Distributed Algebraic Connectivity Maximization for Robotic Networks: A Heuristic Approach

Andrea Simonetto, Tamás Keviczky, and Robert Babuška

Abstract. We consider a weighted communication graph in a network of mobile robots, and its associated Laplacian whose entries depend on the pairwise distance between the robots. We propose a heuristic distributed solution for the maximization of the algebraic connectivity of the graph by moving the robots to appropriate positions. Our approach is optimization-based and can be extended to handle various constraints, such as the robots' dynamics. Our proposed distributed solution uses local algorithms that utilize information only from nearby neighboring robots. Numerical simulations show the applicability and effectiveness of the algorithm and indicate that in certain cases the proposed distributed solution can perform better than the centralized version.

1 Introduction

Groups of autonomous mobile robots that communicate with one another to achieve a common goal are considered as a key enabling technology in several applications ranging from underwater and space exploration [1, 2], to search and rescue [3], fire monitoring [4] and other surveillance applications [5]. These robotic teams are envisioned to possess on-board processing capability, but the common task can only be achieved through information exchange among the members and possibly a base station. Such multi-vehicle teams are thus often referred to as robotic networks. Among the several engineering and research questions these applications pose, maintaining connectivity between the individual robots and increasing the communication quality given the environmental constraints and objectives, have fundamental importance. Many different types of coordination and control frameworks that have been proposed recently for cooperating robotic teams rely on some type of agreement protocol or consensus process that leads to coordinated team actions [6, 7, 8].

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A. Martinoli et al. (Eds.): Distributed Autonomous Robotic Systems, STAR 83, pp. 267–279.
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Since these protocols typically assume only local communication among "neighboring" units, the interconnection topology of the underlying communication graph influences their effectiveness profoundly. Motivated by the significant role it plays in the performance of many distributed control methods, we study distributed solutions for maximizing the algebraic connectivity of the communication graph (often denoted as λ_2) in mobile robotic networks. This parameter is the second smallest eigenvalue of the communication graph's Laplacian matrix, and it dictates the convergence properties of consensus protocols [9, 10]. We focus on distance-based connectivity maximization with minimum separation constraints, as opposed to ensuring line-of-sight connectivity in an obstacle-rich environment [11]. Maximization of λ_2 is also important for collaborative target tracking [12], where a network of mobile robots strive for increased accuracy of the joint position estimate of one or more moving objects [13, 14, 15, 16]. Besides an increase in accuracy, a positive λ_2 also ensures that the network stays connected during the collective motion.

A few examples of decentralized λ_2 maximization have appeared in the literature so far. These are typically either limited to only specific scenarios, or imply heavy communication requirements. Often the proposed approaches are not derived from a centralized solution, in other words the formulated local problems are not directly related to the solution of the centralized one. Without such a consistency, there are typically no guarantees that the algebraic connectivity is maximized. The approach in [17] uses a two-step distributed solution, which relies on supergradients and potential functions. The required communication load scales with the square of the graph diameter. Other approaches proposed in [18] and [19] make use of auctions and game theory, respectively, and consider maintaining connectedness of the graph as the main priority. Although the communication requirement is limited in these algorithms, they are designed via a bottom-up approach, i.e., starting from local problems, and a potential increase of λ_2 is usually a simple by-product of their solution without analytical guarantees.

In this paper, we present a heuristic distributed approach for the λ_2 maximization problem as formulated by [20, 21, 12] in a centralized framework. Our perspective is model-based optimization, which allows additional constraints (e.g., the dynamics of the robots) to be included explicitly in the problem formulation. Moreover, we believe that this approach can eventually lead to a certain type of consistency with regards to the centralized solution. The proposed solution can also be extended to incorporate other interesting scenarios, such as collaborative target tracking. The proposed distributed approach relies on local problems that are solved by each robot using information only from nearby neighbors. Specifically, two communication policies are introduced to respect the potentially limited communication and computation capabilities of the robots. Simulation results support the efficacy of the approach and show interesting properties of the algorithms. For instance, given the nonlinear/nonconvex nature of the problem, in certain scenarios the distributed solutions converge to a higher λ_2 value than the centralized ones.

