Designing Systematic Stable Fuzzy Logic Controllers by Fuzzy Lyapunov Synthesis

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Abstract. Fuzzy logic handles information imprecision using intermediate expressions to define assessments. Fuzzy Systems are intelligent models whose main application has been in Control Engineering applications. Stability is one of the most important issues of control systems. This determines the system to respond in an acceptable way. This work is based on the fuzzy Lyapunov synthesis in the design of fuzzy controllers, to verify the system's stability. The stability will be studied on Mamdani and Sugeno fuzzy systems .The case study presented is a system of a cylindrical tank of water, where we aim to maintain a certain level of water, which is regulated through the controls applied to the water outlet valve of the tank. The method is also tested using an inverted pendulum, which is an unstable system, which can fall at any time unless an appropriate force is applied control.

1 Introduction

One of the main areas of application of fuzzy logic has undoubtedly been the automatic process control, mainly due to the special feature of fuzzy systems that operate in the same numerical and linguistic framework.

Fuzzy systems have shown ability to resolve problems on several application domains. At present time, there is a growing interest to improve the fuzzy systems with learning and adaptation capacities.

Fuzzy systems have been successfully applied to classification problems, control and in a considerable amount of applications. On most of the cases, the key to success has been the ability of the fuzzy systems of incorporate human expert's knowledge.

One of the main problems that man has come across to in the study of the theory of dynamic systems control, is the stability problem. Throughout the years several criterions have been developed to evaluate the stability in the fuzzy controllers.

An effective method to create stable fuzzy controllers is the use of fuzzy Lyapunov synthesis method, where we can design the linguistic rules bases of such controllers.

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With time, some methods were formalized, such as the fuzzy Lyapunov synthesis [1] to measure the stability of the only control systems that were applied until the end of the 90s, which are the Type-1 fuzzy control systems.

In this work, the proposed method of fuzzy Lyapunov synthesis is described, with emphasis on its application to the systematic design of fuzzy controllers, where the objective is to achieve the stability in such controllers using this method. Where is shown that as the fuzzy Lyapunov synthesis it's a valid tool for evaluate the stability in Type-1 fuzzy control systems.

In this work the results for the design of FLC (fuzzy logic controller-FLS) applying fuzzy Lyapunov synthesis are presented, a concept that is based in the computation with words paradigm [6], with the purpose of provide evidence of the systems strength.

Also, this work we present a dynamic model for the study cases, such method will guarantee the stability when applied to the water tank case and the inverted pendulum.

2 Type-1 Fuzzy Sets and Systems

The Fuzzy sets are defined based on the operating characteristics of systems. A fuzzy set in the universe U is characterized by the membership function A (x) which takes the interval [0, 1], unlike classical sets take the value zero or one {0, 1}.

The formal definition of a fuzzy set and membership function is as follows:

"If X is a collection of objects denoted generically by x, then a fuzzy set in X is defined as a set of ordered pairs"

The fuzzy set can be represented by:

$$A = \{(\mu_A(x), x) \,/\, x \,\varepsilon\, U\} \tag{1}$$

Where μA (x) is the degree of membership. The membership functions for the fuzzy set A.

A membership function with parameters p (x) the element x is a follows:

$$\mu_A(x) = \mu_A(p_1(x), p_2(x), \ldots, p_n(x)) \tag{2}$$

An enumeration of pairs defined on discrete elements of the set is as follows:

$$A = \textstyle\sum_{x \in U} \mu_A(x)/X \tag{3}$$

Where ∑ is not a sum, but an aggregation of pairs, and $\mu_A(x)/X$ does not represent any ratio, but a couple (possible/ cell).

Each fuzzy system is associated with a set of rules with regard to IF-THEN linguistic interpretations and can be expressed as follows:

$$R^m\text{: If } u_1 \text{ is } A_1{}^m \text{ and} \ldots u_p \text{ is } A_p{}^m\text{. Then v es } B^m \tag{4}$$

With m = 1,2, ... M

And where $A^m$and B^m are fuzzy sets in $U \subset R$ (real numbers) and $V \subset R$ respectively, u= $(u_1, u_2, \ldots u_n)$

$\in U_1 x U_2 x \ldots x U_n$ and $v \in V$. and $x = x_1, x_2, \ldots, x_n \in U$e and $\in V$ are the specific numerical values of u and v, also respectively.

