Binary Medium Model for Rock Sample

Enlong Liu* and Jianhai Zhang

College of Water Resource and Hydropower
Sichuan University
Chengdu, 610065, P.R. China
liuenlong@scu.edu.cn

Summary. Rock materials exhibit strain softening at low confining pressure. How to accurately describe the strain softening of rock materials is very important for rock engineering. A new model, referred to as Binary Medium Model for rock matrials, is formulated under the theoretical framework of breakage mechanics for geomaterials. In the model, rock materials is conceptualized as binary medium consisting of bonded blocks and weakened bands which are idealized as bonded elements and frictional elements respectively. During the loading process the bonded elements gradually fracture and transform to be the frictional elements. The deformation properties of the bonded elements are described by ideal elastic-brittle materials and those of the frictional elements are described by hardening elasto-plastic materials. Two sets of breakage parameters, i.e., breakage ratios and strain concentration coefficients, describing the influence of rock structure on the process of failure, are introduced. The proposed model has been used to predict the behavior of sandstone sample in triaxial compression test. By making comparisons of predictions with experimental data it is demonstrated that the new model provides satisfactory modeling of many important features of the behavior of rock materials.

Keywords: binary medium model, rock materials, strain softening, breakage parameters.

1 Introduction

The failure processes and constitutive properties of rock materials are one of the important problems investigated by the researchers studying on solid mechanics, materials science, geophysics and geological engineering, and the research performed has very profound help in constructing the projects related to rock engineering [1]. There are many constitutive models for rock materials proposed to describe their stress-strain relationship. Based on the results of Drucker and Cam-clay model, Sandler et al. proposed the Cap model for rock materials and the associated algorithm including a FORTRAN subroutine [2, 3]. With the development of damage mechanics and fracture mechanics, many researchers established many constitutive models for rock materials [4-6] including the fracture-damage model,

* Corresponding author.

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the coupled elasto-plasticity and damage model, the elasto-brittle-fractal damage model and the statistical damage model. How to describe reasonably the strain softening under low confining pressure and strain hardening under high confining pressure of rock materials, however, is a hot topic and challenging theoretical problem.

A new constitutive model, referred to as Binary Medium Model for rock materials, is formulated here. Rock materials are conceptualized as binary medium consisting of bonded blocks (bonded elements) and weakened bands (frictional elements) here. The deformation properties of the bonded elements are described by ideal elastic-brittle materials and those of the frictional elements are described by hardening elasto-plastic materials. Finally the proposed one has been used to predict the behavior of sandstone sample in triaxial compression test.

2 Binary Medium Model for Rock Materials

The rock materials, idealized as binary medium materials, are heterogeneous materials whose constitutive equations can be formulated by using the mesomechanics theory. For a representative element volume (REV), which can be considered as continuum medium macroscopically and includes infinite meso-characteristics microscopically, we can obtain the following relations by the homogenization theory of heterogeneous materials [7]

$$\{\sigma\} = (1-\lambda)\{\sigma\}_b + \lambda\{\sigma\}_f \tag{1}$$

$$\{\varepsilon\} = (1-\lambda)\{\varepsilon\}_b + \lambda\{\varepsilon\}_f \tag{2}$$

in which $\{\sigma\}$, $\{\varepsilon\}$ are respectively the average stress and strain of the element, $\{\sigma\}_b$, $\{\varepsilon\}_b$ are respectively the local stress and strain of the bonded element, $\{\sigma\}_f$, $\{\varepsilon\}_f$ are respectively the local stress and strain of the frictional element, and λ is the volumetric ratio of frictional elements.

We define

$$\sigma_m = \sigma_{kk}/3, \quad \sigma_s = \sqrt{3s_{ij}s_{ij}/2} \tag{3}$$

and

$$\varepsilon_v = \varepsilon_{kk} \quad , \qquad \varepsilon_s = \sqrt{2e_{ij}e_{ij}/3} \tag{4}$$

where $s_{ij} = \sigma_{ij} - \sigma_{kk}\delta_{ij}$, $e_{ij} = \varepsilon_{ij} - \varepsilon_{kk}\delta_{ij}/3$, and δ_{ij} is kronecker symbol.

In the following the stress can be divided into the mean stress and deviatoric stress and the corresponding strain can be divided into the volumetric strain and generalized shear strain respectively. By introducing breakage ratio λ_v and λ_s which reflect respectively the influence of mean stress and deviatoric stress, we can rewrite equation (1) and (2) as the following

$$\sigma_m = (1-\lambda_v)\sigma_{mb} + \lambda_v \sigma_{mf} \tag{5}$$

$$\varepsilon_v = (1-\lambda_v)\varepsilon_{vb} + \lambda_v \varepsilon_{vf} \tag{6}$$

$$\sigma_s = (1-\lambda_s)\sigma_{sb} + \lambda_s \sigma_{sf} \tag{7}$$

$$\varepsilon_s = (1-\lambda_s)\varepsilon_{sb} + \lambda_s \varepsilon_{sf} \tag{8}$$

where the subscript *b* denotes the bonded element and the subscript *f* denotes the frictional element.

