otherwise.} \end{cases} \tag{3}$$

Then, the changes of a distance between two cars is equal to the relative velocity, that is, $d_n^t - d_n^{t-1} = v_{n+1}^t - v_n^t$ always holds (Fig 1 helps you understand easier). Now, we denote $d_n^t - d_n^{t-1}$ by Δd_n^t. Then, we substitute these equations into a equation (3) and obtain

$$v_n^{t+1} = \begin{cases} v_n^t + 1 + \min\left(0, \Delta d_n^t + \lfloor \frac{d_n^t}{c} \rfloor\right) & (v_n^t < v_{\max} \text{ and } v_n^t < d_n^t) \\ v_n^t + \min\left(0, \Delta d_n^t + \lfloor \frac{d_n^t}{c} \rfloor + 1\right) & \text{otherwise.} \end{cases} \tag{4}$$

Actually there is the case of a driver slowing down one cell per second speed at R4, but in this subsection, we consider only the case that no car brakes at R4. Since no car brakes at R4, it increases the risk of collision when a speed value increases. Thus, if we look at the case with the highest risk and find that no collision between vehicles occurs, then there will be no collision among vehicles in any other cases. Supposing the equation (4) holds, we discuss a statement if $v_n^{t+1} \leq d_n^t$ when $v_n^t \leq d_n^{t-1}$ at any n and any t. First, in addition to the equation (4), if $v_n^t < d_n^t$ is satisfied, the statement is true. Because $v_n^{t+1} \leq v_n^t + 1$ for any t and any n, and then, $v_n^t < d_n^t \Leftrightarrow v_n^t + 1 \leq d_n^t$.

Secondly, we discuss the case $v_n^t = d_n^t$. In this case, we can obtain the following: $v_n^{t+1} = v_n^t + \min\left(0, \Delta d_n^t + \lfloor \frac{d_n^t}{c} \rfloor + 1\right) \leq v_n^t$. Using this inequality, it is easy to see that $v_n^{t+1} \leq v_n^t = d_n^t$.

Finally, we look at the case $v_n^t > d_n^t$. In this case, whether it is possible to crash to the vehicle ahead depends on a condition of a parameter c. Figure 2 shows the relevance of d_n^t and v_n^t. Drivers can take the combination of { d_n^t, v_n^t } within the region colored in gray. In the black region, the condition that $v_n^{t+1} > d_n^t$ when $v_n^t \leq d_n^{t-1}$ is satisfied because of the following reason. The inequalities $v_n^t \le d_n^{t-1}$ and $v_n^{t+1} > d_n^t$ can be put into $v_n^t \le d_n^t - \Delta d_n^t$ and $v_n^t > d_n^t - \Delta d_n^t - \lfloor \frac{d_n^t}{c} \rfloor$, respectively. In the black region, both $v_n^t \le d_n^{t-1}$ and $v_n^{t+1} > d_n^t$ are satisfied. So, the sufficient condition to be collision free "$v_n^{t+1} \le d_n^t$ if $v_n^t \le d_n^{t-1}$" is not satisfied. In order to know the condition of c, we see whether there exists a common region between the gray and black area. As the case there are no such region, the sufficient condition $v_n^{t+1} > d_n^t$ when $v_n^t \le d_n^{t-1}$ always holds in the gray region, therefore we find any collision does not occur. To check that, it is sufficient to focus on a line $v_n^t = v_{\max}$ and the point $(c,\ c - \Delta d_n^t)$, which is the coordinate both d_n^t and v_n^t are smallest in the black region (the bigger point in Fig 2). We get the condition by comparing the value $c - \Delta d_n^t$ and $v_{\max}$, as follows:

$$c > \Delta d_n^t + v_{\max} = v_{\max} - 1. \tag{5}$$

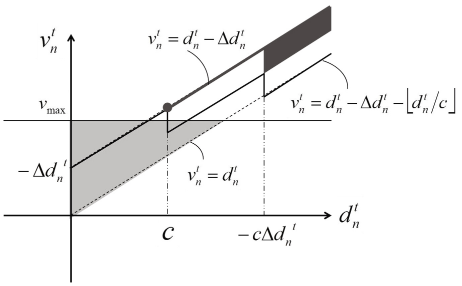

Fig. 2. The relevance of d_n^t and v_n^t: It shows the region of the combination of { d_n^t, v_n^t } drivers can take (colored in gray), and also shows the region where it is possible to crash to the vehicle ahead, that is, $v_n^{t+1} > d_n^t$ when $v_n^t \le d_n^{t-1}$ (colored in black)

