Possibilistic Reasoning in Multi-Context Systems: Preliminary Report

Yifan Jin, Kewen Wang, and Lian Wen

School of Information and Communication Technology
Griffith University, Brisbane, QLD 4116, Australia
yifan.jin@griffithuni.edu.au,
{k.wang,l.wen}@griffith.edu.au

Abstract. This paper makes the first attempt to establish a framework for possibilistic reasoning in (nonmonotonic) multi-context systems, called possibilistic MCS. We first introduce the syntax for possibilistic MCS and then define its equilibrium semantics based on Brewka and Eiter's nonmonotonic multi-context systems. Then we investigate several properties and develop a fixoint theory for possibilistic MCS.

1 Introduction

Sharing and reasoning about information in a distributed and heterogeneous environment is becoming more important than ever with the advent of the web and of ubiquitous connectivity. In many cases, such information is not organized as a unique, homogeneous and coherent knowledge base, but is scattered in a large set of local and inter-related contexts. As a result, advanced information systems for the web should be able to deal with such heterogeneity. Moreover, this kind of information is usually incomplete and the information flow between different sources can be quite diverse. During the last decade, there have been extensive efforts in resolving this challenge and in particular, multi-context systems are regarded a promising tool for formalizing and processing heterogeneous and incomplete information [2; 3]. In artificial intelligence, a context is either a situation in the general sense of the term or a part of knowledge or both. Informally, a multi-context system is a formal description of the information available in a number of contexts and specifies the information flow between those contexts. Several logical approaches to context systems have been proposed, most notably McCarthy's propositional logic of context [10] and the multi-context systems devised by Giunchiglia and Serafini [7]. We note that multi-context systems are different from multi-agent systems in that, unlike an agent, a context is not autonomous in general while there is information flow between contexts.

Several different logic-based approahces to MCS have been proposed, e.g. in [10] the contexts are based on classical monotonic reasoning and in [12] and [5] the contexts allow for reasoning based on the absence of information from a context, and in [4] a formalism of heterogeneous nonmonotonic multi-context systems is introduced, which is capable of combining arbitrary monotonic and nonmonotonic logics.

On the other hand, possibility logic, which is developed from Zadeh's possibility theory [14], provides a useful framework for representing states of partial ignorance

P. Anthony, M. Ishizuka, and D. Lukose (Eds.): PRICAI 2012, LNAI 7458, pp. 180–193, 2012.

owing to the use of a dual pair of possibility and necessity measures [6]. We note that some efforts have been made to merge possibilistic reasoning in multiple-source information, e. g. [1], but MCS is different from the frameworks for merging multiple-source information as MCS aims to provide a suitable framework for performing distributed reasoning across multiple information sources.

To our best knowledge, *the problem of incorporating possibilistic reasoning into MCS has not been studied yet*. This paper makes the first attempt to establish a framework for combining nonmonotonic MCS and possibilistic reasoning. We first introduce the syntax for our possibilistic MCS and then define the equilibrium semantics based on Brewka and Eiter's nonmonotonic MCS in [4]. In our framework, each context is represented as a possibilistic logic program [11]. Then we investigate several properties and develop algorithms for the possibilistic MCS.

We proceed, in section 2, with a brief review of possibilistic normal logic program with answer set semantic. In section 3 we introduce the poss-MCS and deal with a part of it, then we extend the result in section 4. Finally, we conclude the work in section 5.

2 Preliminary

We first recall some basics of possibilistic logic and then introduce the syntax and semantics for possibilistic logic programs proposed in [11]. We deal with propositional logic and logic programs. Throughout the paper, a possibilistic concept is denoted $\overline{X}$ while its classical counterpart is denoted X.

We assume that Σ is a set of atoms. A (classical) interpretation I is a subset of Σ. An atom a is true under I if $a \in I$; otherwise, a is false under I. By 2^{Σ} we denote the set of all interpretations on Σ, i. e. the power set of Σ.

A *possibilistic formula* $\overline{\phi}$ on Σ is a pair $(\phi, [\alpha])$ where ϕ is a propositional formula on Σ and $\alpha \in [0, 1]$. Informally, $(\phi, [\alpha])$ expresses that the formula ϕ is certain at least to the level α. This degree α is evaluated by a necessity measure but it is not a probability. The higher is the level, the more certain is the formula. In particular, a possibilistic formula $\overline{\phi}$ is called a *possibilistic atom* if ϕ is an atom. A possibilistic knowledge base (poss-KB) $\overline{K}$ on Σ is a finite set of possibilistic formula on Σ. If $\overline{K} = \{(\phi_1, [\alpha_1]), \ldots, (\phi_n, [\alpha_n])\}$ ($n \geq 0$), then the classical part of $\overline{K}$ is denoted $K = \{\phi_1, \ldots, \phi_n\}$.

The basic part of the semantics for possibilistic logic is the *possibility distributions*, each of which is a mapping from 2^{Σ} to the interval $[0, 1]$.

