The Dynamics of Disproportionality Index for Cellular Automata Based Sociophysical Models

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Abstract. The cellular automata approach to the analysis of the electorate voting is presented. We show the analysis of the proportionality of elections by using different update rules for the cellular automata representing the voters space. There exist many methods of performing system update which can be generally classified into two classes: outward, when the individual's opnion is spread over his neighbors and inward when the individual is under pressure of his environment. In the paper we show the relaxation, stability and the Gallagher index value for at the successive stages of CA simulation run. We find that the majority of methods leads to similar results however few of them is promising when trying to reproduce some social phenomena.

Keywords: Cellular automata, Sociophysical models, Disproportionality.

1 Introduction

Among different characteristics of the voting process, one of the fundamental is the question about the representativeness of the results of elections. In this general notion we encompass especially the problem of proportionality of elections. There exist a lot of different numerical indexes which try to describe this effect [1]. One of the most popular is the Gallagher index

$$G = \sqrt{\frac{1}{2}(V_i^2 - S_i^2)} \tag{1}$$

In the above equation V is the percentage number of votes obtained by the given party while S is the number of mandates (seats) awarded to this party. The value of G index depends mainly on the voting system (FPTP, proportional) and the stability of political system. Some typical values can be found in our previous paper where the problem was studied using CA method [2].

In the paper we are going to study the dynamics of G index changes. We will consider the typical multi-seat constituency where 11 deputies are elected. This value corresponds to the typical constituency in Poland. For such a constituency

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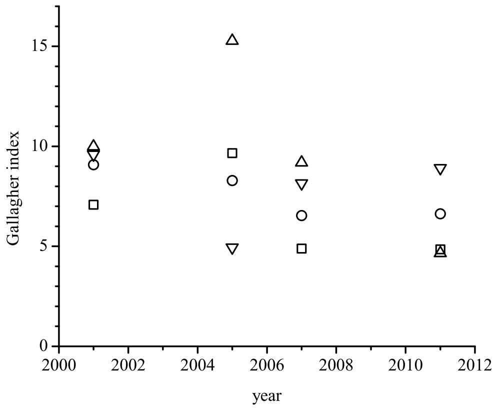

Fig. 1. The dynamics of Gallagher index changes for selected 11-mandate constituencies in Poland. Private calculations.

the disproportionality index can be also calculated and its value is higher than for the whole country. In the Fig.1 we show the changes of Gallagher index for the 4 selected polish constituencies during the last 4 elections (proportional system).

We do not call here the detailed locations of these constituencies but they significantly differ one from another according to the sociological structure of electors. We took into account only those parties which exceeded 1% of votes in the particular constituency.

It is hard to present unambiguous conclusions related to the observed results. The most stable case is represented by circles, for all others it exist minima or maxima and their positions are uncorrelated.

We will try to find whether we can find the Cellular Automata system which can, even approximately, reproduce the effects visible in the Fig.1.

2 Model

The details of the calculation model has been presented in our earlier paper [2]. Let us mention here only main features of this model as well as those assumptions which are different from the ones presented there. We use here only two-dimensional sample. Its size equals 81×99 what corresponds to the 8019 cells. There are several reasons to make such a choice. We can directly compare the results to those obtained in earlier paper, the number is divisible by 11 what enables to study the system with equal division into one-seat constituencies and finally among the samples with equal sizes we consider this one for which the stabilization is fastest.

We study the 4-state model which corresponds to the division of the space of opinions by the Nolan's diagram. Every person's opinion is located in the selected quadrant of two-dimensional plane divided according to the economic and social beliefs. Certainly, such a description should involve more detailed analysis related to the actual, exact position of person on the chart, but for simplicity reasons often just 4-state approach enabling only general localization is used. Such an approach is similar to the mentioned in [3] ACLS model (authoritarian/conservative/libertarian/socialist).

Originally we have studied four different methods of performing the assignment of mandates(seats) to the parties according to the number of votes they obtained and according to the political system. There were: (1) FPTP, (2) two-round majority system, (3) d'Hondt system and (4) d'Hondt with the threshold 5%. It turned out that the results obtained by using similar methods of seats assignment (1 and 2, 3 and 4) do not differ much therefore in the current paper we perform the calculations only for (1) and (3) methods.

