# **A Generalized Critical State Model for Gas Hydrate-Bearing Sediments**

Rong-tao Yan, Chang-fu Wei\* , Hou-zhen Wei, Hui-hui Tian, and Er-ling Wu

State Key Laboratory of Geomechanics and Geotechnical Engineering, Institute of Rock and Soil Mechanics, Chinese Academy of Sciences, Wuhan 430071, China cfwei@whrsm.ac.cn

**Summary.** Methane gas hydrate, usually found beneath permafrost and in marine continental margin sediments worldwide, has attracted interest as a possible energy resource and as a potential agent in climate change and seafloor instability. Here, in order to describe the mechanical behavior of gas hydrate-bearing sediments (GHBS), an elastoplastic model is proposed based on the framework of a critical state model. The presence of gas hydrate can increase yield stress, enhancing the cohesion, peak strength and stiffness of the sediments. Therefore, GHBS is considered as the bonded material in the proposed model. A new bonding strength parameter is introduced into the yield function to evaluate the effect of gas hydrate on the yield behavior of the earth materials. Bonding strength of the GHBS subjected to mechanical loading and/or hydrate dissociation may be drastically reduced, causing strain softening after peak strength. Dilatancy is assumed to be a function of the bonding strength and the stress ratio, instead of the single parameter stress ratio, which can reflect the direction of plastic strain increment more realistically. The proposed model can transform into the modified Cam Clay model when hydrate saturation reduces to zero. Finally, the proposed model has been used to predict the stress-strain behaviors of GHBS in triaxial tests, and it is demonstrated that the proposed model has the capability to describe the behavior of GHBS.

**Keywords:** gas hydrate, sediments, critical state model.

# **1 Introduction**

Gas hydrate, a crystalline solid composed of natural gas and hydrogen-bonded water molecules, is formed at the relatively high pressure and low temperature conditions present along many continental margins and in the permafrost regions. As a potential source of fuel, hydrate dissociation can destabilize hydrate layer and potentially result in large landslides and soil [failu](#page-7-0)res [1]. Due to the practical importance and fundamental interest of analyzing submarine stability, more and more research efforts have been devoted to investigating the response of gas-hydratebearing sediments (GHBS) to thermal and mechanical disturbances[1, 2].

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<sup>\*</sup> Corresponding author.

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Gas hydrate may cement sediments when it reaches at sufficient concentrations, and the strength and stiffness tend to increase with hydrate saturation *s*<sub>1</sub>. Besides, growth habit can have a profound effect on sediment properties. In contrast, when hydrate in the sediment dissociates, the strength and stiffness will decrease, which induces a landslide[3-7].

In order to estimate the slope stability, it is essential to develop a numeric simulator including formation and/or dissociation behavior of hydrate, thermal properties of the reservoirs, permeability and mechanical behaviors of the reservoirs and so on[2]. Therefore, a constitutive model which can describes stress-strain relationship of hydrate- bearing sediments have to be developed.

In this paper, a general elastoplastic constitutive model is developed to describe the mechanical behavior of GHBS subjected to mechanical loading and/or hydrate dissociation in the framework of the modified Cam clay model, and the performance of the constitutive model is demonstrated by comparing the theoretical simulations with experimental data on the mechanical behavior of GHBS.

### **2 Structure of the Constitutive Model**

There are three distribution modes of hydrate in sediments: 1) pore-filling, Hydrate grows freely in the pore spaces without bridging neighboring particles; 2) loading-bearing, Hydrate bridges the neighboring grains and contributes mechanical stability to the grains skeleton by becoming part of the load-bearing framework; 3) cementation, Hydrate grows at inter-granular contacts. The pore-filling mode has no effect on the strength of GHBS at low hydrate saturation. It may become a loading-bearing mode when hydrate saturation is above a critical saturation 25-30% [8].