For the bonded element, the stress-strain relation can be assumed as follows

$$d\sigma_{mb} = K_{mvb} d\varepsilon_{vb} + K_{msb} d\varepsilon_{sb} \tag{9}$$

$$d\sigma_{sb} = K_{svb} d\varepsilon_{vb} + K_{ssb} d\varepsilon_{sb} \tag{10}$$

and for the frictional element, the stress-strain relation can be assumed as follows

$$d\sigma_{mf} = K_{mvf} d\varepsilon_{vf} + K_{msf} d\varepsilon_{sf} \tag{11}$$

$$d\sigma_{sf} = K_{svf} d\varepsilon_{vf} + K_{ssf} d\varepsilon_{sf} \tag{12}$$

in which K_{mvb}, K_{msb}, K_{svb}, K_{ssb}, K_{mvf}, K_{msf}, K_{svf} and K_{ssf} can be determined by the deformation properties of the bonded element and the frictional element.

Introducing strain concentration coefficients c_v and c_s, which satisfy the following expressions

$$\varepsilon_{vb} = c_v \varepsilon_v \tag{13}$$

$$\varepsilon_{sb} = c_s \varepsilon_s \tag{14}$$

From equation (13) and (14) we can obtain

$$d\varepsilon_{vb} = B_v d\varepsilon_v \tag{15}$$

$$d\varepsilon_{sb} = B_s d\varepsilon_s \tag{16}$$

in which $B_v = c_v^0 + \dfrac{\partial c_v}{\partial \varepsilon_v}\varepsilon_v^0$ and $B_s = c_s^0 + \dfrac{\partial c_s}{\partial \varepsilon_s}\varepsilon_s^0$.

Through equations (5) to (16), we can obtain the following equations

$$
\begin{aligned}
d\sigma_m &= [(1-\lambda_v^0)K_{mvb}B_v + K_{mvf} - K_{mvf}(1-\lambda_v^0)B_v]d\varepsilon_v \\
&\quad + [(1-\lambda_v^0)K_{msb}B_s + \frac{\lambda_v^0}{\lambda_s^0}K_{msf} - \frac{\lambda_v^0}{\lambda_s^0}K_{msf}(1-\lambda_s^0)B_s]d\varepsilon_s \\
&\quad - K_{mvf}d\lambda_v\frac{1}{\lambda_v^0}(1-c_v^0)\varepsilon_v^0 - \frac{\lambda_v^0}{\lambda_s^0}d\lambda_s K_{msf}\frac{1}{\lambda_s^0}(1-c_s^0)\varepsilon_s^0 + \frac{d\lambda_v}{\lambda_v^0}(\sigma_m^0 - \sigma_{mb}^0)
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
d\sigma_s &= [(1-\lambda_s^0)K_{svb}B_v + \frac{\lambda_s^0}{\lambda_v^0}K_{svf} - \frac{\lambda_s^0}{\lambda_v^0}K_{svf}(1-\lambda_v^0)B_v]d\varepsilon_v \\
&\quad + [(1-\lambda_s^0)K_{ssb}B_s + K_{ssf} - K_{ssf}(1-\lambda_s^0)B_s]d\varepsilon_s \\
&\quad - \frac{\lambda_s^0}{\lambda_v^0}K_{svf}d\lambda_v\frac{1}{\lambda_v^0}(1-c_v^0)\varepsilon_v^0 - d\lambda_s K_{ssf}\frac{1}{\lambda_s^0}(1-c_s^0)\varepsilon_s^0 + \frac{d\lambda_s}{\lambda_s^0}(\sigma_s^0 - \sigma_{sb}^0)
\end{aligned} \tag{18}
$$

where the superscript 0 indicates the initial value in a incremental step.

3 Determination of Model Paramters

The stress-strain properties of the bonded element can be assumed as be elastic-brittle, which has the following expression

$$d\sigma_{mb} = K_b d\varepsilon_{vb} \tag{19}$$

$$d\sigma_{sb} = 3G_b d\varepsilon_{sb} \tag{20}$$

where K_b and G_b are the bulk modulus and shear modulus of the bonded element respectively, which can be determined by the initial value of the sample tested.

The frictional element is transferred from the bonded element and there are no bonding between them, whose stress-strain properties can be described by the constitutive characteristics of sample broken totally subjected to loading. Here we use the elastic-plastic relationship of the sample to describe it.

