Gravitational Search Algorithm Design of Posicast PID Control Systems

P.B. de Moura Oliveira[1], E.J. Solteiro Pires[2], and Paulo Novais[3]

[1] INESC TEC - INESC Technology and Science (formerly INESC Porto)
Department of Engineering, School of Sciences and Technology,
5001–801 Vila Real, Portugal
{oliveira,epires}@utad.pt
[2] Departamento de Informática, Universidade do Minho 4710-057 Braga, Portugal
pjon@di.uminho.pt

Abstract. The gravitational search algorithm is proposed to design PID control structures. The controller design is performed considering the objectives of set-point tracking and disturbance rejection, minimizing the integral of the absolute error criterion. A two-degrees-of-freedom control configuration with a feedforward prefilter inserted outside the PID feedback loop is used to improve system performance for both design criteria. The prefilter used is a Posicast three-step shaper designed simultaneously with a PID controller. Simulation results are presented which show the merit of the proposed technique.

1 Introduction

Controllers based on proportional, integrative and derivative (PID) modes are applied within the majority of industrial control loops. Despite the development of more complex control methodologies, there are several reasons for the success and resilience of PID control, such as simplicity, performance and reliability in a wide range of system dynamics [1]. Thus, the development of new PID control based schemes and design methodologies are relevant research issues. Two classical control system design goals are input reference tracking and disturbance rejection. Optimal PID controller settings for set-point tracking can result in poor disturbance rejection and vice-versa, i.e., optimal disturbance rejection PID settings can result in poor set-point tracking. The design of PID controllers both for set-point tracking and disturbance rejection can be improved using two-degrees-of-freedom (2DOF) configurations [2]. A well known 2DOF configuration uses a feedforward prefilter applied to the input reference signal and a PID controller within the feedback loop. The ideal design of such 2DOF controllers requires simultaneous optimization of system response both for set-point tracking and disturbance rejection.

The GSA algorithm was proposed by Rashedi et al. [3] which reported the advantages of using this algorithm in optimizing a set of benchmark unimodal and multimodal functions. In [3] a comparison was presented between GSA, particle swarm optimization (PSO) and a real genetic algorithm (RGA), showing that GSA performs better that PSO and RGA in the tested function set. Since its proposal GSA has been reported successfully in solving several problems [4,5,6]. In this paper the

V. Snasel et al. (Eds.): SOCO Models in Industrial & Environmental Appl., AISC 188, pp. 191–199.
springerlink.com

gravitational search algorithm (GSA) is proposed to design 2DOF control configuration in which the prefilter is a three-step Posicast input shaper and the feedback loop is a PID controller.

2 Gravitational Search Algorithm

The GSA was proposed originally by [3], and it is inspired in the natural interaction forces between masses. Accordingly to Newton's law of gravity, the gravitational force, F, between two particles in the universe can be represented by:

$$F = G\frac{M_1 M_2}{R^2} \tag{1}$$

where: M_1 and M_2 are the two particles masses, G is the gravitational constant and R is the distance between the two particles. Newton's well known second law relates force with acceleration, a, and mass, M, as:

$$F = Ma \Leftrightarrow a = \frac{F}{M} \tag{2}$$

Considering a swarm of particles (or population), X, of size s, in which every element represents a potential solution for a given search and optimization problem, moving in a n-dimensional space, with vector x representing the particle position. The force between particles i and j, for dimension d and iteration t is represented by [3]:

$$F_{ij}^d = G(t)\frac{M_i(t)M_j(t)}{R_{ij}(t)+\varepsilon}(x_j^d(t) - x_i^d(t)) \tag{3}$$

where: ε is a small constant, and the gravitational constant can be defined in every iteration by:

$$G(t) = G(t_0)\frac{t_0{}^{\beta}}{t} \quad \beta < 1 \tag{4}$$

with: $G(t_0)$ representing the initial gravitational constant, and R representing the Euclidian distance between the two particles. The use of R instead of R^2 in (1) was proposed by [3] based on experimental tests. The total force that acts in each particle i for a certain dimension, d, is evaluated by:

$$F_i^d(t) = \sum_{j \in K_{best},\, j \neq i}^{s} \varphi_{1j} F_{ij}(t) \tag{5}$$