For the proportional system of mandates assignment, the further method of dividing the sample into constituencies is certainly not important. We want however to study the influence of such division on the results obtained for majority system. Also following our earlier study [2] we introduce four types of divisions according to the two features: number of cells in each constituency and their compactness. In the further part of paper this option will be also restricted but some initial plots are prepared for all four possibilities.

The crucial algorithm determining the behavior of system is the method of performing the cell content update. Earlier we applied only the so-called Stauffer's rule III [4]. In the paper authors presented three rules for two-dimensional CA update. They are indeed the tries of generalization of seminal Sznajd's scheme [5] onto the two-dimensional case. With the first two schemes it is assumed that a structure constructed of several cells is considered. According to the rule I we consider the "plaquette" 2×2 and the opinion of cells from inside the plaquette is expanded onto 8 neighboring cells only if all cells has the same state. According to the version a (rule Ia) the same state as possessed by the plaquette cells is expanded while with the rule Ib we use the opposite one. Since in the Stauffer's paper only two-state CA was used, here we have to redefine the notion opposite. As the opposite state we understand the state which lies in the opposite (not in the neighboring) quadrant. By using the rule II we consider the 1×2 set of cells trying to persuade their opinions into 6 neighbors. Here, the distinction into 3 possibilities (IIa, IIb, IIc) is related to the behavior of CA in situation where two initial states are different. If initial states are same always their state is copied, it doesn't exist the situation when the oppositions are copied. The rule III is the direct enhancement of typical one-dimensional behavior [5] for the 2×2 subset of cells where the chosen state influences the cells in the same row and column.

The Sznajd/Stauffer's update is the example of method where the opinion is spread into its neighbors. In order to take into account also another class of update algorithms we take into account the Glauber mechanism which considers the state cell as dependent on the state of its neighbors. The discussion about the difference between both techniques, usually called outward/inward (or outflow/inflow) can be often observed [6,7]. The Glauber mechanism is connected to the physical image of interactions between different states when the probability of state change is given by the formula

$$prob = \frac{1}{1 + exp(\frac{\Delta E}{k_B T})}. \tag{2}$$

It shows the next difference between both methods used. While Stauffer's rules are purely deterministic, the Glauber's rule is probabilistic. As it can be seen in the formula 2 we have some physically interpreted quantities like energy change, Boltzmann constant and temperature. Actually the energy change (ΔE) has to be calculated in the way similar to some magnetic multi-state models (e.g Potts model). For simplicity let us consider the formula which enables to calculate the energy change when the state s_i is changed

$$\Delta E = J * \sum_j \Delta s_i * s_j \tag{3}$$

where summation is over neighboring states and Δs_i equals 1 if we change between the states neighboring on the Nolan chart, and $\Delta s_i = 2$ if states are opposite. The dynamics of the system depends on the parameter $\beta = \frac{J}{k_B T}$ which has to be defined for every run.

Additionally we decided to study two time regimes of update. According to the first one we work always on the same array. The "Monte Carlo Sweep" is then the number of successive updates performed on the sample. In our approach the number equals the size of array. In the second method, let us call it synchronous, we try to simulate the concurrent update therefore the states of cells are set according to the array remembered at the beginning of the Monte Carlo Sweep. This mechanism is used only for Stauffer's update rules.

3 Results and Conclusions

The presentation of results we start from the analysis of the relaxation time. In the Fig. 2 there is shown the percentage number of runs which are unrelaxed after given Monte Carlo Sweeps. The plots are shown only for the Stauffer's update since for all simulations with the Glauber dynamics, independently on the β parameter the system does not relax to the steady state. The maximum number of sweeps equals 100000. The data in Fig.2 and Tab.1 are collected from the 100 independent runs for every version of rule.

In the Table 1 we summarize the data concerning the relaxation time. All values are shown in Monte Carlo sweeps. The main conclusion which follows the analysis of Fig.2 and Tab.1 is that except of just a few cases, the course of curves is similar. The relaxation point is usually about $10^3 - 10^4$, except of Stauffer's rule Ib. This effect can be expected since the lack of fixed points for this particular rule has been mentioned in earlier paper [4]. Certainly, our model is a little more complicated than the one used in the original paper due to increase of number of states. We can expect however the similar features where taking into account the existence or nonexistence of fixed points.