A Mamdani fuzzy system consists of 4 basic elements: the fuzzifier, the rule base, the knowledge base and inference system defuzzifier. While in the Sugeno fuzzy system the rule base operates differently than the Mamdani systems because the consequent of these rules is no longer a linguistic label but is a function of input that has the system at any given time.

3 Overview of the Problem

3.1 Description of Then of the Water Tank

Let us consider the problem of designing a stable fuzzy controller (FLC) for a cylindrical water tank, based on a Takagi-Sugeno controller.

The tank has an inlet and outlet pipe. It can change the valve controlling the water that is entering, but the flow going out depends on the diameter of the outlet pipe (which is constant) and the pressure in the tank (which varies with the level of water). The system has many nonlinear characteristics.

A controller for the water level in the tank needs to know the current water level and needs to be able to calibrate the valve. The input to the controller is the error in the water level (the desired water level minus the current water level) and its output is the velocity at which the valve opens or closes.

3.1.1 Dynamic Model

The dynamic behavior of the system is governed by the following differential equation:

$$\dot{x} = q(t) - p(x)u \tag{5}$$

where:

$\dot{x}$: Change in the amount of water in the tank.
q (t): is the flow of water entering the tank.
u: is the variable that controls both opening the drain valve.
p (x)u: is the flow of water leaving the tank.

The aim is to design the rule base for constructing a fuzzy controller:

$$u = u(x; p_s, q_s) \tag{6}$$

x: is the quantity of water in the tank
p_s: is the nominal output steady flow of water.
q_s: is the nominal steady flow of water intrusion.

Capable of regulating the amount of water in the tank x (t) to a desired nominal amount x_s:

$$x(t) \rightarrow x_s \tag{7}$$

The system has three modes of operation: x is low, x is normal and x is high.

3.1.2 Stability Analysis

To consider fuzzy Lyapunov synthesis, we assume the following:

- We assume that the functional relationship (5) is known.
- But p (x) is not known explicitly and the only knowledge we have of p (x) is: p (x) ≥0 for all x.
- The value p_s= p (x_s) is known.

Theorem 1 (Asymptotic stability [3])
An equilibrium point x = 0 is asymptotically stable at $t = t_0$ if

1. x = 0 is stable, and
2. x= 0 is locally attractive; i.e., there exists $\delta(t0)$ such that

$$\|x(t0)\| < \delta \Rightarrow \lim_{t\to\infty} x(t) = 0. \tag{8}$$

Consider the nonlinear system with an equilibrium point at the origin, f(0)=(0) then the origin is asymptotically stable if there exists a scalar Lyapunov function V(x) with continuous partial derivatives such that

- V(x) is positive definite
- $\dot{V}$ (x) is negative definite

To determine the rules of control in each of the three modes, the fuzzy controller design proceeds as follows.

Let us introduce the Lyapunov function candidate.

$$V = \frac{1}{2}(x - x_s)^2 \tag{9}$$

Differentiating V we have:

$$\dot{V} = (x - x_s)\dot{x} = (x - x_s)(q(t) - p(x)u) \tag{10}$$

Negative equation for each mode of operation and stability to the tank system, so we need to be met:

$$\begin{gathered}\dot{V} < 0 \\ \dot{V} \leq 0\end{gathered} \tag{11}$$

If x is low, then $(x - x_s)$< 0, therefore negative for $\dot{V}$ we need (q (t)> p (x) u) is fulfilled.

But we know that q(t) and p (x) are non-negative, so we set, u = 0.

If x is high, then $(x - x_s)$>0, thus making $\dot{V}$ negative needs that $q(t) < p(x)u$ is met and set $u = u_{max}$ where $u_{max}$is the openness of the outlet valve.