Here GHBS is considered as a bonded soil regardless of the hydrate formation mode. Based on the previous researches for bonded soils[9-11], a constitutive framework for GHBS is presented based on the framework of the modified Cam clay model. It is assumed that the deformation of the GHBS is infinitesimal and the strain increment can be decomposed into an elastic part  $d\varepsilon^{\epsilon}$  and a plastic part  $d\varepsilon^{\prime}$ , i.e.,

$$
d\varepsilon = d\varepsilon^e + d\varepsilon^p \tag{1}
$$

# *2.1 Elastic Behavior*

According to the theory of elastoplasticity, only elastic deformation occurs for stress excursions within yield boundary. The elastic response is expressed as:

$$
d\varepsilon_{\nu}^{e} = \left[\kappa_{\nu}/(1+e)p\right]dp\tag{2-a}
$$

$$
d\varepsilon_s^\varepsilon = dq/3G = [3(1-2\nu)(1+e)p] \cdot dq/[2(1+\nu)\kappa_h]
$$
 (2-b)

where  $\kappa_h$  is the gradient of unloading or swelling line of GHBS,  $e_v$  is the void ratio and  $\nu$  is the Poisson's ratio and independent of hydrate saturation[4]. Due to the cementation of hydrates to soils, the stiffness may be enhanced [7]. Therefor  $\kappa$ <sup>*h*</sup> is assumed to be a function of hydrate saturation  $\delta$ <sub>*si*</sub>, i.e.,

$$
\kappa_{h} = \kappa_{0} \cdot e^{\zeta \cdot (S_{h} - 0.25 \cdot L)}
$$
\n(3)

Here,  $\kappa$  is the gradient of unloading or swelling line for soils without hydrates,  $\zeta$  is a material parameter and can be acquired by fitting Eq. (3) with experimental data. *L* is the index of hydrates formation mode,  $L = 0$  for cementation and  $L = 1$  for pore filling.

#### *2.2 Yield Function for GHBS*

In order to account for the influence of hydrate formation on the mechanical behavior for GHBS, we introduce a parameter called bonding strength  $p_b$  to modify the mean effective stress as

$$
p^* = p + p_b \tag{4}
$$

Thus, the modified stress ratio can be written as

$$
\eta^* = q/p^* = q/(p + p_b) \tag{5}
$$

Based on the framework of the modified Cam clay model, the yield function for GHBS is given by [10-11]

$$
f = p^*(p^* - p_b - p_c - p_m) + q^2 / M^2
$$
 (6)

$$
p_m = k \cdot p_b \tag{7}
$$

In Equations (6)-(7),  $p_c$  plays a role similar to the preconsolidation pressure for uncemented soils,  $p_m$  controls the growth rate of the size of the initial elastic domain resulting from the intergranular bonding, and  $k$  is a model parameter and assumed to be 1.0 here for simplicity. *M* is the gradient of the failure envelope in the q-p plane, and a lot of experimental results demonstrated that *M* is independent of hydrate saturation [3-6]. The yield surface for GHBS is illustrated in the Fig. 1.



**Fig. 1.** Effect of hydrate cementation yield and decementation on yielding

### *2.3 Harding Rule and Decementation Law*

Based on the results of triaxial compression tests [4-7], GHBS samples clearly show the effects of decementation and dilatance. To capture these features, deviatoric plastic strain,  $\mathcal{E}_s^p$ , is introduced into the hardening rule, which is expressed as

$$
dp_c = \left[ (1+e) p_c / (\lambda_h - \kappa_h) \right] \cdot [d\varepsilon_v^p + M(1 - e^{-\xi \cdot p_b}) d\varepsilon_s^p]
$$
 (8)

where  $\lambda_h$  is the plastic compressibility coefficient of GHBS. Sultan & Garziglia [8] suggested that

$$
\lambda_{h} = \lambda_{0} [1 - (1 - \lambda_{1} / \lambda_{0}) (1 - \exp(-\beta \cdot (S_{h} - 0.25 \cdot L)))]
$$
\n(9)

Here,  $\lambda_0$  is the plastic compressibility coefficient of the soils without hydrates,  $\lambda$ <sub>1</sub> is the plastic compressibility coefficient of hydrate phase and equal to 0.00147 [17],  $\beta$  defines the slope of change of the compressibility with the hydrate saturation.

A decementation process can be divided into two steps: (1) Local decementation takes place before the peak strength, and (2) the collapse of hydrate cementation occurs once the peak strength is arrived.

Here, the content of decementation is assumed to be related to the plastic deviatoric strain  $\varepsilon_s^p$ , and the following equation is proposed to describe the phenomenon, i.e.,

$$
dp_{b} = p_{b}(-\rho_{b} \cdot d\varepsilon_{s}^{p})
$$
\n(10)

where  $\rho_b$  is a material parameter. In order to distinguish the two steps of decementation,  $\rho_h$  is assumed a value of  $k_1$  or  $k_2$ , which is shown in Fig.2.