The paper is organized as follows. Section 2 formulates the centralized problem as suggested by [20, 21, 12]. The proposed distributed approach and communication policies are described in Section 3. Numerical simulations are shown in Section 4 to assess the performance of the distributed solutions with respect to centralized schemes. Conclusions and open issues are discussed in Section 5.

2 Problem Formulation

We consider a network of N agents. The agents represent mobile robots and the network encodes undirected communication links, meaning that if two agents are connected, they can communicate with each other. As a general notation $a_i(k)$ represents the value of the variable a for agent i at time k. Let $x(k) \in \mathbb{R}^{2N}$ be the collection of the agents' positions on a 2-D plane, i.e., $x(k) = (x_1^\top(k), \ldots, x_N^\top(k))^\top$. Although our scheme can be extended to more complicated robot dynamics, for simplicity of exposition we consider agents with the following discrete-time dynamics

$$x_i(k) = x_i(k-1) + v_i(k-1)\Delta t \tag{1}$$

where $v_i(k)$ is the velocity control input and Δt the sampling time. We use graph-theoretical tools to model the network. The set $\mathscr{S}$ contains the indices of the mobile agents (nodes), with cardinality $N = |\mathscr{S}|$. We use $\mathscr{E}$ to indicate the set of communication links, i.e., the edges $\{(i,j)|i,j \in \mathscr{S}\}$. The graph $\mathscr{G}$ is then expressed as $\mathscr{G} = (\mathscr{S}, \mathscr{E})$. Let the graph be connected initially, the agent clocks synchronized, and assume perfect communication (no delays or packet losses). The agents with which agent i communicates are called neighbors and are contained in the set $\mathscr{N}_i$. Note that node i is not included in the set $\mathscr{N}_i$. We define $\mathscr{J}_i = \mathscr{N}_i \cup \{i\}$ and $N_i = |\mathscr{J}_i|$.

We define a set of Laplacian matrices $\mathscr{L}$ associated with $\mathscr{G}$ as

$$\mathscr{L} = \{L \in \mathbb{R}^{N \times N} | L = L^\top, \ell_{ij} = 0 \text{ iff } (i,j) \notin \mathscr{E}, L\mathbf{1} = \mathbf{0}\}$$

The entries of a Laplacian matrix L are defined as

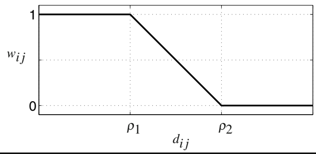
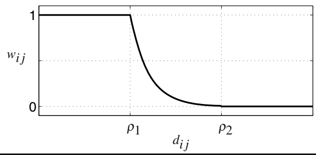
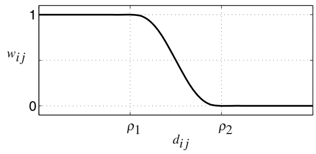
$$\ell_{ij} := \begin{cases} 0 & (i,j) \notin \mathscr{E} \\ -w_{ij} & (i,j) \in \mathscr{E}, i \neq j \\ \sum_{l \neq i} w_{il} & i = j \end{cases} \tag{2}$$

where the positive weights w_{ij} represent the "connection strength" between agents i and j. The weights themselves depend on the physical distance between the agents. For this purpose we introduce the square distance matrix D, whose entries d_{ij} are defined as

$$d_{ij} = ||x_i(k) - x_j(k)||^2. \tag{3}$$

The value of the normalized weights will be 1 representing a "strong connection" if d_{ij} is less than a certain threshold, i.e., $d_{ij} \leq \rho_1$, with $\rho_1 > 0$. On the other hand, agents will not be connected at all ($w_{ij} = 0$) for $d_{ij} > \rho_2$, with $\rho_2 > \rho_1$. For $\rho_1 < d_{ij} \leq \rho_2$ the agents are connected with a connection strength that decreases smoothly with their distance. Typically, spatially decaying functions are used for the weights w_{ij} [22], and a few of them are shown in Table 1. Case (1) is a linear representation which is continuous but not differentiable, case (2) is the exponential function of [12], which is not differentiable and also discontinuous at ρ_2, while case (3) is a

Table 1 Possible choices of weighting functions. Case (1) is a linear representation, case (2) is the exponential function of [12], while case (3) is a 5-th order polynomial description.