For the last case when x is normal, when designing the Takagi-Sugeno fuzzy controller, each rule is given by the following fuzzy rule as follows $u = k_1 x + k_2$ for some constant k1, k2. Substituting $u = k_1 x + k_2$ in the fuzzy control system we obtain:

$$\dot{V} = (x - x_s)(q(t) - p(x)(k_1 x + k_2))$$
$$= (x - x_s)(q(t) - p(x)(k_1(x - x_s)k_1 x_s + k_2))$$
$$= k_1 p(x)(x - x_2)^2 + (x - x_s)\,(q(t) - p(x)(k_1 x_s + k_2)) \quad (12)$$

The first term in the case of $(k_1 p(x)(x - x_2)^2)$ is not positive for any k_1>0. Therefore, to make negative $\dot{V}$ is needed to clear the second term that is $q(t) - p(x)(k_1 x_s + k_2) = 0$, or it could be $k_2 = \frac{q(t)}{p(x)} - k_1 x_s$. Then $q(t) - p(x)$are unknown, we approach using q_s and p_s therefore we obtain $k_2 = \frac{q_s}{p_s} - k_1 x_s$

Then we have that when x is normal, $u = k_1 x + k_2 = k_1 x + \frac{q_s}{p_s} - k_1 x_s$for some constant $k_1 > 0$.

Recall that the outlet valve of the tank water is not always negative, that is, always leaving at least a minimum amount of water, $k_1$therefore be fulfilled: $k_1 = (0 - x_s) + \frac{q_s}{p_s} \geq 0$, or:

$$k_1 \leq \frac{q_s}{p_s x_s} \quad (13)$$

In summary, using Fuzzy Lyapunov synthesis we obtain the following rules of Takagi-Sugeno control for the water tank system. Where now the output linguistic variables are as shown below:

- If x is low then $u = 0$
- If x normal then $u = (x - x_s) + \frac{q_s}{p_s}$
- If x high then u = u

Using Fuzzy Lyapunov synthesis and fuzzy control rules Takagi-Sugeno type obtained for the System Water Tank, help us reach the Mamdani type rules where the problem statement in terms of the input variable would be virtually same, that is, the input variables are given in linguistic variables and x is the amount of water in the tank which as noted may be low, normal or high and now add a further input variable, which will flow_current call and this variable can be negative or positive, this depends on the nominal steady flow denoted by $q_s$input. Now as for the output variables there is a difference with the Takagi-Sugeno type controller where the output variables are mathematical functions such as: If x is normal then $u = k_1(x - x_s) + \frac{q_s}{p_s}$

The Conditions can be converted to fuzzy rules as follows:

- Si x is low then $u = 0$
- Si x is normal then $u = (x - x_s) + \frac{q_s}{p_s}$
- Si x is high then $u = u$

(14)

- IF nivel is low THEN valve is close_fast
- IF nivel is normal THEN valve is no_change
- IF nivel is high THEN valve is open_fast

(15)

3.2 Description of the Inverted Pendulum

The Inverted Pendulum plant consists of a cart and a pendulum. The regulator's objective is to move the cart to its commanding position, without causing the pendulum to tip over.

3.2.1 Dynamic Model

The inverted pendulum control has a huge variety of problems that have made it one of the concrete systems for testing control laws discussed in more recent times. In the inverted pendulum control there are basically two problems: the problem of local stability around the equilibrium position, which is analogous to the problem of the juggler who intends to keep a stick in the tip of a finger, and the problem of lifting the pendulum from its rest position to the position where it is kept straight upwards.

Let us consider the problem of designing a stable fuzzy controller for the well-known inverted pendulum problem. The state variables are as follows [9]:

$x_1 = \theta$- The angle of the pendulum, and
$x_2 = \dot{\theta}$ - Angular velocity

The system dynamic equations, which are assumed unknown, are shown below:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x_1, x_2) + g(x_1, x_2)u \end{aligned} \tag{16}$$

Where:

$$f(x_1, x_2) = \frac{9.8 \sin x_1 - \frac{m l x_2^2 \cos x_1 \sin x_1}{m_c + m}}{l\left(\frac{4}{3} - \frac{m \cos^2 x_1}{m_c + m}\right)} \tag{17}$$

m_c: is the mass of the carriage
M: is the mass of the rod
l: the length of the rod
U: is the applied force (control).

3.2.2 Stability Analysis

To apply fuzzy Lyapunov synthesis, we assume the following:

- The system has two degrees of freedom, θ and $\dot{\theta}$ for us $x_1$and $x_2$called respectively. Therefore $\dot{x}_1 = x_2$.
- $\dot{x}_2$ is proportional to the control signal u, that is, when u increases (decreases) $\dot{x}_2$increases (decreases).