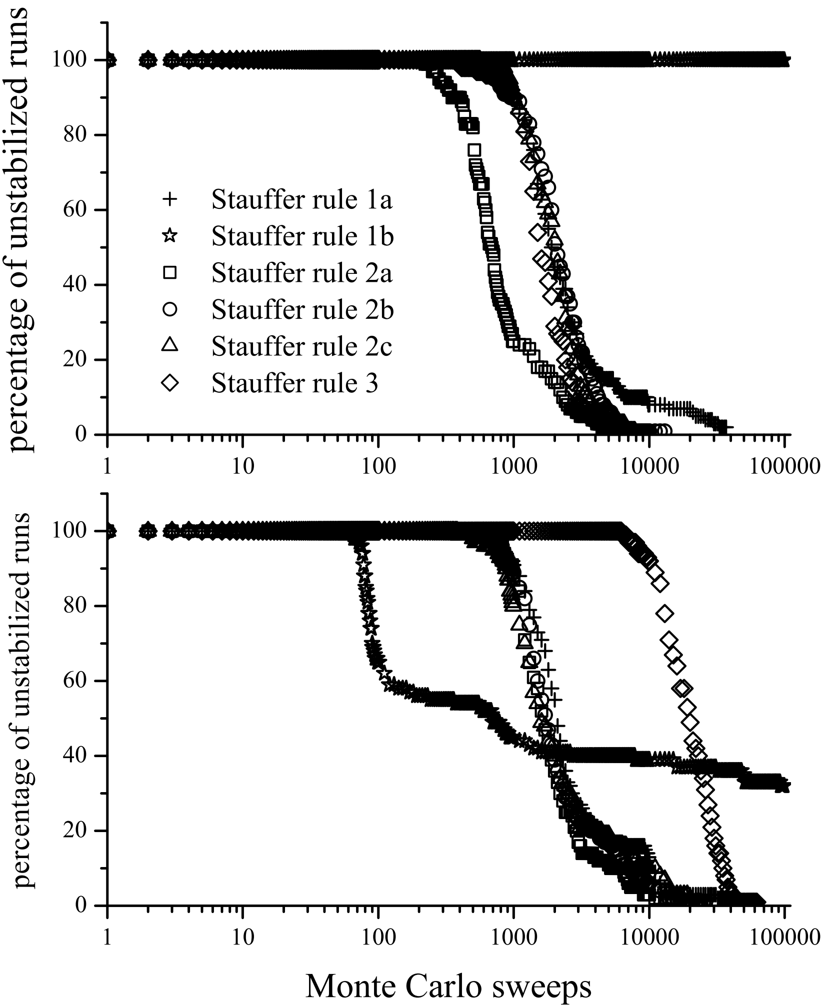

Fig. 2. The percentage number of runs which are unrelaxed after given Monte Carlo Sweeps. Upper plot - unsynchronized update. Lower plot - synchronized update.

Table 1. Average relaxation time for different versions of system update

	unsynchronized	synchronized
Stauffer rule Ia	$(4.3 \pm 7.3) * 10^3$	$(4.5 \pm 7.7) * 10^3$
Stauffer rule Ib	$---$	$(3.5 \pm 4.6) * 10^4$
Stauffer rule IIa	$(1.1 \pm 1.0) * 10^3$	$(2.5 \pm 2.8) * 10^3$
Stauffer rule IIb	$(2.5 \pm 1.7) * 10^3$	$(3.1 \pm 3.8) * 10^3$
Stauffer rule IIc	$(2.2 \pm 1.2) * 10^3$	$(3.4 \pm 4.5) * 10^3$
Stauffer rule III	$(1.84 \pm 0.9) * 10^3$	$(2.14 \pm 1.0) * 10^4$

The fixed points for the 4-state model belong to the two types: all states can be the same and the two states can occupy approximately half of cells each (please note that the size of sample is odd, so these numbers can not be equal). We do not want here to use the magnetic analogy and call the ferro- antiferro- or ferrimagnetic configurations since for the multi-state systems the description is more complicated than for simple 2-state and the additional interpretation of states should be taken into account. The majority of runs (about 80%) leads to a uniform ordering of states.