in which, φ_{1j} represents an uniform randomly generated number in the interval [0,1] and K_{best} is the set of best particles, with size set to k_0 at the beginning of the search procedure and decreased linearly over time. The acceleration of mass i, called law of motion [3], is represented by:

$$a_i = \frac{F_i^d(t)}{M_i(t)} \tag{6}$$

with M_i representing the inertia mass for particle *i,* evaluated with:

$$M_i = \frac{m_i(t)}{\sum_{j=1}^{s} m_j(t)} \tag{7}$$

and

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \tag{8}$$

where: *fit*, *best*, *worst* represent respectively the: current, best and worst fitness values for particle *i* in iteration *t*. The velocity and position of each particle are up-dated accordingly to the following equations:

$$v_i^d(t+1) = \varphi_{2i}\, v_i^d(t) + a_i^d(t) \tag{9}$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \tag{10}$$

with φ_{2j} representing an uniform randomly generated number in the interval [0,1].

3 PID Control Design: Problem Statement

A general PID control structure for single-input single-output systems can be illustrated using the classical block diagram presented in Figure 1. Two of the more relevant control design objectives are set-point tracking and disturbance rejection.

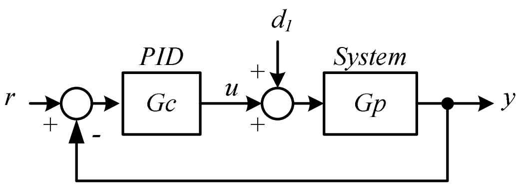


Fig. 1. PID control configuration

For some types of system dynamics optimum set-point tracking can be achieved by using an open-loop control feedforward configuration. The modification of the reference input in order to improve system tracking can be implemented by using command shaping techniques [8,13]. The input shaping concept was originally proposed [7] to control underdamped systems. However the same technique can be used for other non-oscillatory system dynamics, as reported in [9].

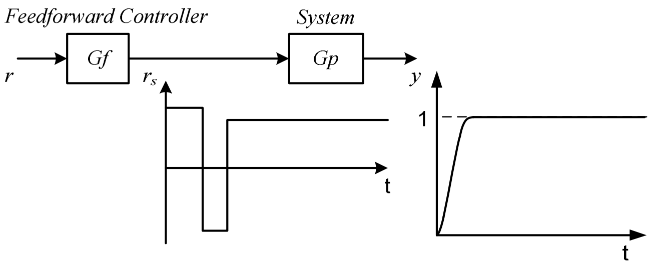


Fig. 2. Feedforward input command shaping

The pre-filter or shaper modifies the input step reference input, *r*, into another signal, r_s, appropriate to cancel some of the system dynamics, in order to achieve a dead-beat response, as illustrated in Figure 2. Cancelling the underdamped complex poles with a feedforward controller was originally proposed by Smith [7], and termed Posicast control. Considering a unit step reference input, the half-cycle Posicast control signal is represented by:

$$r_s(t) = A_1 r(t) + A_2 r(t - t_1) \tag{11}$$

with:

$$\begin{aligned} A_1 + A_2 &= 1 \\ t_1 &= \frac{T_d}{2} \end{aligned} \tag{12}$$

and: *r* representing an unit reference input step, r_s the shaped signal, A_1 and A_2 the first and second step amplitudes and T_d the undamped time period. Equation (11) represented in the Laplace complex domain results in the following shaper transfer function:

$$G_f(s) = A_1 + A_2 e^{-t_1 s} = A_1 + (1 - A_1) e^{-t_1 s} \tag{13}$$

The control signal represented by (11) can be obtained convolving the unit step input with a sequence of two impulses. This is known as a zero-vibration shaper [10], represented in the continuous time-domain as:

$$ZV(t) = A_1 \delta(t) + (1 - A_1)\delta(t - t_1) \tag{14}$$

where δ(t) is the Dirac delta function. The amplitude of the first step or impulse is a function of the overshoot, *Mp*:

$$A_1 = \frac{1}{1 + M_p} = \frac{1}{1 + e^{-\frac{\zeta \pi \omega_n}{\omega_d}}} \tag{15}$$

with: ζ representing the damping factor, ω_n and ω_d representing the undamped and damped natural frequencies, respectively. This study considers a Posicast shaper with three steps, represented by:

$$G_{fts}(s) = A_1 + A_2 e^{-t_1 s} + A_3 e^{-t_2 s} \quad 0 < t_1 < t_2 \tag{16}$$

$$A_1 + A_2 + A_3 = 1 \quad A_1 \geq 1, A_2 < 0 \tag{17}$$

As systems can be subjected to disturbances and model uncertainty, the ideal feed-forward control configuration presented in Figure 2 is usually combined with a feedback control loop. This combination can be accomplished using several control configurations, incorporating the feedforward controller (called hereby shaper) inside or outside the feedback loop. This problem has been addressed by [8], showing that using the shaper inside the loop is not advantageous for rejecting input disturbances. Thus, the control configuration used in this study is a two-degrees-of-freedom (2DOF) configuration presented in Figure 3, with the Posicast input command shaper (PICS) outside the loop.

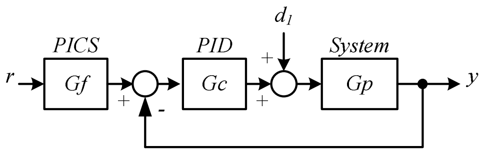


Fig. 3. Two-degrees-control configuration with input command shaping

The 2DOF controller design can be accomplished using several methodologies. The PID controller can be designed first for achieving good disturbance rejection, and then the PICS can be designed to enhance set-point tracking. The former is a sequential design procedure. However, this can result in low performance. Indeed, both feed-forward input shaper and feedback controller should be designed simultaneously. This type of methodology, also called concurrent design [11], is particularly useful when both design objectives are conflicting.

4 GSA Design of PID Control Structures

4.1 PID Design for Set-Point Tracking

The PID controller used in this study is governed by the following equation:

$$G_c(s) = \frac{s^2 K_d + sK_p + K_i}{s}\left(\frac{1}{1+sT_f}\right) \tag{18}$$

where: K_p, K_i and K_d represent the proportional, integrative and derivative gains, respectively, and T_f the filter time constant. The GSA is proposed to design the control structures described in the previous section. The first case is the PID control design considering the control configuration presented in Figure 1, for set-point tracking minimizing the integral of absolute error criterion:

$$IAE = \int_o^T |e(t)|\,dt, \quad with \quad e(t) = r(t) - y(t) \tag{19}$$

The algorithm used is presented in Figure 4, based on the original GSA [3], with some minor adaptations. In this case, if the swarm initialization is performed using a totally random procedure, some controller PID gains will make the system unstable. In these unstable cases the IAE value is disproportional high compared with stable cases, which makes the GSA to perform badly. Thus, to avoid unstable particles incorporating the first population, the swarm is initialized randomly using a candidate interviewing procedure. A randomly generated particle is allowed to be part of the initial swarm if it fulfills a predefined minimum IAE threshold.

```
t = 0
initialize swarm X(t)
while(!(termination criterion))
  evaluate X(t)
  update G, best, worst
  evaluate particles M and a
  update particles velocity and position
  t = t + 1
end
```

Fig. 4. Gravitational Search Algorithm for PID controller design

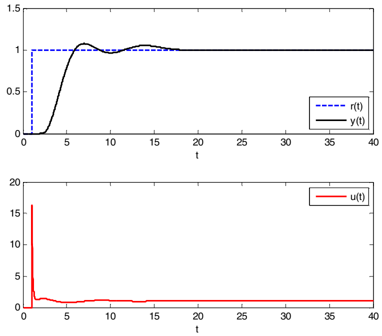
The gravitational constant is updated using:

$$G(t) = G_0\left(1 - \frac{t}{n\_it}\right) \tag{20}$$

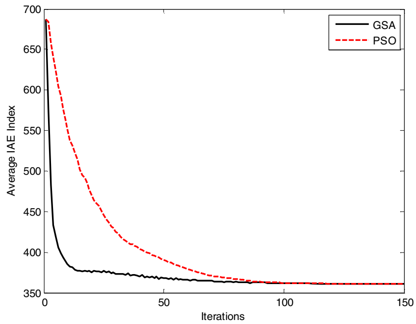
where *n_it* represents the total number of iterations. This linear decreasing equation was found better for this application, among other possibilities tested by experimentation. The evaluation of each particle mass and acceleration are evaluated with equations (7) and (6), respectively. Particle velocity and position are updated with (9) and (10), respectively. The simulation experiment considers the control of a fourth order system with time delay represented by the model:

$$G_p(s) = \frac{1}{(1+s)^4} e^{-s} \tag{21}$$

Each swarm particle encodes the PID gains parameters $\{K_p, K_i, K_d\}$ and the filter constant was set to 0.1. The search interval was equal both for initialization and search defined by: $0.1 \le K_p, K_i\ K_d \le 5$. The initialization threshold was set to an IAE of 1000. The total number of iterations (*n_it=150*) was the search termination criterion used. This number was deliberately set low as the aim here is not to achieve the optimal PID settings but good settings in a short evolutionary time period. The value used for the initial gravitational constant was *Go=0.5*. The best PID gains achieved were {K_p=*1.06*, K_i=*0.34*, K_d=*1.63*}, results in an IAE=361. Figure 5-a) presents the unit step system response and respective control signal.



a) Responses for Gp, with GSA PID gains for set-point tracking.



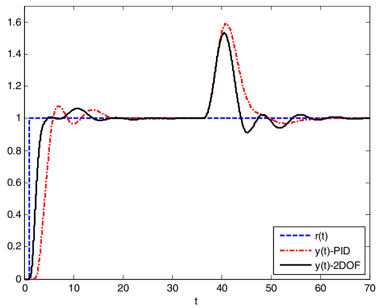
b) Comparison between the GSA and PSO.

Fig. 5. Simulation results for set-point tracking

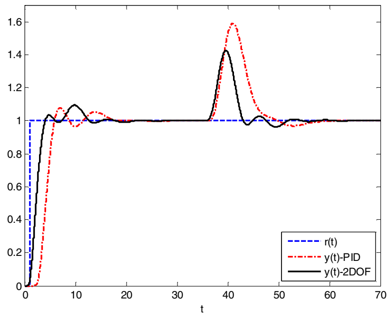
Figure 5-b) presents a comparison between GSA algorithm and a standard Particle Swarm Optimization (PSO) algorithm, showing the evolution of the mean value of the ITAE index from a set of 20 runs. The PSO parameters in terms of swarm size and number of objective function evaluations were the same as used in the GSA. The cognitive and social constants were set to 2 and the inertia weight was linearly decreased between 0.9 and 0.4 over the 150 iterations. For this parameters set Figure 5-b) clearly shows that the GSA convergence rate is faster in an early stage of the run. However, the PSO could be set to a faster convergence rate by reducing the higher limit for the inertia weight. The achieved value for the average fitness value is the same, which indicates that both algorithms converged for the same value in all trial runs. An interesting feature shown in Figure 5-b) is that the average fitness trend for GSA is more irregular than the PSO. This may prove relevant in escaping search traps such as local optima.

4.2 PID Design for Set-Point Tracking and Disturbance Rejection

If the PID gains derived for set-point tracking are applied to disturbance rejection the performance achieved is not good, as illustrated in Figure 6.a). To improved disturbance rejection, the 2DOF configuration presented in Figure 3 is used, with and three-step input shaper represented by (16) and a PID controller. The design is performed considering the simultaneous optimization of both pre-filter and PID controller. The optimization procedure considers an input step applied to the reference input first, and an input step applied to the input disturbance input, d_1, when the system as settled its tracking (in this case t=35s). The cost function used is the ITAE and each swarm particle encodes both the prefilter parameters and PID gains $\{A_1, A_2, t_2, \mu, K_p, K_i, K_d\}$ subjected to the amplitude constraint (17). The search intervals for the PID gains are the same as before and for the three-step Posicast shaper: $1 \leq A_1 \leq 2$, $-2 \leq A_2 \leq 0$, $0.5 \leq t_1 \leq 6$ and $1.1 \leq \mu \leq 4$. Parameter A_3 is evaluated using the amplitude constraint (17) and $t_1 = t_2/\mu$. Figure 6-a) presents the simulation results comparing the PID configuration with the 2DOF with Posicast input shaping. No limits were imposed to the actuator signal and the parameters achieved for the prefilter were $\{A_1=2.0,\ A_2=-1.59,\ A_3=0.59,\ t_1=0.61,\ t_2=2.43\}$, and PID gains $\{K_p=1.55,\ K_i=0.55,\ K_d=2.45\}$, IAE=538. The results with the 2DOF clearly improved the single's PID, with an IAE=687, accounting with the unit step disturbance. Figure 6-b) presents the simulation results comparing the 2DOF with actuator saturation limits $-15 \leq u(t) \leq +15$, and the PID controller was implemented using a anti-windup scheme based on the conditioning technique [12,13]. The parameters achieved for the prefilter were $\{A_1=2,\ A_2=-0.1,\ A_3=-0.9,\ t_1=0.45\ t_2=0.5\}$, and PID gains $\{K_p=1.96,\ K_i=0.68,\ K_d=2.68\}$, with IAE=450. The plots presented in Figure 6-b) show an significant improvement both compared to the PID as well as the PID without the anti-windup scheme.