**Fig. 2.** Schematic diagram for variation of  $p_k$  with plastic strain

# 2.4 Defining  $p_h$

Here it is proposed that  $p<sub>h</sub>$  varies with hydrate saturation  $S<sub>h</sub>$  according to the following relationship:

$$
p_{b} = A \cdot (S_{b} - 0.25 \cdot L)^{B} + C \tag{11}
$$

where  $A, B, C$  are model parameters, which can be determined by fitting Eq. (11) with experimental data. *L* is the index of hydrates formation mode,  $L=0$ for cementation and  $L = 1$  for pore filling.

# *2.5 Flow Rule*

Li and Dafalias [13] pointed out that dilatancy *d* depends not only upon stress ratio  $\eta$ , but also upon the internal state of material. Hereby,  $p_h$  is introduced into the dilatancy *d* as an internal state variable, and an equation is given to describe the dilatancy *d* for GHBS:

$$
d = d(\eta^*, p_{\nu}) = Me^{-\xi \cdot p_{\nu}} - \eta^* \tag{12}
$$

where  $\xi$  is a model parameter.

### **3 Parameter Determination**

Summarily, 12 parameters are introduced into the proposed model, and they are  $\lambda_0, \kappa_0, \nu, M, \zeta, \beta, A, B, C, \zeta, k_1, k_2$ . The first four model parameters  $(\lambda_0, \kappa_0, \nu, M)$ are intrinsic soil properties, which are independent of the hydrate saturation. These parameters have the same meaning as those adopted in the Modified Cam Clay model, and can be easily determined.  $\zeta$  and  $\beta$  are determined by fitting the  $k_{h} - S_{h}$  and  $\lambda_{h} - S_{h}$  curves with the results of isotropic compression/swelling tests on the GHBS samples at two different hydrate saturation.  $A, B, C$  can be calculated from the curve of  $p_i - S_i$  at three different hydrate saturation, the  $p_i$ of the sample can be deduced from the initial yield point on the stress-strain curve.

Parameter  $\xi$  significantly affects the plastic strain direction, but has no distinctive effect on the stress-strain relationship. Parameter  $k_i$  represents local decementation and has slight effect on the peak strength and the dilatancy. Parameter  $k<sub>z</sub>$  influences the strain-softening behavior in such a way that the deviatoric stress decreases more rapidly with a larger  $k_1$ . Parameters  $\xi$ ,  $k_1$  and  $k_2$  can be estimated from the stress-strain relationship.

### **4 Verification of the GHBS Model**

The constitutive model developed was used to simulate the results of triaxial drained tests, which were performed by Masui et al. [14]. The model parameters are listed in Table 1. In Figures 3 and 4, the model simulations are compared with experimental data.

Parameters	Values			Values	
	Type A	Type B	parameters	Type A	Type B
$\boldsymbol{e}$	0.65	0.65	B	0.7	0.7
$\lambda_{0}$	0.02	0.02	$\mathcal C$	0.9	0.55
$K_0$	0.012	0.012	$\boldsymbol{M}$	1.2	1.2
	2.0	2.0	ξ	0.6	0.42
	$-1.395$	$-1.395$	$k_{1}$	0.2	0.2
$\mathcal V$	0.15	0.15	$k_{2}$	5	5
А	3.5	8.255			

**Table 1.** Model parameters of the GHBS model for GHBS (from Masui et al.,2008)[14]

In Figure 3, the soil samples with different hydrate saturations (marked as type A) are formed by pore filling; in Figure 4, the soil samples with different hydrate saturations (marked as type B) are formed by cementation. It can be seen that the model predictions agree very well with the experimental data, showing that the proposed model is capable of capturing the main features of the behavior of GHBS, such as the increase of stiffness, peak shear strength, dilatancy and strain softening effect.



**Fig. 3.** Comparison of experimental and simulated triaxial drained test results (Type A) [14]. Circles represent experimental data, and solid curves represent simulations.



**Fig. 4.** Comparison of experimental and simulated triaxial drained test results (Type B) [14]. Circles represent experimental data, and solid curves represent simulations.

# **5 Conclusions**

In this study, an elastoplastic model is proposed based on the framework of the modified Cam clay model, in which GHBS is considered as a bonded material. A new bonding strength parameter is introduced into the yield function to evaluate the effect of gas hydrate on the yield behavior of the GHBS, and dilatancy is assumed to be a function of the bonding strength and the stress ratio. It is demonstrated that the proposed model can captures very well the main features of the behavior of GHBS samples.

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