The three interesting cases which can be observed on plots are: both implementations of rule Ib (the unsynchronized case has been described before) and the synchronized update for rule III. For the synchronized update of rule Ib we obtain the relaxation and the ratio of final orderings of states conforms to the one mentioned before (80% uniform - 20% two-state). The extrapolation of the curve shows that we can expect that all runs lead to the fixed point but the time of relaxation can be even few decades longer than presented on the plot. The shape of curve for rule III presents the similar dependence as other curves but the average lifetime is significantly longer (the difference is of order of one decade).

Comparing the values from Tab.1 we can point some differences which are the effects of the change of update method. The analysis of standard deviations shows that these values are for synchronized update larger when compared to averages as for unsynchronized update. It is the effect of so-called long tails of relaxation time distributions.

The crucial interest in our paper is related to the study of disproportionality generated by different update schemes. Figures 3-5 shows the values of Gallagher index averaged over all runs which are active at the particular Monte Carlo sweep. They are not averaged over sweeps what could smooth the curves. In Fig.3 we show the results for rule III and different methods of constituency construction. The upper plot is indeed the repetition of plots from our paper [2]. The plots confirm that the crucial process influencing the result is the coherence of constituencies and not their strength. Further we will present only one plot for compact and one for spread type of constituency.

The results for different methods of update are shown in Figs.4 and 5

We can observe that qualitative behavior which was noticed when considering rule III is repeated for almost all rules visible in figures. Usually, shortly after

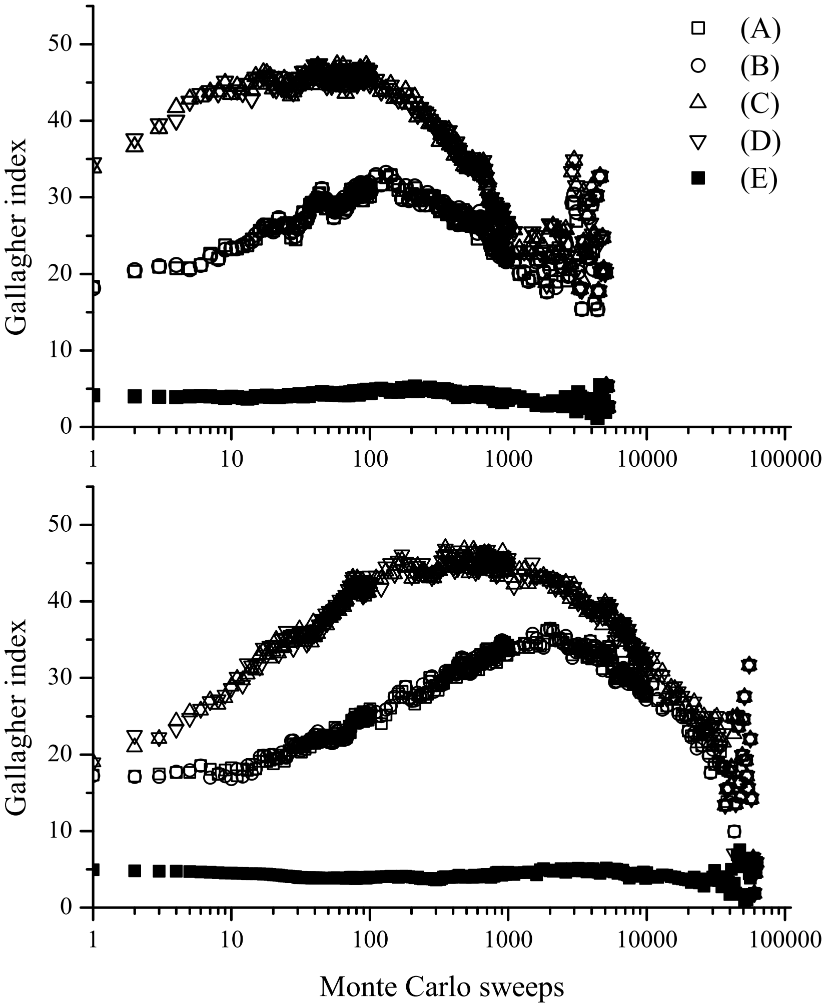

Fig. 3. The dependence of Gallagher index on the index of Monte Carlo sweep for the Stauffer's rule III. (A)-compact and equal constituencies; (B)-compact and not equal; (C)-spread and equal; (D)-spread and not equal. Upper plot - unsynchronized update. Lower plot - synchronized update.