Theorem 1 (Asymptotic Stability [3]). An equilibrium point x = 0 is asymptotically stable at $t = t_0$ if

1 x = 0 is stable, and
2 x = 0 is locally attractive; i.e., there exists $\delta(t0)$ such that

$$\|x(t0)\| < \delta \Rightarrow \lim_{t\to\infty} x(t) = 0 \quad (18)$$

Consider the nonlinear system with an equilibrium point at the origin, f(0)=0 then the origin is asymptotically stable if there exists a Lyapunov function V (x) with continuous derivatives such that:

- V(x) is positive definite
- $\dot{V}$ (x) is negative definite

The fuzzy controller design proceeds as follows.

Let us introduce the Lyapunov function candidate

$$V(x_{1,}x_2) = \frac{1}{2}(x_1^2 + x_2^2) \quad (19)$$

which is positive-definite and radially unbounded function. The time derivative of$V(x_{1,}x_2)$ results in:

$$\dot{V} = x_1\dot{x}_1 + x_2\dot{x}_2 = x_1x_2 + x_2\dot{x}_2 \quad (20)$$

To guarantee stability of the equilibrium point we require that:

$$x_1x_2 + x_2\dot{x}_2 < 0 \quad (21)$$

We can now derive sufficient conditions so that inequality (21) holds: If and have opposite signs, then and (21) will hold if; ifx_1 and $x_2$are both positive, then $x_1x_2 < 0$ (21) will hold if $\dot{x}_2 = 0$; $x_1$if $x_2$and are both negative, then (21) will hold if $\dot{x}_2 < -x_1$; if x_1 and x_2 are both negative, then (21) will hold if $\dot{x}_2 < -x_1$.

Can clearly see that V is positive definite.

With this knowledge about the system, we can now derive sufficient conditions to ensure that (20) is fulfilled, these conditions can be viewed as follows:

- If x_1 and $x_2$have opposite signs, then x_1 x_2 and (21) is satisfied if $\dot{x}_2 = 0$
- If x_1 and x_2 and are both positive, (21) is satisfied if $\dot{x}_2 = -x_1$
- If x_1 and x_2 and are both negative, (21) is satisfied if $\dot{x}_2 => -x_1$

Conditions can be transferred from conditions for them to be converted to fuzzy rules created, are as follows:

- IF $x_1$is positive AND x_2 is positive, THEN $\dot{x}_2$ is negative big
- IF $x_1$is negative AND $x_2$is negative, THEN $\dot{x}_2$ is positive big
- IF $x_1$is positive AND$x_2$is negative, THEN $\dot{x}_2$is zero
- IF $x_1$is negative AND $x_2$is positive, THEN $\dot{x}_2$iszero

It is important to note that the partitions or fuzzy granulation x_1, $x_2$and *u* elegantly derive the expression (19), because $V = x_2(x_1 + \dot{x}_2)$, and as we need $\dot{V}$ is negative, it is natural to examine the signs of $x_1$and x_2, therefore, the obvious fuzzy partition is positive, negative. The partition for x_2 or *u* called large negative, zero, positive big is obtained in a similar way when we give linguistic values positive or negative for $x_1$and $x_1$in (20).

To ensure that $\dot{x}_1 < -x_1$ y $\dot{x}_2 > -x_1$ is satisfied even if we do not know the exact magnitude of x_1,only that it is positive or negative, we must give $\dot{x}_2$ or u large negative values and large positive.

Obviously, we can also start a predefined partition given to the linguistic variables and then try to understand each value in the expression for $\dot{V}$ you're using and from this find the rules, which is a somewhat more complex task.

Either way, whatever is done first, so far we have shown that the Fuzzy Lyapunov synthesis [5] transforms the classical Lyapunov approach from the world of conventional mathematics to the world of fuzzy system or the Computing with Words paradigm [10].

4 Results for Case 1

We show in this section the simulations of the dynamic model of the plant the water tank.

A controller for the water level in a tank has to know the current water level has to be able to set the valve.

The controller input is the water level error (desired water level minus the actual water level), and its output is the speed at which the valve opens or closes.

We can change the valve controlling the water flow, but the output rate depends on the diameter of the outlet pipe (which is constant) and the pressure in the tank (which varies with the level of water). The system has some very nonlinear characteristics.

4.1 Mamdani Fuzzy Controller

Simulation results for the Mamdani fuzzy controller are shown in Figures 1 and 2.