Given a possibility distribution π, for each interpretation ω, $\pi(\omega)$ represents the degree of compatibility of the interpretation ω with the available information (or beliefs) about the real world.

A possibility distribution π defines two different weights for propositional formulas. For each propositional formula ϕ, we define

– Possibility degree: $\Pi(\phi) = max\{\pi(\omega) \mid \omega \models \phi\}$.
– Necessity degree: $N(\phi) = 1 - \Pi(\neg\phi)$.

The possibility degree $\Pi(\phi)$ evaluates the extent to which ϕ is consistent with the available beliefs expressed by π. Thus the possibility degree is also referred to as the consistent degree. The necessity degree $N(\phi)$, also called certainty degree evaluates the extent to which ϕ is entailed by the available beliefs expressed by π.

We say a possibility distribution is *compatible* with a poss-KB $\overline{K}$ if, $N(\phi) \geq \alpha$ for every $(\phi, [\alpha]) \in \overline{K}$. Generally, there may exist several possibility distributions compatible with $\overline{K}$. The most desirable distribution is usually selected by the minimum specificity principle [13]. A possibility distribution π is said to be the *least specific distribution* among all compatible distributions if there is no possibility distribution π' such that it is compatible with $\overline{K}$, $\pi' \neq \pi$, and $\forall\omega$, $\pi'(\omega) \geq \pi(\omega)$.

Definition 1. *Let Σ be a finite set of atoms. A possibilistic atom is $\overline{a} = (a, [\alpha])$. where $a \in \Sigma$ and $\alpha \in [0, 1]$.*

The classical projection of $\overline{a}$ is the atom a and $n(a) = \alpha$ is the necessity degree of the possibilistic atom $\overline{a}$.

Definition 2. *A possibilistic normal logic program (or poss-program) is a set of possibilistic rules of the form:*

$$\overline{r} = a \leftarrow a_1, \ldots, a_m, not\ b_1, \ldots, not\ b_n,\ [\alpha]. \tag{1}$$

where $m \geq 0$, $n \geq 0$, $\{a_1, \ldots, a_m, b_1, \ldots, b_n, a\} \subseteq \Sigma$, and $n(\overline{r}) = \alpha \in [0, 1]$.

The symbol "not" denotes the *default negation* and for each atom b_i, $not\ b_i$ is a negative literal.

Similar to possibilistic propositional logic, the classical projection r of a possibilistic rule $\overline{r}$ is the classical rule $a \leftarrow a_1, \ldots, a_m, not\ b_1, \ldots, not\ b_n$. Also, α represents the certainty level of the information described by the rule $\overline{r}$.

Given a rule $\overline{r}$ of the form (1), its head is defined as $head(\overline{r}) = a$ and its body is $body(\overline{r}) = body^+(\overline{r}) \cup not\ body^-(\overline{r})$ where $body^+(\overline{r}) = \{a_1, \ldots, a_m\}$, $body^-(r) = \{b_1, \ldots, b_n\}$.

The positive projection of $\overline{r}$ is $\overline{r}^+ = head(\overline{r}) \leftarrow body^+(\overline{r}), [\alpha]$.

The set of all rules of $\overline{P}$ with the head a is $H(\overline{P}, a) = \{\overline{r} \in \overline{P} \mid head(\overline{r}) = a\}$.

If a poss-program $\overline{P}$ does not contain any default negation (i. e. $body^-(\overline{P}) = \emptyset$), then $\overline{P}$ is called a *definite poss-program*.

We first introduce the semantics for definite poss-programs.

The reduct of a poss-program $\overline{P}$ w. r. t. a set A of atoms is the definite poss-program defined by:

$$\overline{P}^A = \{\overline{r}^+ \mid \overline{r} \in \overline{P}, body^-(\overline{r}) \cap A = \emptyset\} \tag{2}$$

We note that the rule $\overline{r}^+$ is actually the possibilistic rule formed by the classical reduct r^+ together with the certainty level of $\overline{r}$.

For a set of atoms $A \subseteq \Sigma$ and a rule $\overline{r}$ in $\overline{P}$, we say $\overline{r}$ is applicable in A if $body^+(\overline{r}) \subseteq A$ and $body^-(\overline{r}) \cap A = \emptyset$. $App(\overline{P}, A)$ denotes the set of rules in poss-program $\overline{P}$ that are applicable in A.

$\overline{P}$ is said to be *grounded* if it can be ordered as a sequence $\langle\overline{r}_1, \ldots, \overline{r}_n\rangle$ such that

$$\forall i,\ 1 \leq i \leq n,\ \overline{r}_i \in App(\overline{P},\ head(\{\overline{r}_1, \ldots, \overline{r}_{i-1}\})) \tag{3}$$

Given a poss-program $\overline{P}$ over a set Σ of atoms, similar to the case of propositional possibilistic logic, the semantics of $\overline{P}$ is also defined through possibility distributions on Σ.