a) Responses for Gp, for single PID and 2DOF Posicast PID.



b) Responses for Gp, for single PID and 2DOF Posicast PID with saturation and anti-windup.

Fig. 6. Simulation results for disturbance rejection

5 Conclusions

The GSA was proposed to optimize PID control structures using the integral of absolute error criterion. Two control configurations were addressed: i) classical feedback loop with PID controller for set-point tracking ii) 2DOF configuration using a feedforward Posicast input command shaper, placed outside the feedback PID loop. Both three-step Posicast parameters and PID gains were designed simultaneously, both for the objectives of set-point tracking and disturbance rejection. The same relevance was given to both objectives. The results presented show that GSA has a faster convergence rate than PSO algorithm for PID design and it can conveniently design both the input shaper and PID controller in the 2DOF configuration, with and without considering controller variable saturation levels. Further research will explore the proposed technique for other process dynamics.

References

1. Åström, K.J., Hägglund, T.: The Future of PID Control. Control Engineering Practice 9(11), 1163–1175 (2001)
2. Araki, M., Taguchi, H.: Two-Degree-of-Freedom PID Controllers. International Journal of Control Automation, and Systems 1(4), 401–411 (2003)
3. Rashedi, E., Nezamabadi-pour, H., Saryazdi, S.: GSA: A Gravitacional Search Algorithm. Information Sciences 179, 2232–2248 (2009)
4. Precup, R.E., David, R.C., Petriu, E.M., Preitl, S., Răda, M.C.: Gravitational Search Algorithms in Fuzzy Control Systems Tuning. Preprints of the 18th IFAC World Congress, pp. 13624–13629 (2011)
5. Khajehzadeh, M., Raihan Taha, M., El-Shafie, A., Eslami, M.: Search for critical failure surface in slope stability analysis by gravitational search algorithm. International Journal of the Physical Sciences 6(21), 5012–5021 (2011); Academic Journals
6. Duman, S., Sonmez, Y., Guvenc, U., Yorukeren, N.: Application of Gravitational Search Algorithm for Optimal Reactive Power Dispatch Problem. In: IEEE Symposium Innovations in Intelligent Systems and Applications (INISTA), pp. 519–523 (2011)
7. Smith, O.J.M.: Posicast Control of Damped Oscillatory Systems. Proc. IRE 45(9), 1249–1255 (1957)
8. Huey, J.R., Sorensen, K.L., Singhose, W.E.: Useful applications of closed-loop signal shaping controllers. Control Engineering Practice 16, 836–846 (2008)
9. Tuttle, T.D.: Creatig Time-Optimal Commands for Linear Systems, PhD Thesis, MIT (1997)
10. Singer, N.C., Seering, W.P.: Preshaping command inputs to reduce system vibration. Journal of Dynamic Systems Measurement and Control 112(3), 76–82 (1990)
11. Chang, P.H., Park, J.: A concurrent design of input shaping technique and a robust control for high-speed/high-precision control of a chip mounter. Control Engineering Practice 9(2001), 1279–1285 (2001)
12. Hanus, K.M., Henrotte, J.L.: Conditioning technique, a general anti-windup and bumpless transfer method. Automatica 23(6), 729–739 (1987)
13. Moura Oliveira, P.B., Vrančić, D.: Underdamped Second-Order Systems Overshoot Control. Accepted for publication in the IFAC Conference on Advances in PID Control, PID 2012, Brecia (2012)