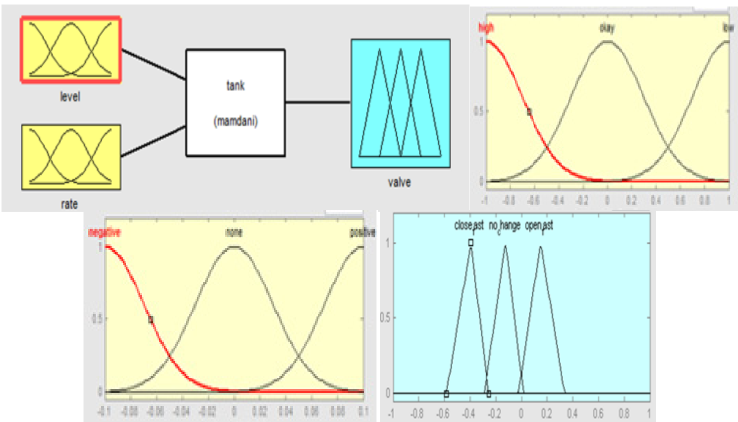

Fig. 1. Set membership functions for Type-1 Mamdani if the water tank

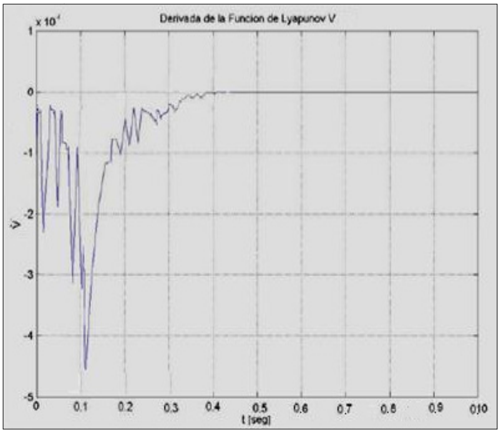

Fig. 2. $\dot{V}$ for the plant with the Mamdani Fuzzy Controller is stabilized in time0.5 s

4.2 Sugeno Fuzzy Controller

Simulation results for the Sugeno fuzzy controller are shown in Figures 3 and 4.

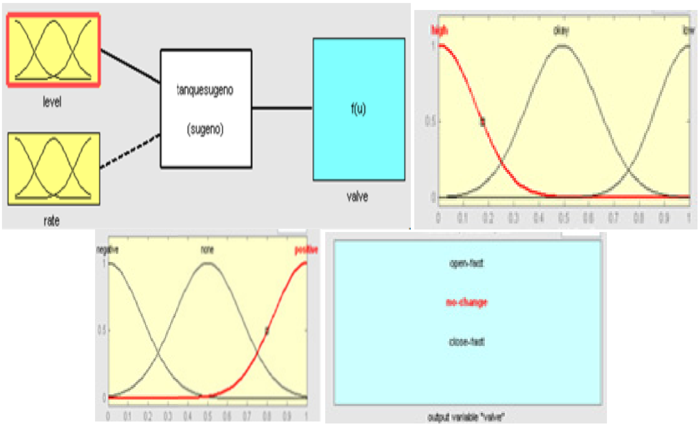

Fig. 3. Set membership functions for Type-1 Sugeno the case of water tank

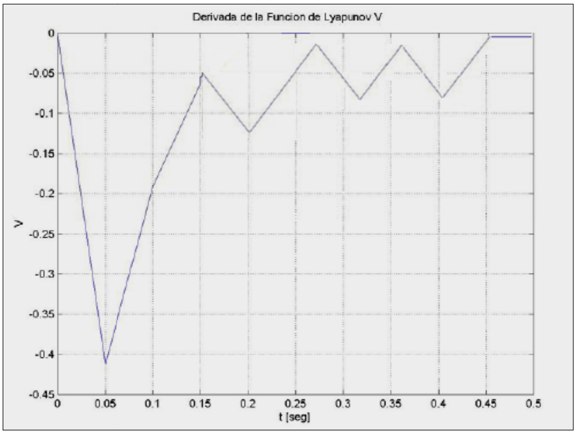

Fig. 4. $\dot{V}$ for the plant with the Sugeno Fuzzy Controller is stabilized in time 0.45 s

We show in table 1 a comparison of the results of bath controllers for case 1.

Table 1. Case 1 Comparison

Fuzzy Controller	Time (s) that stabilizes Comparative simulations of energy	Error
Mamdani	0.5s	0.0358
Sugeno	0.45s	0.0348

5 Results for Case 2

The Inverted Pendulum consists of a cart and a pendulum.