The yield surface can be assumed as

$$f_f = \frac{\sigma_{mf}}{\sigma_{sf}} - M_f = 0 \tag{21}$$

in which M_f are critical stress ratio. The ratio of the plastic volumetric strain to the plastic shear strain of the frictional element can be expressed as

$$d_f = \frac{\Delta\varepsilon_{vf}^p}{\Delta\varepsilon_{sf}^p} = M_f - \eta_f \tag{22}$$

where $\eta_f = \sigma_{mf} / \sigma_{sf}$ is the effective stress ratio. By equation (21) and (22), we can obtain

$$\Delta\sigma_{mf} = (K_f + A_f K_f^2 \eta_f d_f) \Delta\varepsilon_{vf} - 3A_f K_f G_f d_f \Delta\varepsilon_{sf} \tag{23}$$

$$\Delta\sigma_{sf} = (3A_f K_f G_f \eta_f)\Delta\varepsilon_{vf} + (3G_f + 9A_f G_f^2)\Delta\varepsilon_{sf} \tag{24}$$

where $K_{mvf} = K_f + A_f K_f^2 \eta_f d_f$, $K_{msf} = -3A_f K_f G_f d_f$, $K_{svf} = 3A_f K_f G_f \eta_f$, $K_{ssf} = 3G_f + 9A_f G_f^2$, $A_f = 1/(K_f \eta_f d_f - 3G_f - H_{pf})$, $K_f = G_f \dfrac{2(1+\nu_f)}{3(1-2\nu_f)}$ is the elastic bulk modulus, $G_f = P_a G_{0f} (\sigma_c / \sigma_r)^{N_R} (\sigma_{mf} / P_a)^{0.5}$ is the elastic shear modulus, $H_{pf} = h_f G_f (\dfrac{M_f}{\eta_f} - 1.0)^{n_f}$ is the hardening modulus, P_a is the atmosphere pressure, σ_r is the residual strength, ν_f is Possion's ratio, G_{0f}, N_R, h_f and n_f are parameters.

The breakage ratios and strain concentration parameters are assumed as follows

$$\lambda_v = 1 - e^{-a_v \varepsilon_v^{n_v}} \tag{25}$$

$$\lambda_s = 1 - e^{-a_s \varepsilon_s^{n_s}} \tag{26}$$

$$c_v = e^{-t_v \varepsilon_v^{r_v}} \tag{27}$$

$$c_s = e^{-t_s \varepsilon_s^{r_s}} \tag{28}$$

in which $a_v, a_s, n_v, n_s, t_v, r_v, t_s$ and r_s are model parameters.

4 Testing Verification

For the sandstone samples tested under the confining pressures 2MPa, 10MPa and 50MPa using the apparatus of MTS815 concrete and rock test system, the triaxial results, tested and predicted, are presented in Fig. 1 and Fig. 2. The parameters used in the model are as follows, $K_b = K_{b0}(\sigma_{3b} / P_a)^{N_K}$, $G_b = G_{b0}(\sigma_{3b} / P_a)^{N_G}$, $K_{b0} = 760.3MPa$,

$G_{b0} = 3893.0MPa$, $N_K = 0.533$, $N_G = 0.285$, $G_{0f} = 4705$, N_R = 1.341, $\nu_f = 0.03$, $h_f = 0.56$, $n_f = 4.2$, $M_f = 2.5$, $a_v = 8.0$, $n_v = 0.97$, $a_s = 180.0$, $n_s = 1.02$, $t_v = 1.0$, $t_s = 0.95$, $r_v = 2.7$ and $r_s = 3.25$.

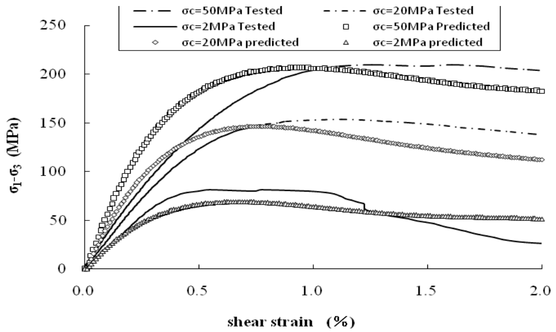

Fig. 1. The deviatoric stress-shear strain relationship

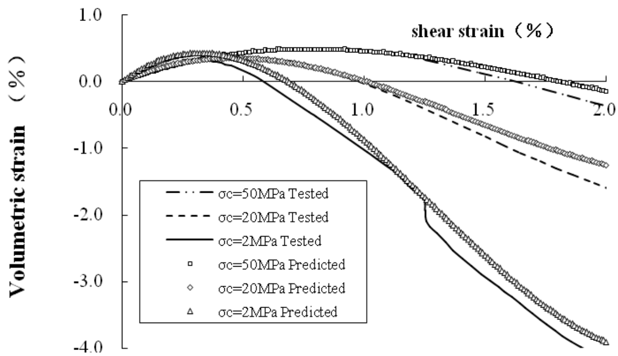

Fig. 2. The volumetric strain-shear strain relationship

5 Conclusions

Under the theoretical framework of breakage mechanics for geomaterials, the rock materials is conceptualized as the binary medium consisting of the bonded elements described by ideal elastic-brittle materials and the frictional elements described by hardening elasto-plastic materials respectively. By introducing breakage parameters including breakage ratios and local strain coefficients, a new constitutive model for rock materials is proposed and verified with the triaxial results of sandstone samples, which demonstrated that the proposed model can simulate the constitutive properties of sandstone sample properly.

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