The compatibility of a possibility distribution with definite poss-program $\overline{P}$ is defined in [11] (Definition 4). There may exist several different possibility distributions that are compatible with a given definite poss-program. Among these compatible distributions, we are particularly interested in the least specific one, which is given in the next result.

Proposition 1. *Let $\overline{P}$ be a definite poss-program. We define a possibilistic distribution $\pi_{\overline{P}}$ for $\overline{P}$ as, for each $A \in 2^{\Sigma}$,*

$$\pi_{\overline{P}}(A) = \begin{cases} 0, & \textit{if } A \nsubseteq head(App(P, A)) \\ 0, & \textit{if } App(P, A) \textit{ is not grounded} \\ 1, & \textit{if } A \textit{ is a model of } P \\ 1 - max\{n(\overline{r}) \mid A \nvDash r\}, & \textit{otherwise.} \end{cases} \quad (4)$$

Then $\pi_{\overline{P}}$ is the least specific distribution compatible with $\overline{P}$.

The least specific distribution for $\overline{P}$ determines its possibilistic measures.

Definition 3. *Let $\overline{P}$ be a definite poss-program and $\pi_{\overline{P}}$ the least specific distribution compatible with $\overline{P}$. Then the possibility and necessity degrees for an atom a is defined by*

$$\Pi_{\overline{P}}(a) = max\{\pi_{\overline{P}}(A) \mid a \in A\}.$$
$$N_{\overline{P}}(a) = 1 - max\{\pi_{\overline{P}}(A) \mid a \notin A\}.$$

$\Pi_{\overline{P}}(a)$ gives the level of consistency of a w. r. t. the definite poss-program $\overline{P}$ and $N_{\overline{P}}(a)$ evaluates the level at which a is inferred from $\overline{P}$. For instance, whenever an atom a belongs to the model of the classical program, its possibility is equal to 1.

The necessity measure allows us to introduce the following definition of the possibilistic model of a definite poss-program.

Definition 4. *Let $\overline{P}$ be a definite poss-program. Then the set*

$$\overline{M}(\overline{P}) = \{(a, N_{\overline{P}}(a)) \mid a \in \Sigma,\ N_{\overline{P}}(a) > 0\} \quad (5)$$

is referred to as its possibilistic model.

So far we have introduced the semantics for definite poss-programs. Now we turn to study the computation of the possibility distribution and possibilistic model for a given poss-program. First we define β-applicability of a rule $\overline{r}$ to capture the certainty of an conclusion that the rule can derive w. r. t. a set $\overline{A}$ of possibilistic atoms.

Definition 5. *Let $\overline{r}$ be a possibilistic rule of the form $c \leftarrow a_1, \ldots, a_n, [\alpha]$ and $\overline{A}$ be a set of possibilistic atoms.*

1. *$\overline{r}$ is β-applicable in $\overline{A}$ with possibility β=$min\{\alpha, \alpha_1, \ldots, \alpha_n\}$ if $\{(a_1, \alpha_1), \ldots, (a_n, \alpha_n)\} \subseteq \overline{A}$.*
2. *$\overline{r}$ is 0-applicable otherwise.*

If the rule body is empty, then the rule is applicable with its own certainty degree and if the body is not satisfied by $\overline{A}$, then the rule is 0-applicable and it is actually not at all applicable w. r. t. $\overline{A}$. So the applicability level of the rule depends on the certainty level of atoms in its body and its own certainty degree.

The set of rules in $\overline{P}$ that have the head a and are applicable w. r. t. $\overline{A}$ is denoted $App(\overline{P}, \overline{A}, a)$:

$$App(\overline{P}, \overline{A}, a) = \{\overline{r} \in H(\overline{P}, a),\ \overline{r} \text{ is } \beta\text{-applicable in } \overline{A},\ \beta > 0\} \tag{6}$$

Having defined the applicability of possibilistic rules, we can generalise the consequence operator of classical logic programs to poss-programs.

Definition 6. *Let $\overline{P}$ be a poss-program, a be an atom and $\overline{A}$ be a set of possibilistic atoms. Then we define the consequence operator for $\overline{P}$ by*

$$\overline{T}_{\overline{P}}(\overline{A}) = \{(a, \delta) \mid a \in head(P), App(\overline{P}, \overline{A}, a) \neq \emptyset, \delta = max\{\beta \mid \overline{r} \text{ is } \beta\text{-applicable in } \overline{A}\}\}$$

Then the iterated operator $\overline{T}^{k}_{\overline{P}}$ is defined by

$$\overline{T}^{0}_{\overline{P}} = \emptyset \text{ and } \overline{T}^{n+1}_{\overline{P}} = \overline{T}_{\overline{P}}(\overline{T}^{n}_{\overline{P}}),\ \forall n \geq 0. \tag{7}$$

$\overline{T}_{\overline{P}}$ has a least fixpoint that is the possibilistic consequences of $\overline{P}$ and it is denoted by $\overline{Cn}(\overline{P})$. We have $\overline{Cn}(\overline{P}) = \overline{M}(\overline{P})$ (see [11] for more details).