The regulator's objective is to move the carriage to its commanding position, without causing the pendulum to tip over.

5.1 Mamdani Fuzzy Controller

Simulation results for the Mamdani fuzzy controller are shown in Figures 5 and 6.

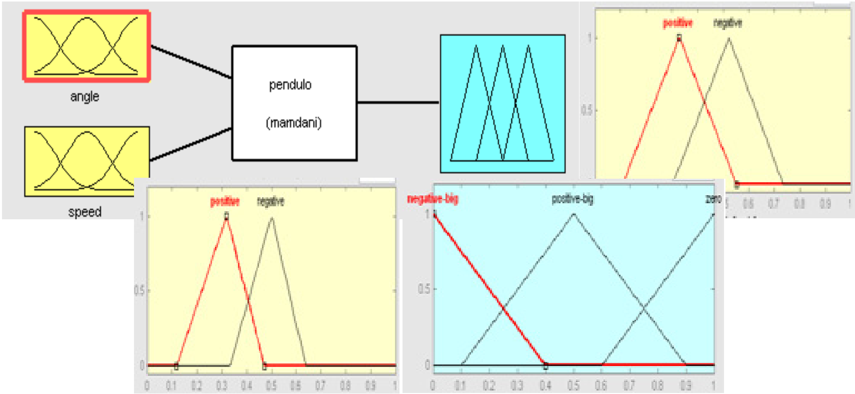

Fig. 5. Set of membership functions for type-1 Mamdani inverted pendulum case

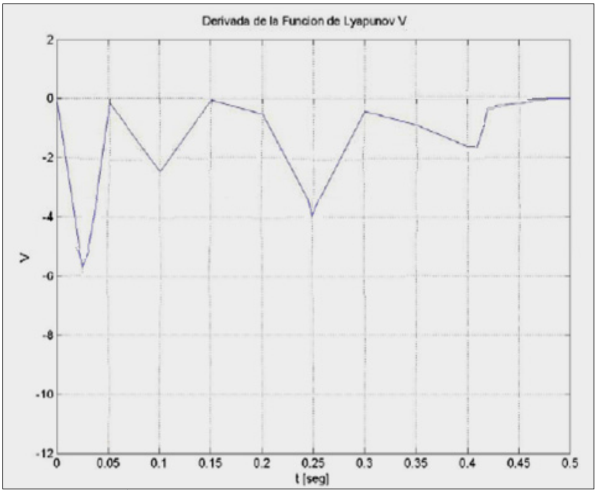

Fig. 6. $\dot{V}$ for the plant with the Mamdani Fuzzy Controller with xd = 0.5 rad, stabilizes in time 0.45s

5.2 *Sugeno Fuzzy Controller*

Simulation results for the Sugeno fuzzy controller are shown in Figures 7 and 8.

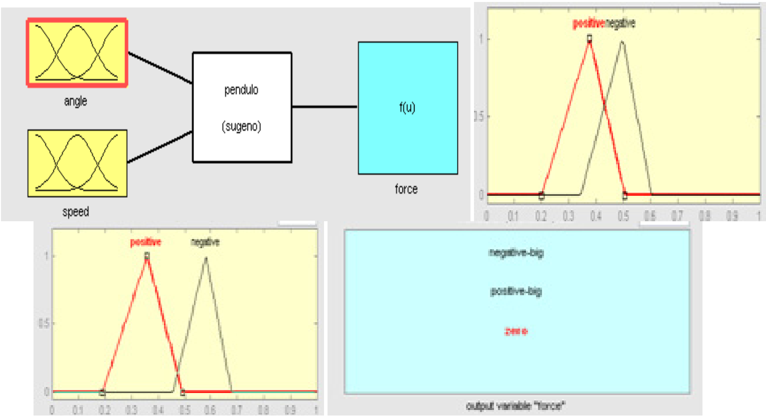

Fig. 7. Set of membership functions for type-1 Sugeno the case of the inverted pendulum

