Dynamical Models for Representing and Building Consensus in Committees

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Abstract. When a committee of experts is formed with the aim to make a decision of social or economic relevance, the various competencies act in order to produce an equilibrium among the features that characterize the alternatives or the objectives, that constitute the choice that the committee is called to make. It is worth to remark that, in some circumstances, the committee behave as a unique body, whose organs, the experts, share the same opinions and select the same choice. When this occurs, the committee has reached unanimous consensus.

More frequently only a majority of the experts agree about a final choice and circumscribe a precise decision to make. Also in this case we speak of consensus reached by, or inside, the committee.

The mechanisms for enhancing, and possibly, reaching consensus are here studied by means of the definition of dynamical models, geometric and game theoretical in nature.

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1 Introduction

Collective decisions are usually given to the responsibility of suitable committees of experts. In particular when the involved acts of choice are related with environmental, social, or economical issues.

Static procedures and dynamic procedures are defined (see e.g. [1], [3], [4]), [5]) in order to organize knowledge, synthesize individual judgments in collective, enhance and evaluate consensus in a group. In fact, an important

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step in a consensus procedure is the evaluation of the degree of consensus, i. e. the number of individuals decision makers that form a majority: the greater the majority, the higher the degree of consensus.

The maximum possible degree of consensus is, of course, unanimity, what is rare, especially if group decisions that have to be made are related with domains which embody complex issues such as environment, social conflicts, financial crises.

When facing subjects of such a complexity the inputs, that promote and breathe life into the debating group, arise from both the necessity of cooperating and the need of competition and fighting.

The individuals, or groups, in the committee move and search for stability; there are seen as points in a space, that change positions and mutual distances. In a metric space distances between opinions are measured and reveal to what extent a decision maker changes his mind.

In this perspective we deal with a game theoretical model for dynamical consensus searching. Indeed, we mean a consensus procedure as shaping a winning coalition in a cooperative game, where:

1. the decision makers are the players,
2. utility transfer is allowed,
3. only suitable coalitions are admissible.

2 Requirements for a Definition of Consensus

Particular characteristics are needed for a better understanding, or a definition, of the concept of consensus. Let us sketch some of these requirements;

1. Sometimes consensus may be reached immediately, just with the presentation or formalization of the problem, either by means of the unanimous and immediate agreement of the decision makers, or by means of a suitable procedure that rules the search for consensus, e. g., a static average procedure [1], 1987], that works like a black box. In both cases it is a matter of *immediate*, or *one shot*, *consensus*.
2. In a *dynamic procedure*, consensus is related, or determined, by suitable behaviors, such as compromise or agreement. Indeed, consensus should develop like the formation of the opinions or convictions during the debates, or discussions, among persons. Therefore a dynamic procedure soliciting consensus results in a trade-off between agreement and compromise, related with individual decision makers or groups inside the committee. Such a behavior gives rise to movements toward consensus. Any movement is leaded by desire and rationality to get a goal. In other words, to simplify, decision makers are seen as bodies or points that move with their ideas and willingness.
3. *Perfect information* is needed for reaching consensus. Any member in the group knows every act and the behavior of any decision maker.

An additional figure plays a role in our model, that is the *supervisor*, or *chairman*. He/she coordinates the group decision process; the action of the supervisor is *technical*.

3 Evaluation of Resources and a Static Model

A group charged of the duty of reach a sufficiently shared decision, i. e. a decision endowed of a suitable consensus, must first know the elements that are the objects of the judgements. In other words the group must *evaluate the resources.*

Then the group proceeds to the *evaluation of the evaluations*, what leads to enhancing consensus.

In order to evaluate the resources and activate consensus enhancing processes, the decision makers turn to Group Decision Support Systems.

In particular, the achievement of consensus is an objective for *Cooperative Work*. In general, decision makers use *Decision Aids*.

An amount s of quantifiable resources, such as raw materials, energy, money for industrial and scientific projects, grants, must be allocated over m projects. To this purpose a committee of n experts is formed. The expert i recommends to allocate the *amount* x_{ij} over the project j. Then we have the constraints

$$x_{i1} + x_{i2} + ... + x_{im} = s, i = 1, 2, ..., n. \tag{1}$$

Set $\mathbf{x_j} = (x_{1j}, x_{2j}, ..., x_{nj})$, and $\mathbf{s} = (s, s, ..., s)$. Then the system of equations (1) assumes the form:

$$\mathbf{x_1} + \mathbf{x_2} + ... + \mathbf{x_m} = \mathbf{s}. \tag{2}$$

The recommendations of each expert must be aggregated, in a unique consensus allocation [1] by a chairman or external authority: again the shares sum up to s.

The aggregated allocation satisfies suitable reasonable requirements, such as:

- the dependence of the project j only from the recommended allocations by the experts for project j;
- if all the experts recommend to reject the project j, then consensus about the allocation of resources to project j is 0.