For poss-programs, it is easy to formalize the notion of stable models by a generalized reduct [8].

Definition 7. *Let $\overline{P}$ be a poss-program and A a set of atoms.*
We say $\overline{A}$ is a stable model of poss-program $\overline{P}$ if $\overline{A} = \overline{Cn}(\overline{P}^{A})$.

The possibility distribution for $\overline{P}$ is defined in terms of its reduct's possibility distribution as follows.

Definition 8. *Let $\overline{P}$ be a possibilistic logic program and A be an atom set, then $\tilde{\pi}_{\overline{P}}$ is the possibility distribution defined by:*

$$\forall A \in 2^{\Sigma},\ \tilde{\pi}_{\overline{P}}(A) = \pi_{\overline{P}^{A}}(A) \tag{8}$$

With these two definitions we can also define the possibility and necessity measures to each atom by Definition 3.

3 Possibilistic Multi-Context Systems

In this section we will first incorporate possibilistic reasoning into multi-context systems (MCS) and then discuss their properties.

3.1 Syntax of Poss-MCS

A possibilistic multi-context system (or poss-MCS) is a collection of contexts where each context is a poss-program with its own knowledge base and bridge rules. In this paper, a possibilistic context $\overline{C}$ is a triple $(\Sigma, \overline{K}, \overline{B})$ where Σ is a set of atoms, $\overline{K}$ is a poss-program, and $\overline{B}$ is a set of possibilistic bridge rules for the context $\overline{C}$. Before formally introducing poss-MCS, we first give the definition of possibilistic bridge rules. Intuitively, a possibilistic bridge rule make it possible to infer new knowledge for a context based on some other contexts. So possibilistic bridge rules provide an effective way for the information flow between related contexts.

Definition 9. *Let $\overline{C}_1, \ldots, \overline{C}_n$ be n possibilistic contexts. A possibilistic bridge rule $\overline{br}_i$ for a context $\overline{C}_i$ ($1 \leq i \leq n$) is of the form*

$$a \leftarrow (C_1 : a_1), \ldots, (C_k : a_k), \mathit{not}\,(C_{k+1} : a_{k+1}), \ldots, \mathit{not}\,(C_n : a_n), [\alpha] \quad (9)$$

where a is an atom in $\overline{C}_i$, each a_j is an atom in context $\overline{C}_j$ for $j = 1, \ldots, n$.

Intuitively, a rule of the form (9) states that the information a is added to context $\overline{C}_i$ with necessity degree α if, for $1 \leq i \leq k$, a_j is present in context $\overline{C}_j$ and for $k+1 \leq j \leq n$, a_j is not provable in $\overline{C}_j$.

By br_i we denote the classical projection of $\overline{br}_i$:

$$a \leftarrow (C_1 : a_1), \ldots, (C_k : a_k), \mathit{not}\,(C_{k+1} : a_{k+1}), \ldots, \mathit{not}\,(C_n : a_n). \quad (10)$$

The necessity degree α of the bridge rule $\overline{br}_i$ is written $n(\overline{br}_i)$.

Definition 10. *A* possibilistic multi-context system*, or just* poss-MCS*, $\overline{M} = (\overline{C}_1, \ldots, \overline{C}_n)$ is a collection of contexts $\overline{C}_i = (\Sigma_i, \overline{K}_i, \overline{B}_i)$, $1 \leq i \leq n$, where each Σ_i is the set of atoms used in context $\overline{C}_i$, $\overline{K}_i$ is a poss-program on Σ_i, and $\overline{B}_i$ is a set of possibilistic bridge rule over atom sets $(\Sigma_1, \ldots, \Sigma_n)$.*

A poss-MCS is definite *if the poss-program and possibilistic bridge rules of each context is definite.*

Definition 11. *A* possibilistic belief set *$\overline{S} = (\overline{S}_1, \ldots, \overline{S}_n)$ is a collection of possibilistic atom sets $\overline{S}_i$ where each $\overline{S}_i$ is a collection of possibilistic atoms $\overline{a_i}$ and $a_i \in \Sigma_i$*

In the next two subsections we will study the semantics for possibilistic definite MCS.

3.2 Model Theory for Definite Poss-MCS

Like poss-programs we will first specify the semantics for definite poss-MCS (i. e. without default negation) and then define the semantics for poss-MCS with default negation by reducing the given poss-MCS to a definite poss-MCS.

For convenience, by a *classical MCS* we mean a multi-context system (MCS) without possibility degrees as in [4]. The semantics of a classical MCS is defined by the set of its equilibria, which characterize acceptable belief sets that an agent may adopt based on the knowledge represented in a knowledge base. Let us first recall the semantics of classical MCS. Let $M = (C_1, \ldots, C_n)$ be a classical MCS with each (classical) context $C_i = (\Sigma_i, K_i, B_i)$, where Σ_i is a set of atoms, K_i is a logic program, and B_i is a set of (classical) bridge rules.