3 Simulation Results

We simulate this model with the parameter $L = 2000$ cells (15 km), open boundary, inflow ratio $\alpha = 0.5$, outflow ratio $\beta = 0.5$, $v_{\max} = 5$ (= 135km/h), $p_0 = 0.75$, $p_d = 0.375$, $p_s = 0.05$, the target of TTC $c = 4.1$, 5, 7, 15, and the calculation time step is 54000 (1 step = 1 second). From the subsection 2.2, using the condition 5, it is found that all cases are collision free. In the case of a car not running out from the end of the course in probability β, the car is on the edge of the road at this time and goes out at the next time. In this situation, we can understand that a new car is entering from branch road. Figure 3 shows the fundamental diagram of this traffic model. The density and the flux observed at the observation points (l) located at every 100 cells. The density is the number of cars located on the points averaged over 300 seconds. The flux is the average number of cars that pass the observation points per time step. The equations are given as follows:

$$\text{Density: } \frac{1}{T}\sum_t^T \sum_n \mathbf{1}\left(x_n^t = l\right)$$
$$\text{Flow: } \frac{1}{T}\sum_t^T \sum_n \mathbf{1}\left(x_n^t \le l \cap x_n^t > l\right).$$

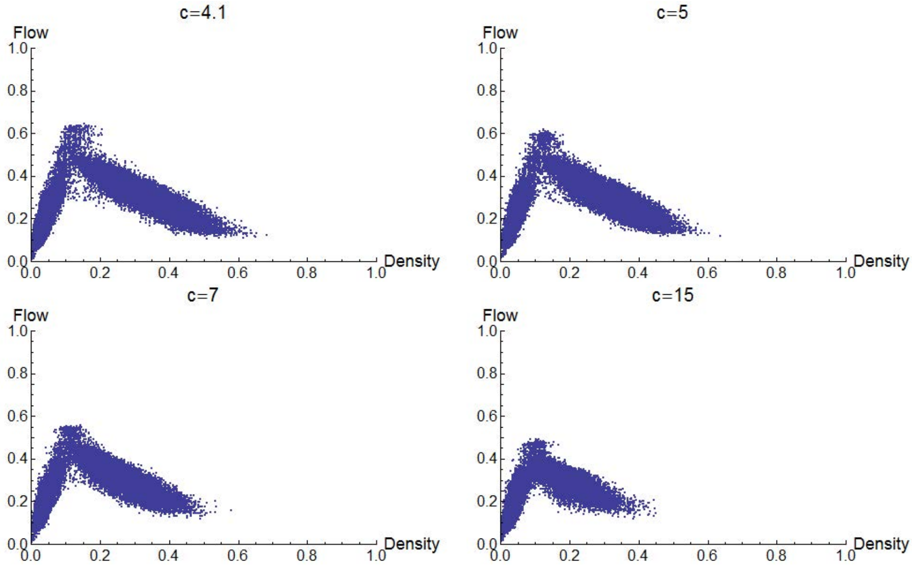

Fig. 3. The fundamental diagrams of identical vehicles in the proposed model. These diagrams are drawn on the same simulation condition except the values of parameter c.

We can see both the free flow and congestion phase in all of diagrams of figure 3. In addition to this, the meta-stable state is seen clearly. It may be derived by the slow-to-start effect. Furthermore, not only these two phases, but also there are dots spread sparsely in the boundary of two phases. They are seen especially in the case of $c = 4.1$ and $c = 5$. In general, the average velocity of vehicles in synchronized flow is between 20 km/h and 60 km/h. In order to calculate the average value in this system, we use the gradient between each of the dots and origin. The gradients just correspond to the average velocity of vehicles. For $c = 4.1$ and $c = 5$ (Fig 3), the average velocity of vehicles at the dots spread sparsely, for example the point in the area [0.15,0.2] $\times$ [0.3,0.4], is surely between 20 km/h and 60 km/h. The result suggests the reproducibility of synchronized flow in this model.