Case	Function	Figure
(1)	$w_{ij} := \begin{cases} 1 & d_{ij} < \rho_1 \\ \frac{1}{\rho_2 - \rho_1}(\rho_2 - d_{ij}) & \rho_1 \le d_{ij} < \rho_2 \\ 0 & d_{ij} \ge \rho_2 \end{cases}$	
(2)	$w_{ij} := \begin{cases} 1 & d_{ij} < \rho_1 \\ \exp\left(-\frac{5(d_{ij} - \rho_1)}{\rho_2 - \rho_1}\right) & \rho_1 \le d_{ij} < \rho_2 \\ 0 & d_{ij} \ge \rho_2 \end{cases}$	
(3)	$w_{ij} := \begin{cases} 1 & d_{ij} < \rho_1 \\ \sum_{p=0}^{5} \alpha_p d_{ij}^p & \rho_1 \le d_{ij} < \rho_2 \\ 0 & d_{ij} \ge \rho_2 \end{cases}$	

polynomial description, which for a suitable choice of the coefficients α_p is both continuous and twice-differentiable.

As a direct consequence of the above definitions, the entries of the Laplacian matrix (2) will depend on the pairwise distance and therefore the position states of the robots, making it state-dependent, which we will denote by $L(x)$. We are interested in the maximization of the algebraic connectivity of the weighted graph by moving the robots to appropriate positions. This goal can be formulated as the following optimization problem [21]:

$$\mathbf{P}(L(x)) : \quad \max_{x,\gamma} \ \gamma \tag{4a}$$

$$\text{s.t.} \quad \gamma > 0 \tag{4b}$$

$$L(x) + \mathbf{1}\mathbf{1}^T \succ \gamma I \tag{4c}$$

where the decision variables are γ and the robot locations x. The optimal value of γ will be the maximum λ_2 for $L(x)$.

This problem would be convex if L was the decision variable, but it is non-convex given that we are optimizing over the positions x and the entries of L are nonlinear functions of x. However, we can obtain an iterative convex approximation following the steps of [20]. First we differentiate (3) with respect to time as

$$2(\dot{x}_i(k-1) - \dot{x}_j(k-1))^\top (x_i(k-1) - x_j(k-1)) = \dot{d}_{ij}(k-1)$$

and then we employ Euler's first-order discretization method to rewrite (3) as

$$d_{ij}(k) = -d_{ij}(k-1) + 2(x_i(k) - x_j(k))^\top (x_i(k-1) - x_j(k-1))$$

In the same way, the weights of the state-dependent Laplacian $L(x)$ are discretized as

$$\begin{aligned} w_{ij}(k) &= w_{ij}(k-1) + \left.\frac{\partial w_{ij}}{\partial d_{ij}}\right|_{d_{ij(k-1)}} (d_{ij}(k) - d_{ij}(k-1)) \\ &= w_{ij}(k-1) + 2\left.\frac{\partial w_{ij}}{\partial d_{ij}}\right|_{d_{ij(k-1)}} (x_i(k) - x_j(k) - x_i(k-1) + x_j(k-1))^\top (x_i(k-1) - x_j(k-1)) \end{aligned}$$

This allows us to consider the maximization of the algebraic connectivity of L as the following iterative convex semi-definite programming (SDP) problem:

$$\mathbf{P}_k\left(L(x), x(k-1), D(k-1), v_{\max}\right):$$

$$\max_{x(k),D(k),\gamma(k)} \quad \gamma(k) \tag{5a}$$

$$\text{s.t.} \quad \mathscr{Q}_1 : \begin{cases} \gamma(k) > 0 \\ L(x(k)) + \mathbf{1}\mathbf{1}^T \succ \gamma(k) I \end{cases} \tag{5b}$$

$$\mathscr{Q}_2 : \begin{cases} \mathscr{Q}_{2.1} : d_{ij}(k) + d_{ij}(k-1) - 2(x_i(k) - x_j(k))^\top (x_i(k-1) - x_j(k-1)) = 0 \\ \mathscr{Q}_{2.2} : \quad d_{ij}(k) > \rho_1, \quad \forall (i,j) \in \mathscr{E} \\ \mathscr{Q}_{2.3} : \quad ||x_i(k) - x_i(k-1)|| \leq v_{\max} \Delta t \quad i = 1, \ldots, N \end{cases} \tag{5c}$$

where the constraint $\mathscr{Q}_{2.2}$ is used both to avoid agents getting too close to each other and to restrict the distances to be positive. This is not automatically ensured by $\mathscr{Q}_{2.1}$ if the agents could move arbitrarily fast. The constraint $\mathscr{Q}_{2.3}$ on the velocity represents the physical limitations of the agents.