A *belief state* of a MCS $M = (C_1, \ldots, C_n)$ is a collection $S = (S_1, \ldots, S_n)$ where each S_i is a set of atoms that $S_i \subseteq \Sigma_i$. A (classical) bridge rule (10) is *applicable* in a belief state S iff for $1 \leq j \leq k$, $a_j \in S_j$ and for $k+1 \leq j \leq n$, $a_j \notin S_j$.

In general, not every belief state is acceptable for an MCS. Usually, the equilibrium semantics selects certain belief states for a given MCS as acceptable belief states. Intuitively, an equilibrium is a belief state $S = (S_1, \ldots, S_n)$ where each context C_i respects all bridge rules applicable in S and accepts S_i. The definition of equilibrium can be found in [4]. There may exist several different equilibrium, among these equilibrium, we are particularly interested in the minimal one. Formally, S is a grounded equilibrium of an MCS $M = (C_1, \ldots, C_n)$ iff for each i ($1 \leq i \leq n$), S_i is an answer set of logic program $P = K_i \cup \{head(r) \mid r \in B_i \text{ is applicable in } S\}$. Then if the logic program P is grounded, we can use $Cn(P)$ to obtain its (unique) answer set, which is the smallest set of atoms closed under P and alternatively, it can be computed as the least fixpoint of the consequence operator T_P: $T_P(A) = head(App(P, A))$ (see [9]).

So for a classical definite MCS, its unique grounded equilibrium is the collection consisting of the least model of each context. The grounded equilibrium of a definite MCS M is denoted $GE(M)$.

Then we clarify the links between the grounded equilibrium S of a definite MCS M and the rules producing it. We see that for each context C_i, S_i is underpinned by a set of applicable rules $App_i(M, S)$, that satisfies a stability condition and that is grounded.

Proposition 2. *Let M be a definite MCS and S be a belief state,*

$$S \text{ is the grounded equilibrium of } M \Leftrightarrow \begin{cases} S_i = head(App_i(M, S)) \\ \bigcup_i App_i(M, S) \text{ is grounded} \end{cases} \tag{11}$$

Now let us turn to the semantics of definite poss-MCS. Thus, we will specify the possibility distribution of belief states for a given definite poss-MCS. As we know that the satisfiability of a rule r is based on its applicability w. r. t. an belief state S and $S \not\models r$ iff $body^+(r) \subseteq S \wedge head(r) \notin S$. But this is not enough to determine the possibility degree of a belief state. For example, if we have a definite MCS $M = (C_1, C_2)$ consisting of two definite MCS. Assume K_1 and K_2 are empty, and B_1 consists of the single bridge rule $p \leftarrow (2 : q)$ and B_2 of the single bridge rule $q \leftarrow (1 : p)$. Now $S = (\{p\}, \{q\})$ satisfies every rule in M. But it is not an equilibrium because the groundedness is not satisfied. Besides, assume that an definite MCS $M = (C_1)$ with a single context, K_1 is $\{a\}$ and B_1 consists of the bridge rule $b \leftarrow (1 : c)$. Now $S = (\{a, b\})$ satisfies every rules in M but it is not an equilibrium because b cannot be produced by any rule from C_1 applicable in S. In these two cases, the possibility of S must be 0 since they cannot be an equilibrium at all, even if they satisfy every rule in their MCS.

Definition 12. *Let $\overline{M} = (\overline{C}_1, \dots, \overline{C}_n)$ be a definite poss-MCS and $S = (S_1, \dots, S_n)$ be a belief state. The possibility distribution $\pi_{\overline{M}} : 2^{\Sigma} \to [0,1]$ for $\overline{M}$ is defined as, for $S \in 2^{\Sigma}$,*

$$\pi_{\overline{M}}(S) = \begin{cases} 0 & \text{if } S \nsubseteq head(\bigcup_i App_i(M,S)) \\ 0 & \text{if } \bigcup_i App_i(M,S) \text{ is not grounded} \\ 1 & \text{if } S \text{ is an equilibrium of } M \\ \pi_{\overline{M}}(S) = 1 - max\{n(\overline{r}) \mid S \nvDash r, \overline{r} \in \overline{B}_i \text{ or } \overline{r} \in \overline{K}_i\}, & \text{otherwise.} \end{cases} \tag{12}$$

The possibility distribution specifies the degree of compatibility of each belief set S with poss-MCS $\overline{M}$.

Recall that $GE(M)$ denotes the grounded equilibrium of a (classical) definite MCS M. Then the possibility distribution for definite poss-MCS has the following useful properties.

Proposition 3. *Let $\overline{M} = (\overline{C}_1, \dots, \overline{C}_n)$ be a definite poss-MCS, $S = (S_1, \dots, S_n)$ be a belief state and $GE(M)$ the grounded equilibrium of M, then*

1. $\pi_{\overline{M}}(S) = 1$ *iff* $S = GE(M)$.
2. *If* $S \supset GE(M)$, *then* $\pi_{\overline{M}}(S) = 0$.
3. *If* $GE(M) \neq \emptyset$, *then* $\pi_{\overline{M}}(\emptyset) = 1 - max\{n(\overline{r}) \mid body^+(\overline{r}) = \emptyset, \overline{r} \in \overline{B}_i$ *or* $\overline{r} \in \overline{K}_i\}$.