Figure 4 shows the plot of cars' position among the amount of time. In all of diagrams, each car's speed is fluctuated, and there are both the spot where cars have each short distance to the car in front and the spot where cars have each long distance. In this state, the points in the two-dimensional of car's density and flux are ranged widely by time and space. These facts obtained by the traffic simulation using this proposed car following model are some of the characters of the synchronized flow. Moreover, there are different patterns by the value of parameter c. In the upper left panel ($c = 4.1$) of figure 4, the shape of whole

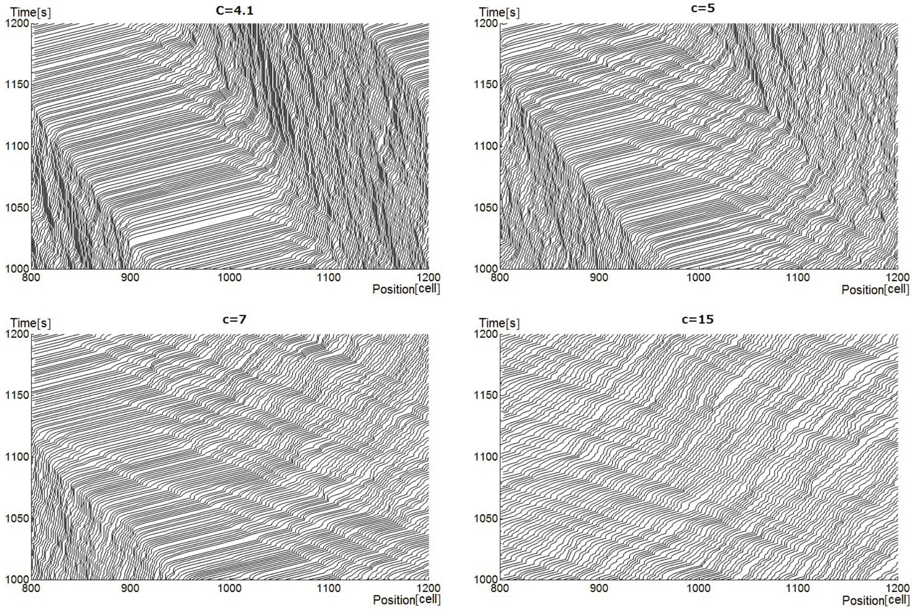

Fig. 4. Time-space diagrams: These figures show the trajectories of vehicles which run between 800 cell and 1000 cell from 1000 seconds to 1200 seconds. The horizontal and vertical axes represent the position and time of vehicle, respectively. Inflow ratio is $\alpha = 0.5$ in all the results. Several patterns of traffic flow are seen in these space-time diagrams.

trajectory of vehicles looks like so-called general pattern (GP) [8]. The other cases of c don't show the pattern like GP, but show several ones that are different from each other. The findings suggest that the model can reproduce some patterns of synchronized flow by changing the parameter c.

4 Conclusion

In this paper, we presented a new CA model considering TTC and performed traffic simulations of identical vehicles in the model. We found that a free flow, meta-stable state and congestion state, which are some of the fundamental properties of traffic flow, were clearly observed in the simulation of this model. In addition to this, we find another area. We calculated the average velocity of vehicles in the area and looked at the the trajectory of vehicles on a inflow parameter. As the consequence, we successfully reproduced some characteristics of the synchronized flow. Furthermore, by using different values of the parameter c, we can see the several patterns of the trajectory of vehicles. It suggests that the proposed model can show different synchronized patterns by changing the parameter c. c is a physically meaningful parameter based on the human mind in safety driving. Thus, we hope that the study about the relevance between these mind and the synchronized flow will proceed. In our future work, it is required to investigate whether it is possible for this model to reproduce the synchronized flow phase in detail.

In addition to this, it is also required to investigate the relation between a model proposed by HK Lee et al. [13] and our proposed model. They reported their model based on the BL iBrake Lightj model reproduces several patterns of the synchronized flow phase which can be seen in real traffic flow [14]. Furthermore, Kerner et al. presented a CA model which explains the physics of synchronized flow [15]. We are interested in comparing with this model.

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