The problem $\mathbf{P}_k(L(x), x(k-1), D(k-1), v_{\max})$ in (5) is solved iteratively in each sampling time step and its decision variables are $x(k)$, $D(k)$, and $\gamma(k)$. The problem formulation depends on the values $x(k-1)$ and $D(k-1)$ from the previous time step $k-1$. Here k stands both for the iteration counter and for the discrete time index since in this problem the two concepts are equivalent. Since $x(k)$ and $D(k)$ are considered independent, there could be a possible inconsistency between the distances and the actual position of the agents. This effect can be diminished if in addition to constraint $\mathscr{Q}_{2.1}$, $D(k-1)$ is recomputed based on $x(k-1)$ before each optimization step. Although the original problem (5) can be proven to converge to a local maximum [20], this property may be lost when recomputing $D(k-1)$. This is due to the fact that the λ_2 of $L(x(k-1))$, based on the recomputed $D(k-1)$, may be smaller than $\gamma(k-1)$. However, in the simulated scenarios we consider in Section 4, the algorithm using recomputed $D(k-1)$ has always converged.

Remark 1. We note that in [20] the requirement that the distances $d_{ij}(k)$ form the entries of a square Euclidean Distance Matrix is considered as an additional convex constraint. This would help in reducing the inconsistency effect between $D(k)$ and $x(k)$. However, our experience indicates that this introduces extra rotational rigidity to the graph in the numerical simulations and as a result it has not been included in our problem setup.

The optimization problem that has been described in this section attempts to solve the connectivity maximization problem in a centralized manner using linearization, discretization and an iterative solution approach. In realistic application scenarios however, computing the desired positions and the corresponding motion commands for the robots cannot be performed in a single centralized location due to computational and communication constraints. In the next section, we describe a solution approach that allows the problem to be solved in a distributed fashion, using local computations and limited communication resources, which increases the flexibility of the robotic network and is thus appealing in practice.

3 The Proposed Distributed Approach

In this section we present a distributed approach to solve (5). First, we introduce necessary notation and definitions, then describe our heuristic method and argue why solving local problems leads to a non-decreasing sequence of algebraic connectivity, when considering the linearized Laplacian of the overall network.

In order to describe the local problems each agent will be solving, we define subgraphs that correspond to the agents and their neighborhood. Let $\mathscr{M}_i$ denote the *enlarged* neighborhood for each agent i defined as

$$\mathscr{M}_i = \bigcup_{l \in \mathscr{J}_i} \mathscr{J}_l, \qquad i = 1, \ldots, N \tag{6}$$

whose cardinality will be M_i. We denote the vector containing all the positions of the agents in the set with $x_{\mathscr{M}_i}$, while we call the set of agents belonging to $\partial\mathscr{M}_i$, the bordering agents of $\mathscr{M}_i$ defined as

$$\partial\mathscr{M}_i = \{l | l \in \mathscr{M}_i, l \notin \mathscr{J}_i\}, \qquad i = 1, \ldots, N \tag{7}$$

Figure 1 provides a graphical illustration of this notation. In some situations we will consider a randomly selected connected subset of $\mathscr{M}_i$ that includes agent i. This set will be denoted by $R(\mathscr{M}_i)$ with cardinality RM_i. Following suit, we also define random versions of the border set $R(\partial\mathscr{M}_i)$ and neighborhood set $R(\mathscr{J}_i)$ with cardinality RN_i as

$$R(\partial\mathscr{M}_i) = \{l | l \in R(\mathscr{M}_i), l \notin \mathscr{J}_i\}, \qquad i = 1, \ldots, N \tag{8}$$

$$R(\mathscr{J}_i) = \{l | l \in R(\mathscr{M}_i), l \notin R(\partial\mathscr{M}_i)\}, \qquad i = 1, \ldots, N \tag{9}$$