Proof. 1. ⇒: Let $\pi_{\overline{M}}(S) = 1$. On the contrary, assume that $S \neq GE(M)$. Then by Equation (12), $\pi_{\overline{M}}(S) = 1$ would only be obtained from the last case:

$$\pi_{\overline{M}}(S) = 1 - max\{n(\overline{r}) \mid S \nvDash r, \overline{r} \in \overline{B}_i \text{ or } \overline{r} \in \overline{K}_i\}.$$

This implies that $S \models r$.

By the first case in Equation (12), we have that $S \subseteq head(\bigcup_i App_i(M,S))$.

By the second case in Equation (12), $\bigcup_i App_i(M,S)$ is grounded.

Therefore, S must be the least equilibrium of M.

⇐:If $S = GE(M)$ by the definition we have $\pi_{\overline{M}}(S) = 1$.

2. Because $S \supset GE(M)$ we have for each $i : S_i \supset head(App_i(M,S)) \vee App(M,S)$ is not grounded by properties 2. So by definition $\pi_{\overline{M}}(S) = 0$.

3. It is obvious that $\emptyset \subseteq head(App_i(M,\emptyset))$ and $App_i(M,\emptyset)$ is grounded. So it can only apply to the forth case of Equation 12.

Definition 13. *Let $\overline{M}$ be a definite poss-MCS and $\pi_{\overline{M}}$ be the possibilistic distribution for $\overline{M}$. The possibility and necessity of an atom in a belief state S is defined by:*

$$\Pi_{\overline{M}}(a_i) = max\{\pi_{\overline{M}}(S) \mid a_i \in S_i\} \tag{13}$$

$$N_{\overline{M}}(a_i) = 1 - max\{\pi_{\overline{M}}(S) \mid a_i \notin S_i\} \tag{14}$$

Proposition 4. *Let $\overline{M}$ be a definite poss-MCS and $S = (S_1, \ldots, S_n)$ is a belief state. Then*

1. $a_i \notin S_i$ *iff* $N_{\overline{M}}(a_i) = 0$.
2. *If* $a_i \in S_i$, *then* $N_{\overline{M}}(a_i) = min\{max\{n(\overline{r}) \mid S_i \nvDash r, \overline{r} \in \overline{B}_i$ *or* $\overline{r} \in \overline{K}_i\} \mid a_i \notin S_i, S_i \subset GE(M)\}$.

Proof. 1. Because $\pi_{\overline{M}}(S) = 1$, $N_{\overline{M}}(a_i) = 0$ is obvious when $a_i \notin S_i$. And when $N_{\overline{M}}(a_i) = 0$ it means there is an S such that $a_i \notin S_i$ and $\pi_{\overline{M}}(S) = 1$, So such an S must be the equilibrium of M. Thus, $a \notin GE(M)$

2.

$$
\begin{aligned}
N_{\overline{M}}(a_i) =& 1 - max\{\pi_{\overline{M}}(S) \mid a_i \notin S_i\} \\
=& 1 - max\{\pi_{\overline{M}}(S) \mid a_i \notin S_i, S_i \subset GE(M)\} \\
& \text{ since by Proposition 3} \\
=& 1 - max\{1 - max\{n(\overline{r}) \mid S_i \nvDash r, \overline{r} \in \overline{B}_i \text{ or } \overline{r} \in \overline{K}_i \mid a_i \notin S_i, S_i \subset GE(M)\} \\
& \text{ since } \pi_{\overline{M}}(S) = 1 - max\{n(\overline{r}) \mid S_i \nvDash r, \overline{r} \in \overline{B}_i \text{ or } \overline{r} \in \overline{K}_i\} \\
=& min\{max\{n(\overline{r}) \mid S_i \nvDash r, \overline{r} \in \overline{B}_i \text{ or } \overline{r} \in \overline{K}_i\} \mid a_i \notin S_i, S_i \subset GE(M)\}.
\end{aligned}
$$

The semantics for definite poss-MCS is determined by its unique possibilistic grounded equilibrium.

Definition 14. *Let $\overline{M}$ be a definite poss-MCS. Then the following set of possibilistic atoms is referred to as the possibilistic grounded equilibrium.*

$$\overline{MD}(\overline{M}) = (\overline{S}_1, \ldots, \overline{S}_n)$$

where $\overline{S}_i = \{(a_i, N_{\overline{M}}(a_i)) \mid a_i \in \Sigma_i, N_{\overline{M}}(a_i) > 0\}$ *for* $i = 1, \ldots, n$.

By the first statement of Proposition 4, it is easy to see the following result holds.