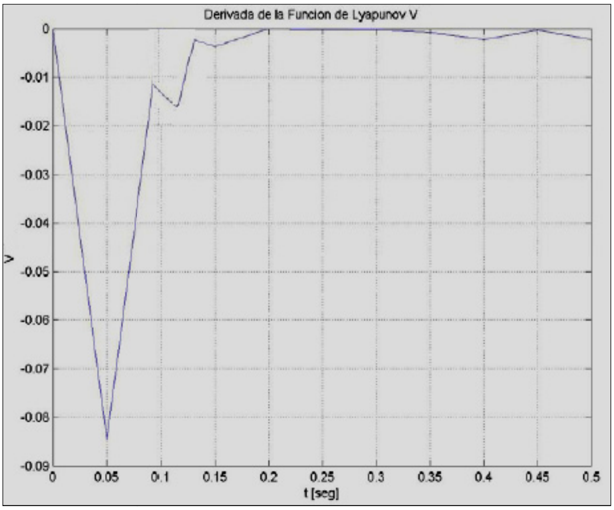

Fig. 8. $\dot{V}$ for the plant with the Fuzzy Controller with $x_d = 0.5$ rad, stabilizes in time 0.2 s

We show in Table 2 a comparison of the results of bath controllers for case 2.

Table 2. Case 2 Comparison

Fuzzy Controller	Time (s) that stabilizes Comparative simulations of energy	Error
Mamdani	0.5s	0.0458
Sugeno	0.45s	0.0356

6 Results of the Statistical Test

In each case, a procedure was developed, which was performed statistical tests "Z score", to test the validity of stability for cases where the Mamdani and Sugeno fuzzy controllers, which allowed the claim about a population parameter, this method is called a hypothesis test for the sample.

6.1 Statistical Test Results for Case 1 Mamdani

Shows descriptive statistics for hypothesis testing, the method of Lyapunov Mamdani and without the method respectively, for the water tank system.

Table 3. Statistical test for Mamdani Fuzzy System for case 1

Fuzzy System	*Sample*	*Null hypothesis* H_0	*Null alternative* H_1	*Significance level* $\propto$	*Average*	*Standard deviation*
Mamdani Lyapunov (n_1)	*30*	$\mu_1 \leq \mu_2$	$\mu_1 > \mu_2$	**0.05**	*0.2795*	*0.415*
Mamdani (n_2)	*30*				*-2.342*	**2.333**

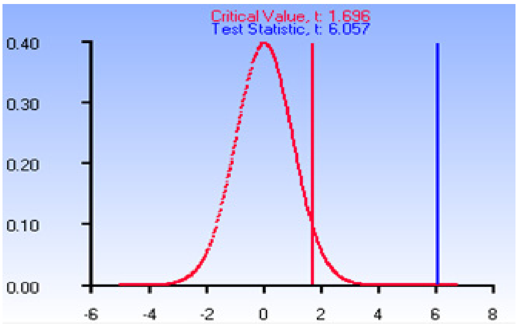

Fig. 9. Graphic of Mamdani hypothesis testing case 1

States that the test of null hypothesis is rejected with a confidence interval of 95%, the value of Z = 6.0571, the alternative hypothesis accepted, which gives us sufficient statistical information to determine that the method of Mamdani

Lyapunov is statistically greater than Mamdani, which there is evidence of the method.

6.2 Statistical Test Results Case 1 Sugeno

This section shows descriptive statistics for hypothesis testing with the method of Lyapunov for Sugeno and without the method respectively, for the water tank system.

Table 4. Statistical test for the Sugeno Fuzzy System for case 1

Fuzzy System	*Sample*	*Null hypo-thesis* H_0	*Null alternative* H_1	*Significance level* $\propto$	*Average*	*Standard deviation*
Sugeno Lyapunov (n_1)	*30*	$\mu_1 \leq \mu_2$	$\mu_1 > \mu_2$	**0.05**	*0.2106*	*0.375*
Sugeno (n_2)	*30*				*-2.145*	**2.1375**

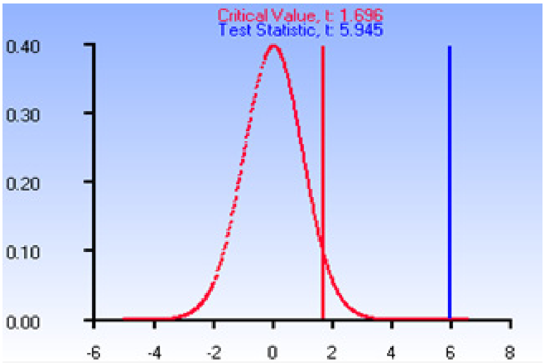

Fig. 10. Graphic of Sugeno hypothesis testing case 1

States that the test of null hypothesis is rejected with a confidence interval of 95%, the value of Z = 5.9453, the alternative hypothesis is accepted, which gives us sufficient statistical information to determine that the Sugeno method of Lyapunov is greater statistically Sugeno, which there is evidence of the method.