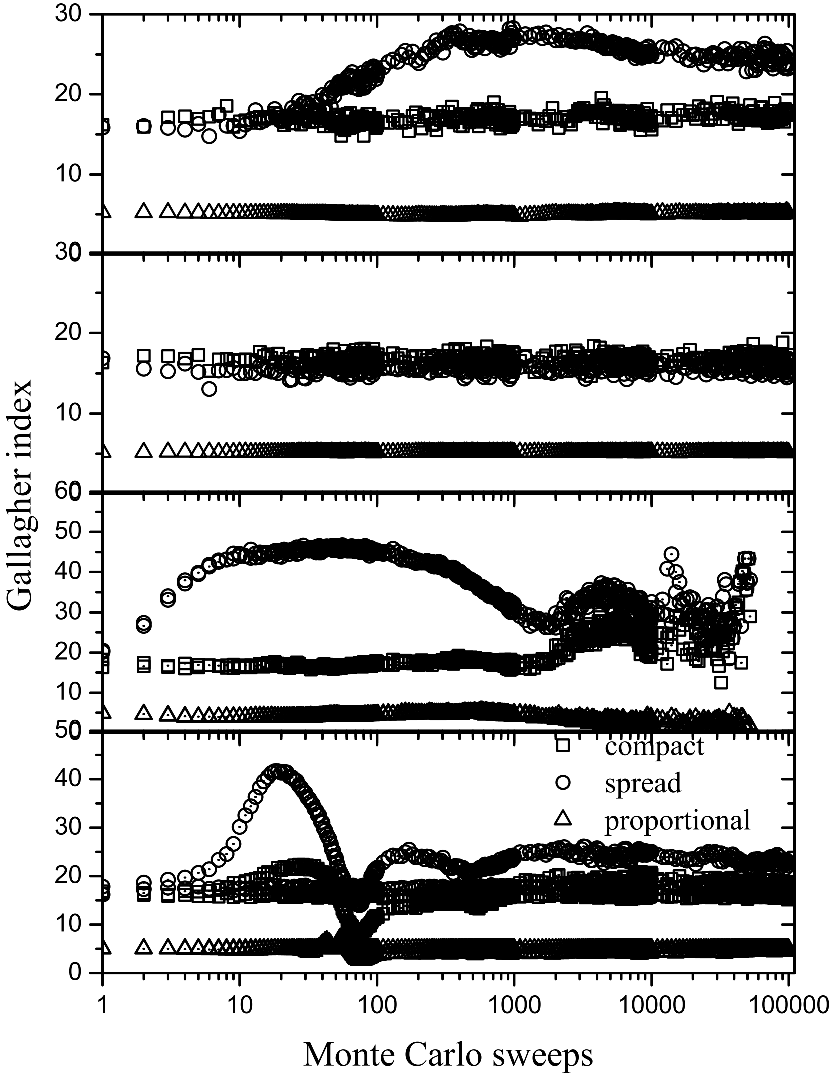

Fig. 4. The dependence of Gallagher index on the index of Monte Carlo sweep for different update rules. From upper: Glauber rule $\beta = 1$; Glauber rule $\beta = 0.5$; Stauffer rule Ia; Stauffer rule Ib. Open symbols correspond to the unsynchronized, dotted - synchronized update.