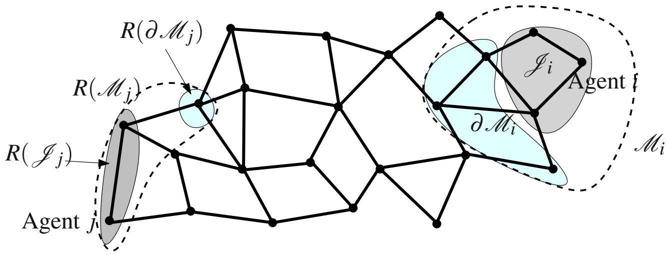


Fig. 1 Notation for the distributed solution

Graphical examples of these definitions are also shown in Figure 1. Finally, we will denote the graph Laplacian associated with subgraph $\mathscr{M}_i$ as L_i with corresponding distance matrix D_i, while the one associated with $R(\mathscr{M}_i)$ as $R(L_i)$ and $R(D_i)$, respectively. We also introduce a scaled maximum velocity $\tilde{v}_{\max,i}$ defined as

$$\tilde{v}_{\max,i} = \left(\sum_{j\in\mathscr{M}_i} \frac{1}{N_j} \right)^{-1} v_{\max}, \quad i = 1,\ldots,N \tag{10}$$

whose value varies from agent to agent. The use of this quantity will be explained later in this section. We consider two possible strategies:

1. Full neighborhood (FN) strategy: the agents are allowed to communicate within the whole enlarged neighborhood $\mathscr{M}_i$. In this case the proposed distributed solution will lead to monotonically increasing connectivity and more importantly, it will respect the constraints on D, meaning that $d_{ij} > \rho_1$ for all i and j. However, the communication requirements and local problem size will increase as the agents get closer to each other and increase their connectivity.
2. Random neighborhood (RN) strategy: the agents are allowed to communicate only with a randomly selected subset of their extended neighborhood $R(\mathscr{M}_i)$. In this case, the overall connectivity may no longer increase monotonically and constraints on D may not always be fulfilled. However, the communication and local problem size can be significantly reduced.

Our algorithms consist of two steps. First, each agent solves the problem $\mathbf{P}^{\text{FN}}_{k,i}$ defined as

$$\mathbf{P}_k(L_i(x_{\mathscr{M}_i}), x_{\mathscr{M}_i}(k-1), D_i(k-1), \tilde{v}_{\max,i}) \tag{11a}$$

$$\text{s.t.} \quad \mathscr{Q}_3:\ x_j(k) = x_j(k-1), \qquad \text{for } j\in\partial\mathscr{M}_i \tag{11b}$$

computing the solution $\hat{x}_{\mathscr{M}_i}(k)$, which is composed of $\hat{x}_{ij}(k)$ for each $j\in\mathscr{M}_i$. Thus, we will call $\hat{x}_{ij}(k)$ the position of agent j as computed by agent i. Note the importance of the extra constraint $\mathscr{Q}_3$ that guarantees monotonically increasing connectivity as will be explained later in this section.

As the second step, the solutions $\hat{x}_{\mathscr{M}_i}(k)$ are shared within the enlarged neighborhood $\mathscr{M}_i$ and averaged according to

$$x_i(k) = x_i(k-1) + \sum_{j\in\mathscr{M}_i} \frac{1}{N_j} \left(\hat{x}_{ji}(k) - x_i(k-1) \right), \qquad i = 1,\ldots,N \tag{12}$$

Algorithm 3 summarizes the method for the FN strategy as described above. For the RN strategy, the algorithm follows the same scheme with the following substitutions: $\mathbf{P}^{\text{FN}}_{k,i} \to \mathbf{P}^{\text{RN}}_{k,i}$, $\mathscr{M}_i \to R(\mathscr{M}_i)$, $\partial\mathscr{M}_i \to R(\partial\mathscr{M}_i)$, $\hat{x}_{\mathscr{M}_i} \to \hat{x}_{R(\mathscr{M}_i)}$, $L_i(\cdot) \to R(L_i(\cdot))$, $D_i \to R(D_i)$, $M_i \to RM_i$, and $N_j \to RN_j$.