Proposition 5. *Let $\overline{M}$ be a definite poss-MCS and M be the classical projection of $\overline{M}$. Then the classical projection of $\overline{MD}(\overline{M})$ is the grounded equilibrium of the definite MCS M.*

Example 1. *Let $\overline{M} = (\overline{C}_1, \overline{C}_2)$ be a definite poss-MCS where $\Sigma_1 = \{a\}$, $\Sigma_2 = \{b, c\}$, $\overline{K}_1 = \{(a, [0.9]), (c \leftarrow b, [0.8])\}$, $\overline{K}_2 = \overline{B}_1 = \emptyset$, $\overline{B}_2 = \{(b \leftarrow 1 : a, [0.7])\}$.*

By Definition 12, $\pi_{\overline{M}}(\{\emptyset\}, \{\emptyset\})$=1 $-$ $max\{0.9\}$=0.1, $\pi_{\overline{M}}(\{\emptyset\}, \{b\})$=1 $-$ $max\{0.9, 0.8\}$=0.1, $\pi_{\overline{M}}(\{\emptyset\}, \{c\}) = 1 - max\{0.9\} = 0.1$, $\pi_{\overline{M}}(\{\emptyset\}, \{b, c\}) = 0$ (not inclusion), $\pi_{\overline{M}}(\{a\}, \{\emptyset\}) = 1 - max\{0.7\} = 0.3$, $\pi_{\overline{M}}(\{a\}, \{b\}) = 1 - max\{0.8, 0.7\} = 0.2$, $\pi_{\overline{M}}(\{a\}, \{c\}) = 0$ (not inclusion), and $\pi_{\overline{M}}(\{a\}, \{b, c\}) = 1$ (the grounded equilibrium).

And thus, by Definition 13, we can get the necessity value for each atom: $N_{\overline{M}}(a) = 1 - max\{0.1\} = 0.9$, $N_{\overline{M}}(b) = 1 - max\{0.1, 0.3, 0\} = 0.7$, and $N_{\overline{M}}(c) = 1 - max\{0.1, 0.3, 0.2\} = 0.7$.

Then by Definition 14 we can get the possibilistic grounded equilibrium $\overline{S} = (\{(a, [0.9])\}, \{(b, [0.7]), (c, [0.7])\})$.

3.3 Fixpoint Theory for Definite Poss-MCS

In the last subsection we introduced the possibilistic grounded equilibrium and the possibilistic distribution of the belief states. In this subsection we will develop a fixpoint theory for the possibilistic grounded equilibrium and thus provide a way for computing the equilibrium.

Similar to Definition 5 for poss-programs, we can define the applicability of possibilistic rules and thus, for an atom $a_i \in \Sigma_i$ and a possibilistic belief state $\overline{S}$ we define

$$App(\overline{M}, \overline{S}, a_i) = \{\overline{r} \in H(\overline{M}, a_i), \overline{r} \text{ is } \beta\text{-applicable in } \overline{S},\ \beta > 0\} \tag{15}$$

The above set is the collection of rules that have the head a_i and are β-applicable in $\overline{S}$.

By modifying the approach in [4], we introduce the following consequence operator for a definite poss-MCS. As we already know from Definition 6, the possibilistic consequences of a poss-program $\overline{P}$ is denoted by $\overline{Cn}(\overline{P})$.

Definition 15. *For each context $\overline{C}_i = (\Sigma_i, \overline{K}_i, \overline{B}_i)$ in a definite poss-MCS $\overline{M} = (\overline{C}_1, \ldots, \overline{C}_n)$, we define $\overline{K}_i^{t+1} = \overline{K}_i^t \cup \{(head(\overline{r}), [\beta]) \mid \overline{r} \in \overline{B}_i$ and is β-applicable in $\overline{E}^t, \beta > 0\}$, where $\overline{K}_i^0 = \overline{K}_i$ for $1 \leq i \leq n$, $\overline{E}^t = (\overline{E}_1^t, \ldots, \overline{E}_n^t)$, $\overline{E}_i^t = \overline{Cn}(\overline{K}_i^t)$ for $t > 0$.*

Since K_i and B_i of each context are finite, the iteration based on $\overline{K}_i^t$ will reach a fixpoint, which is denoted $\overline{K}_i^{\infty}$. Then we have the following proposition.

Proposition 6. *Let $\overline{M} = (\overline{C}_1, \ldots, \overline{C}_n)$ be a definite poss-MCS with $\overline{C}_i = (\Sigma_i, \overline{K}_i, \overline{B}_i)$ for $1 \leq i \leq n$ and $\overline{S} = (\overline{S}_1, \ldots, \overline{S}_n)$ be the grounded equilibrium for $\overline{M}$. Then*

$$\overline{Cn}(\overline{K}_i^{\infty}) = \overline{S}_i.$$

The key idea above is that, for each knowledge base $\overline{K}_i^t$, we use the operator $\overline{Cn}$ to obtain its possibilistic answer set, and then by the first item in Definition 15, add atoms from bridge rules that are derivale from $\overline{K}_i^t$ to get $\overline{K}_i^{t+1}$. Then, we apply the operator $\overline{Cn}$ again. Repeat this process until we reach the fixpoint.