Then we assume, for every project j, there is an *aggregation function* $f_j = f_j(y_1, y_2, ..., y_n)$, with values in the set R^+ of nonnegative real numbers, where $y_i = x_{ij} \in [0, s]$ is the allocation proposed by the expert i.

It seems reasonable to assume the following requirements on the functions f_j:

1. $f_j(\mathbf{0}) = 0$;
2. f_j is increasing with respect to every variable;
3. $\sum_{j=1}^{m} \mathbf{x_j} = \mathbf{s} \Rightarrow \sum_{j=1}^{m} f_j(\mathbf{x_j}) = s$.

An important result is the following Aczel's theorem:

Theorem [1] The general solution $(f_1(\mathbf{y}), f_2(\mathbf{y}), ..., f_m(\mathbf{y}))$ of the system of conditions 1., 2., 3., is given, for $m > 2$, by

$$f_1 = f_2 = ... = f_m = \sum_{i=1}^{n} \alpha_i y_i, \tag{3}$$

where:

$$\forall i, \alpha_i \geq 0, \sum_{i=1}^{n} \alpha_i = 1. \tag{4}$$

4 Geometrical Representation and Dynamical Model

A dynamical model for building consensus has an operational semantics close to the meetings and discussions in the real life (see [3], [6], [7], [8], [9], [13], and [16]).

Let us sketch a description of a behavioural model of a group of experts that must reach a satisfying internal agreement, or consensus, in order to provide for an aid to the decision to be made, for instance, in a political framework.

Let us suppose that any expert has the opportunity to express his evolving opinion. A coordinator (*chairman*, *supervisor*) solicits the experts to get a satisfying level of consensus, by iterating a loop:

1. at a certain moment the coordinator summarizes the discussion,
2. he makes the point of the current state of consensus,
3. confirms characteristic opinions,
4. stresses conflicting points of view,
5. diplomatically reminds the necessity to maintain united the group.

As a result, the members that are at the opposition, fearing to be emarginated by some majority, react asking for clarifying, or modifying, their opinions in order to weaken the tension between their viewpoints and what defines the current status of consensus. This dynamics runs until the degree of consensus does not seem sufficient to the coordinator, or alternatively, the consensus does not seem, at present, achievable and discussion is postponed.

The growth of consensus is due also to the increase of communication, database, knowledge, technology.

In some models (see, e. g., [3]) the Analytic Hierarchy Process (AHP) [17] is assumed to be the system used by all the experts for ranking alternatives. Of course different evaluation schemes can be considered (see, e. g., [2]).

Let us describe a model of collective decision making and the related consensus achievement procedure (see also, e. g. [3], [9], [10] and [11]). Let D denote the set of the decision makers of a committee, A the set of alternatives, and C the set of the accepted criteria. Let any decision maker $d_i \in D$ be able to assess the relevance of each criterion. Precisely, every d_i assigns a function

$$h_i : C \to [0,1] \tag{5}$$

such that

$$\sum_{c \in C} h_i(c) = 1. \tag{6}$$

Remark that h_i just denotes the *evaluation* or *weight function* that the decision maker d_i assigns to every criterion $c \in C$.

Furthermore let us consider the function

$$g_i : A \times C \to [0,1] \tag{7}$$

such that $g_i(a,c)$ is the value of the alternative a with respect to the criterion c, in the perspective of d_i.

The values $h_i(c)$ and $g_i(a,c)$ can be determined by suitable procedures, such as dealt with, e. g., in [2] or [17]. Let n, p, m, denote the numbers of the elements of D, C and A, respectively. The values $(h_i(c))_{c \in C}$ denote the evaluation of the p-tuple of the criteria by the decision maker d_i and the values

$$(g_i(c,a))_{c \in C, a \in A} \tag{8}$$

denote the matrix $p \times m$ whose elements are the evaluations, made by d_i, of the alternatives with respect to each criterion in C.

Function $f_i : A \to [0,1]$, defined by the scalar product

$$(f_i(a))_{a \in A} = ((h_i(c))_{c \in C}) \cdot (g_i(c,a))_{c \in C, a \in A}) \tag{9}$$

is the evaluation, made by d_i , of the set of alternatives $a \in A$.

Dynamics of consensus enhancing process is managed by an external supervisor that has at his disposal a metric μ, e. g. an Euclidean metric, that acts between couples of decision makers d_i and d_jj, i. e., between individual rankings of alternatives, defined by

$$\mu_{ij} = \mu(d_i, d_j) = \sqrt{\frac{\sum_{a \in A} (f_i(a) - f_j(a))^2}{|A|}}. \tag{10}$$

If the functions h_i, g_i range in $[0,1]$, then $0 \leq d(i,j) \leq 1$. Hence the decision maker set D is represented by a set of points of the unit cube in a Euclidean space E^m.

The supervisor observes, at any step of the decision making process, the position of each member in the committee and informs the more peripheral expert about the opportunity to revise his judgement.