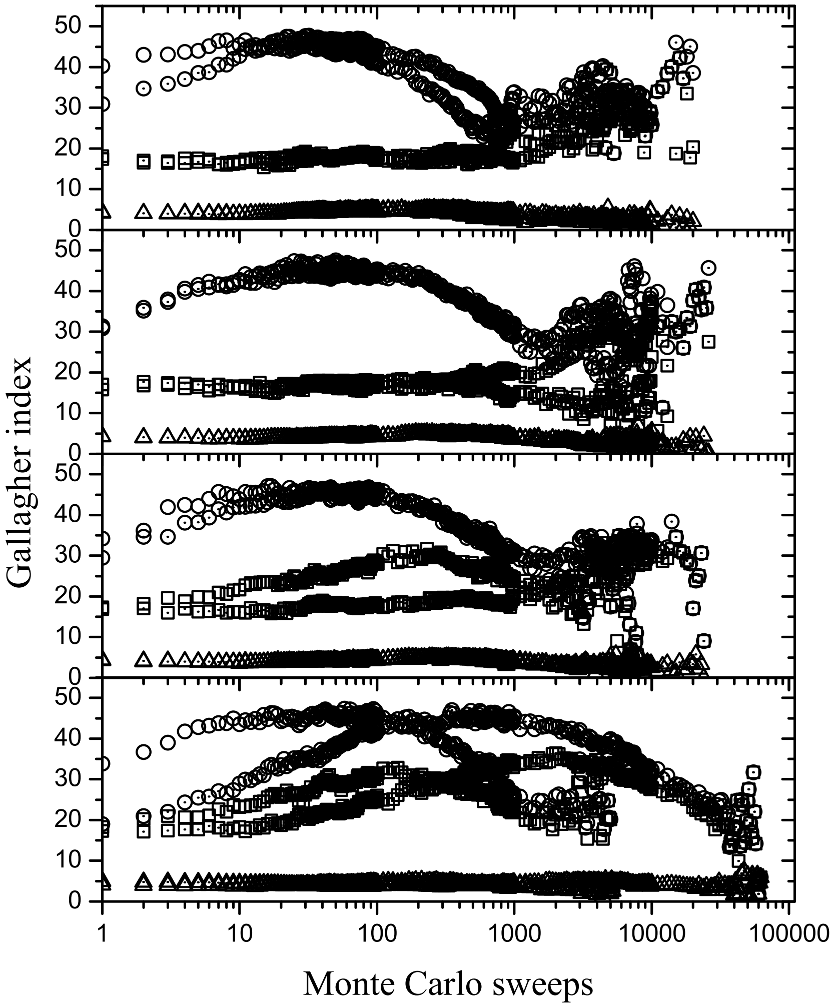

Fig. 5. The dependence of Gallagher index on the index of Monte Carlo sweep for different update rules. From upper: Stauffer rule IIa; Stauffer rule IIb; Stauffer rule IIc; Stauffer rule III. Open symbols correspond to the unsynchronized, dotted - synchronized update.

simulation starts, there exist a maximum. Afterwards the short oscillations occur just before the relaxation. Actually, it seems that these oscillations should be rather related to the still decreasing number of data which undergo averaging, that to the real effect. There certainly exist some quantitative differences. The compact division leads to the lower Gallagher values than the spread ones. The position of maximum can be for synchronized update shifted to the greater values of Monte Carlo sweep index. Sometimes the maximum does not exist or exist only for the one of update methods.

The two interesting cases are: Glauber rule for the lower temperature (greater β)and the Stauffer rule Ib. For the Glauber rule we observe the maximum (only for spread division) and further the system probably stabilizes. For the Stauffer rule (only for synchronized update) the strong oscillations are observed from the beginning of the process. Please notice that the period of these oscillations increases what can be not obvious on the plot in the logarithmic scale. We expect also that a system does not reach the state with the constant Gallagher index value. It is important that some oscillations occur here also for the d'Hondt system of seats awarding. It seems that it still exists the tendency to change it. We think that by enhancing the size of samples we can obtain the systems which can well reproduce the behavior shown in Fig.1.

It seems that when performing a tries to model the electorate behavior we have to pay more attention to these rules which do not lead to the relaxation of system and do not stabilize the values of disproportionality indexes. Among the presented rules there are two which satisfy these requirements: Glauber's rule with higher β parameter and Stauffer's rule Ib with synchronized update.

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