The heuristics presented in the above algorithm lead to a solution with monotonically increasing connectivity, i.e., if we consider the resulting global position vector $x(k) = (x_1^\top(k),\ldots,x_N^\top(k))^\top$, the algebraic connectivity of the corresponding global linearized Laplacian $L(x(k))$ would be monotonically increasing in each iteration. In order to justify our algorithm and ensure this property, the extra constraint $\mathscr{Q}_3$ on the border set is necessary. It allows us to show that $L(x(k)) - L(x(k-1)) \succeq 0$ where $x(k)$ is the collection of the locally averaged $x_i(k)$ solutions. This property

Algorithm 1. Distributed Algebraic Connectivity Maximization for FN strategy

1: Input: $x_i(k-1), x_j(k-1),\ j \in \mathscr{M}_i$
2: Compute: $d_{ij}(k-1)$ from input based on (3)
3: Solve: $\mathbf{P}^{\text{FN}}_{k,i}$ computing $\hat{x}_{ij}(k),\ j \in \mathscr{M}_i$
4: Communicate: $\hat{x}_{ij}(k)$ among members of $\mathscr{M}_i$
5: Average: $x_i(k) = x_i(k-1) + \sum_{j \in \mathscr{M}_i} \frac{1}{N_j}\big(\hat{x}_{ji}(k) - x_i(k-1)\big)$
6: Output: $x_i(k)$

follows from the following line of arguments. Consider the local problem $\mathbf{P}^{\text{FN}}_{k,i}$ and its solution comprised of $\hat{x}_{ij}(k)$ for all $j \in \mathscr{M}_i$. Construct a global vector as

$$\hat{x}^{(i)}(k) = (x_1^\top(k-1), \ldots, \hat{x}_{ij}^\top(k), \ldots, x_N^\top(k-1))^\top \tag{13}$$

where we keep those agent positions that have not been optimized fixed, and we update the rest from the solution of the local problem. It is relatively straightforward to see that due to constraint $\mathscr{Q}_3$, $L(\hat{x}^{(i)}(k)) - L(x(k-1)) \succeq 0$, meaning that the new positions $\hat{x}^{(i)}(k)$ do not decrease the algebraic connectivity of the Laplacian matrix. This trivially implies $(L(\hat{x}^{(i)}(k)) - L(x(k-1)))/N_i \succeq 0$ for all i. Thus summing over all agents leads to

$$\sum_{i=1}^{N} \frac{1}{N_i}(L(\hat{x}^{(i)}(k)) - L(x(k-1))) \succeq 0 \tag{14}$$

Considering the weighted sum $x_i(k)$ in (12), and the associated global vector $x(k)$, it can be shown that

$$L(x(k)) = \sum_{i=1}^{N} \frac{1}{N_i} L(\hat{x}^{(i)}(k)) \tag{15}$$

which leads to the desired monotonicity property

$$L(x(k)) \succeq \sum_{i=1}^{N} \frac{1}{N_i} L(x(k-1)) \succeq L(x(k-1)) \tag{16}$$

given that $\sum_{i=1}^{N} \frac{1}{N_i} \geq 1$. With similar arguments, it is possible to argue that feasibility of the local problem constraints imply feasibility of the centralized problem. The above discussion also elucidates the reason for scaling the maximum velocity in the local problems by $(\sum_{j \in \mathscr{M}_i} \frac{1}{N_j})^{-1}$.

Remark 2. In our numerical experiments we have not encountered any infeasibility when using the FN strategy and the original $\mathscr{Q}_{2.2}$ constraint in the local problems (mainly due to the particular choice of the weighting functions and only a few neighbors for each robot). However, in principle, the constraint $\mathscr{Q}_{2.2}$ should be tightened as well by introducing local $\tilde{\rho}_{1_{ij}} \leq \rho_1$. We are currently investigating the most suitable way of incorporating these tightened constraints in the local problem formulations.