6.3 Statistical Test Results for Case 2 Mamdani

Shows descriptive statistics for the hypothesis test with the Mamdani Lyapunov method without the method respectively, for the inverted pendulum system.

Table 5. Statistical test for Mamdani Fuzzy System for case 2

Fuzzy System	*Sample*	*Null hypothesis* H_0	*Null alternative* H_1	*Significance level* $\propto$	*Average*	*Standard deviation*
Mamdani Lyapunov (n_1)	*30*	$\mu_1 \leq \mu_2$	$\mu_1 > \mu_2$	**0.05**	*0.144*	*0.270*
Mamdani (n_2)	*30*				*-2.815*	**2.8052**

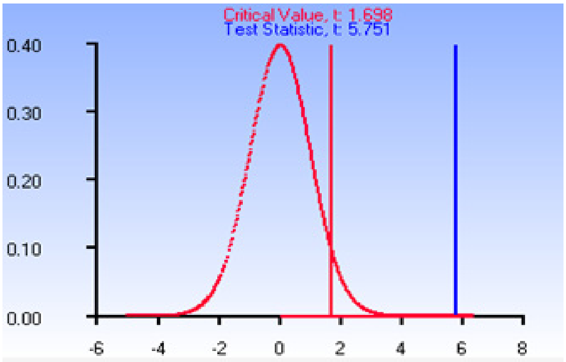

Fig. 11. Plot of Mamdani hypothesis testing for case 2

States that the test of null hypothesis is rejected with a confidence interval of 95%, the value of Z = 5.751, the alternative hypothesis is accepted, which gives us sufficient statistical information to determine that the Mamdani method of Lyapunov is greater statistically Mamdani, which there is evidence of the method.

6.4 Statistical Test Results Case 2 Sugeno

Shows descriptive statistics for hypothesis testing with the method of Lyapunov Sugeno and without the method respectively, for the inverted pendulum system.

Table 5. Statistical test for the Sugeno Fuzzy System for case2

Fuzzy System	*Sample*	*Null hypo-thesis* H_0	*Null alternative* H_1	*Significance level* $\propto$	*Average*	*Standard deviation*
Sugeno Lyapunov (n_1)	*30*	$\mu_1 \leq \mu_2$	$\mu_1 > \mu_2$	**0.05**	*0.181*	*0.346*
Sugeno (n_2)	*30*				*-2.612*	**2.5047**

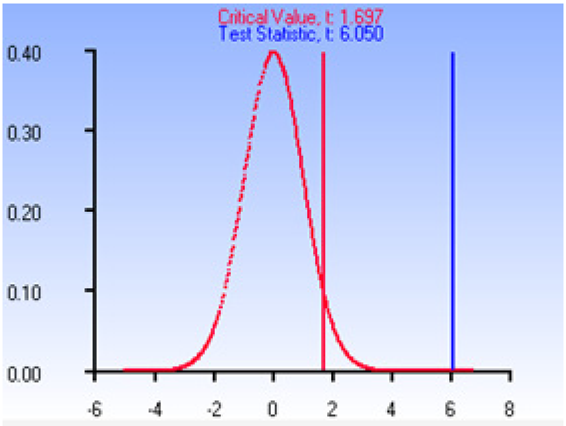

Fig. 12. Plot of Sugeno hypothesis testing case 2

States that the test of null hypothesis is rejected with a confidence interval of 95%, the value of Z = 6.0502, provides statistical information sufficient to determine that the Sugeno method of Lyapunov is greater statistically Sugeno, which evidence of the methods.

7 Conclusions

The main objective of this research was to propose a systematic methodology to design stable fuzzy controllers to solve different cases.

With the proposed method stability of control was achieved in two cases based on simulation results.

The problems for the two cases are resolved as expected; this statement is consistent with simulations, where fuzzy controllers are designed following the method of fuzzy Lyapunov synthesis to achieve the solution to the problem.

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