If we set

$$\mu^* = \max\{\mu(i,j)|i,j \in D\},$$

then a measure of the degree of consensus γ can be defined as the complement to one of the maximum distance between two positions of the experts:

$$\gamma = 1 - \mu^* = 1 - \max\{\mu_{ij}|d_i, d_j \in D\}. \tag{11}$$

5 A Game Theoretic Point of View

We now relate consensus with the construction of a winning coalition in a cooperative game where players are decision makers and utility transfer is allowed. A different game theoretic point of view in dealing with consensus is developed in [12].

We assume given an integer k such that $1 < n/2 < k \leq n$. Let us define a *majority at level* k as a set of decision makers having at least k elements.

Moreover we assume that the members of the committee are points of a metric space (S, μ). In particular, following the notation given in Sec. 6, every decision maker d_i is a point $(f_i(a))_{a \in A}$ of the space R^m with the metric μ given by (10).

Let now δ be a positive real number. We say that q members in the committee agree at level δ if they belong to a ball of diameter in the metric space (S, μ). Reaching consensus can be interpreted as a cooperative game with side payments in which the admissible coalitions are the ones contained in at least a ball of diameter δ of the space (S, μ).

The important concept of admissible coalition was considered in [14]. Admissibility was related with constraints that were in nature ethical, social, etc. Our point of view is a geometrical interpretation of admissibility concept in order to describe possible modifications of coalitions; what is studied, with different methods, also in [14].

Let K be the set of the admissible coalitions. The set K is not empty because every singleton is contained in at least a ball of diameter δ. We can introduce the following classification of the elements of K. In order to get consensus, a coalition H, whose elements are in number of $|H|$, belonging to K, is said to be:

- *winning*, if $|H| \geq k$;
- *losing*, if the coalition $H^c = D - H \in K$, contains a winning coalition;
- *quasi-losing*, if $|H^c| \geq k$, but H^c does not contain any winning coalition;
- *blocking*, if $|H| < k$ and $|H^c| < k$.

It is worth observing that, while in simple games [18] the whole group of players is a winning coalition, in our framework a coalition with at least

k members is winning if and only if it is included in a ball of diameter δ. Winning and losing coalitions were studied in [18], where all coalitions are considered admissible; whereas we assume as admissible only the coalitions satisfying suitable geometric constraints.

Unlike the classical game theory in which we look for minimal winning coalitions [18], finding the consensus means to look for maximal winning coalitions.

We introduce the following further

Definition. Every maximal winning coalition of K is said to be a *solution* of the consensus reaching problem.

One of the following cases occurs:

1. $D \in K$;
2. $D \notin K$, but the consensus problem has a unique solution;
3. $D \notin K$ and the consensus problem has at least two solutions;
4. $D \notin K$ and the consensus problem has no solutions.

In the case (1.) the consensus is reached and the global score of every alternative is obtained by considering a mean of the scores assigned by the decision makers in D.

In the case (2.), if H is the unique maximal solution, the chairman either assumes H as the set D^* of decision makers that give rise to the group decision, or tries to enlarge the set D^*, by persuading some members of $H^c = D - H$ to change their evaluations.

Let us use the procedure introduced in [3] and [9], we call the *Bastian procedure.*

If an element of H^c moves in a maximal winning coalition B, it is possible that B does not contain H. If it happens we fall in the case (3.) and the coalition H may be broken.

In the case (3.) we can use the Bastian procedure as a dynamical procedure to enlarge maximal winning coalitions. The aim is to obtain a unique final maximal winning coalition D^*.

A situation can happen where the players asked by the chairman to change their evaluations can make strategic choices in order to break some coalitions, and cut off some other players to participate to the final aggregation of evaluations.

An alternative to the Bastian procedure is to decide that the maximal winning coalition to enlarge is the one with the maximum number of players. If there are more winning coalitions with these properties, the coalition to be enlarged is the one, if it is unique, that is included in a ball of minimum diameter.

We can also consider fuzzy coalitions [15] and their fuzzy width.

Also in the case (4.) we can use the Bastian procedure, but it is not sure that there is a step in which we have at least a winning coalition, and so it is possible that there is not a solution of the consensus reaching problem.

Some difficulties for the role of the chairman can arise by the blocking coalitions. These coalitions may prevent to obtain solutions in the case (4.) and may give rise to serious problems to the power of the chairman by using the mentioned procedures. Then the existence of blocking coalitions may induce some corrections to our procedures, such as an activation of a form of bargaining or an evaluation of the power of these coalitions.

6 Conclusion

Let us further interpret our model in terms of a metaphor. The way to construct consensus about a social decision usually depends on the particular subject. Psychological and individual propensities and needs are routed in the behaviours of any decision maker in the committee. Each member in the group embark on his way, or programme, but soon he/she has to take into account also all the others' ways that become more or less apparent in time.

Topology provides mainly for the syntax, that explains the formal rules of changing opinions; while game theory plays the role of a semantics when gives intrinsic meanings and motivations to the members of a coalition to modify their thought or feeling.

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