Finally, we present a justification for the choice of the enlarged neighborhood set $\mathscr{M}_i$ using the following example. Consider the interconnection shown in Figure 2 and assume that instead of the enlarged neighborhood $\mathscr{M}_i$, the smaller neighborhood $\mathscr{J}_i$ is used. In that case the consistency constraint in $\mathbf{P}_{k,i}^{\mathrm{FN}}$ requires agents $i+1$ and $i-1$ to be fixed. It is easy to see that if the distance between the agents is already at the lower limit $\sqrt{\rho_1}$, all the agents will remain stationary. However, if we considered the enlarged neighborhood $\mathscr{M}_i$ in the local problem instead, the situation would be different. The bordering agents 1 and N would rotate towards the center of the string, to connect with 3 and $N-2$, respectively.

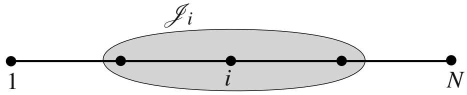


Fig. 2 Illustrative example for justifying the choice of the extended neighborhood set

4 Simulation Results

In this section, we present numerical simulation results to illustrate how the different algorithms perform with respect to the centralized scheme. In particular, we will analyze first the FN strategy and observe that, in some cases, it converges to a higher λ_2 value than the centralized solution. Then we proceed to investigate RN strategies, which lead to reduced communication load for the price of losing the monotonically increasing connectivity property and persistent feasibility of the minimum distance constraint $\mathscr{Q}_{2.2}$. We use the benchmark problem of [20] to relate our results to the literature. This scenario starts with $N=6$ agents on a line forming a connected graph. The initial position vector is $x_i(0) = [1+1.05(i-1), y_i]^\top$, with $y_i \sim (0,\sigma)$, meaning that y_i is drawn from a Gaussian distribution $(0,\sigma)$, with mean 0 and standard deviation $\sigma = 0.1$. Randomness is added to test the algorithms' sensitivity to slightly different initial conditions. The other simulation parameters include a weight function of type (3) in Table 1, $\rho_1 = 0.5$, $\rho_2 = 2$, velocity bound of 0.2, and final time $T = 100$. We collected 50 simulation runs for 4 different cases: (1) FN strategy, (2,3,4) RN strategy with the ratio RM_i/M_i set to $0.75, 0.50$, and 0.25, respectively. We call r_{λ_2} the ratio between the converged λ_2 of the distributed solution and the one from the centralized solution. Therefore, if $r_{\lambda_2} > 1$ the distributed solution has better performance than the centralized one. In Figure 3 an example of the trajectories of the centralized and the distributed solutions for the FN strategy is depicted. The initial positions are marked with squares. The final positions are marked with circles. The bold lines represent the final communication graph and the thin lines the agent trajectories. The values of $\sqrt{\rho_1}$ and $\sqrt{\rho_2}$ are also depicted for comparison. Figure 4 shows, in the same simulation, the evolution of the algebraic connectivity as a function of the sampling time k. We can observe that although in this case the centralized solution converges faster to the final configuration, the distributed approach eventually converges to a higher final algebraic connectivity value. We can also notice "plateaus" during the convergence of the algebraic connectivity, where the agents are rotating and λ_2 is not changing significantly.

Figure 5 represents the ratio between the final distributed and centralized solutions, i.e., r_{λ_2} in all four cases. The disconnected label refers to situations in which

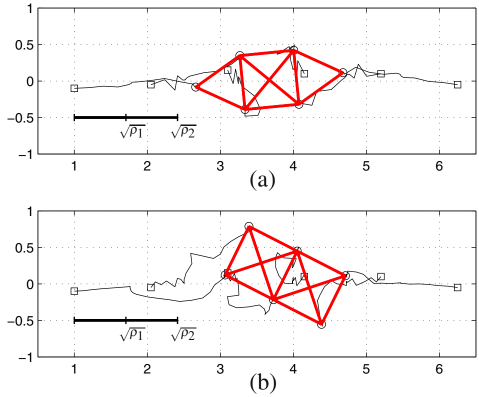


Fig. 3 Trajectories for the centralized solution (a) and for the distributed approach using the FN strategy (b). The initial positions are marked with squares. The final positions are marked with circles. The bold lines represent the final communication graph and the thin lines the agent trajectories.

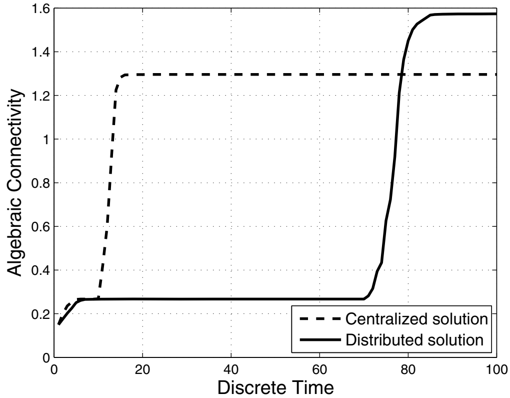


Fig. 4 Algebraic connectivity as a function of discrete time k for both the centralized and the distributed solutions

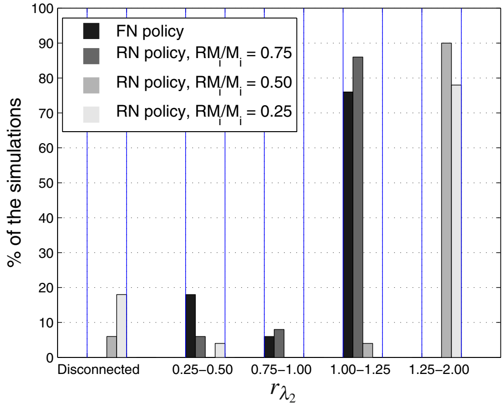


Fig. 5 Ratio between the final distributed and centralized solutions, i.e., r_{λ_2} and the percentage of corresponding simulations. (There are no cases that fall in the segment $0.50-0.75$).

the RN strategies lead to a disconnected graph. From the simulation results, we can observe that the FN strategy has performance comparable to the centralized solution in most cases. It may even converge to a higher λ_2 value in some instances, and it could get stuck in local minima in certain cases, which are not present in the centralized algorithm. Investigation of these local minima is a topic of ongoing research. The behavior of the RN strategies differs from the FN strategy for two reasons: one is the absence of a monotonically increasing connectivity property and the other is a possible infeasibility of the local problems. One consequence of this is that decreasing the ratio RM_i/M_i is more likely to lead to an increasing number of disconnected final graphs. We can also observe a clear increase in performance for $RM_i/M_i = 0.50$ and 0.25. This can be expected since the minimum distance constraints are no longer enforced in a consistent manner, and some agents are allowed to be arbitrarily close to each other if they are excluded from the local problem formulation. This also means that, in some cases, local problems can become infeasible. Infeasible local problems were handled in the simulations by keeping the previous positions, i.e., $x_i(k) = x_i(k-1)$. For small RM_i/M_i ratios this led to all local problems eventually becoming infeasible and all pairwise distances becoming smaller than $\sqrt{\rho_1}$. The ratio RM_i/M_i can be considered as a tuning parameter to obtain a reduction of communication. On one hand, for $RM_i/M_i = 1$ we have possibly high communication load, on the other hand for $RM_i/M_i \to 0$, we have limited communication for the price of sacrificing the monotonically increasing connectivity property. This loss however does not necessarily lead to either disconnected graphs or distances smaller than $\sqrt{\rho_1}$. The choice of 0.75 serves as an example for this phenomenon.

5 Future Developments and Open Questions

We have presented a heuristic distributed solution for the maximization of algebraic connectivity in a network of mobile robots. The method is optimization-based and can be further extended by including other types of constraints. Our approach may be used to obtain a monotonically increasing connectivity property and it can be easily understood based on the existing centralized solution. We presented simulation results for different communication strategies to assess the performance of the method, and we highlighted cases in which the distributed solution converges to a higher λ_2 value than the centralized scheme, along with cases in which it converges to local minima. Several open issues still remain and will be the focus of our future research. In particular, a study of the inconsistency between real and linearized distance $D(k)$, a more realistic dynamical model for the agents, and an investigation of the theoretical properties of both the FN and RN strategies will be considered along with experimental validations. Furthermore, we will investigate possible dual decomposition methods to distribute (5) among the robots, while expecting that the resulting iterative solutions could compromise real-time applicability. Such a dual decomposition approach would typically provide primal feasible solutions only asymptotically, and would require investigating various issues, such as the duality gap. On a longer time scale, other interesting topics of research are how to extend the problem formulation to handle more realistic scenarios, such as obstacle avoidance and environment-dependent connectivity.

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