Let us consider an example.

Example 2. *Let $\overline{M} = (\overline{C}_1, \overline{C}_2, \overline{C}_3)$ be a definite poss-MCS, where*
- $\overline{K}_1 = \{(a, [0.9])\}$, $\overline{B}_1 = \emptyset$;
- $\overline{K}_2 = \emptyset$, $\overline{B}_2 = \{(b \leftarrow (1 : a), [0.8])\}$;
- $\overline{K}_3 = \{(c, [0.7]), (d \leftarrow c, [0.6]), (f \leftarrow e, [0.5])\}$, $\overline{B}_3 = \{(e \leftarrow (2 : b), [0.4])\}$.

At the beginning we will start with $\overline{K}_i^0$ for each context.
For context $\overline{C}_1$ with $\overline{K}_1^0 = \overline{K}_1 = \{(a, [0.9])\}$:
$\overline{T}_{1,0}^0 = \emptyset,\ \overline{T}_{1,0}^1 = \overline{T}_{1,0}(\emptyset) = \{(a,\ [0.9])\},\ \overline{T}_{1,0}^2 = \{(a,\ [0.9])\}$.
For context $\overline{C}_2$ with $\overline{K}_2^0 = \overline{K}_2 = \emptyset$:
$\overline{T}_{2,0}^0 = \emptyset,\ \overline{T}_{2,0}^1 = \overline{T}_{2,0}^0 = \emptyset$.
For context $\overline{C}_3$ with $\overline{K}_3^0 = \overline{K}_3 = \{(c,\ [0.7]), (d \leftarrow c,\ [0.6]), (f \leftarrow e,\ [0.5])\}$:

$\overline{T}^0_{3,0} = \emptyset$, $\overline{T}^1_{3,0} = \overline{T}_{3,0}(\emptyset) = \{(c, [0.7])\}$, $\overline{T}^2_{3,0} = \overline{T}_{3,0}(\{(c, [0.7])\}) = \{(c, [0.7]), (d, [0.6])\}$, $\overline{T}^3_{3,0} = \overline{T}^2_{3,0} = \{(c, [0.7]), (d, [0.6])\}$.

Thus, $\overline{E}^0 = \{(a, [0.9]), (c, [0.7]), (d, [0.6])\}$.

Then starting from the fixpoint of $T_{S_i,0}$, for each context $\overline{C}_i$ we have:

For context 1 with $\overline{K}^1_1 = \{(a, [0.9])\}$: $\overline{T}^0_{1,1} = \overline{T}^2_{1,0} = \{(a, [0.9])\}$.

For context 2 with $\overline{K}^1_2 = \{(b, [0.8])\}$: $\overline{T}^0_{2,1} = \overline{T}^1_{2,0} = \emptyset$, $\overline{T}^1_{2,1} = T_{2,1}(\emptyset) = \{(b, [0.8])\}$, $\overline{T}^2_{2,1} = \overline{T}_{2,1}(\{(b, [0.8])\}) = \{(b, [0.8])\}$.

For context 3 with $\overline{K}^1_3 = \{(c, [0.7]), (d \leftarrow c, [0.6]), (f \leftarrow e, [0.5])\}$:
$\overline{T}^0_{3,1} = \overline{T}^2_{3,0} = \{(c, [0.7]), (d, [0.6])\}$.

Thus, $\overline{E}^1 = \{(a, [0.9]), (b, [0.8]), (c, [0.7]), (d, [0.6])\}$.

Then from the fixpoint of $T_{S_i,1}$, for each context $\overline{C}_i$:

For context 1 with $\overline{K}^2_1 = \{(a, [0.9])\}$: $\overline{T}^0_{1,2} = \overline{T}^0_{1,1} = \{(a, [0.9])\}$

For context 2 with $\overline{K}^2_2 = \{(b, [0.8])\}$:
$\overline{T}^0_{2,2}$=$\overline{T}^2_{2,1}$=$\{(b, [0.8])\}$, $\overline{T}^1_{2,2}$=$\overline{T}_{2,2}(\{(b, [0.8])\})$=$\{(b, [0.8])\}$.

For context 3 with $\overline{K}^2_3 = \{(c, [0.7]), (d \leftarrow c, [0.6]), (f \leftarrow e, [0.5]), (e, [0.4])\}$:
$\overline{T}^0_{3,2}$=$\overline{T}^0_{3,1}$=$\{(c, [0.7]), (d, [0.6])\}$, $\overline{T}^1_{3,2}$=$\overline{T}_{3,2}(\{(c, [0.7]), (d, [0.6])\})$=$\{(c, [0.7]), (d, [0.6]), (e, [0.4])\}$,
$\overline{T}^2_{3,2}$=$\overline{T}_{3,2}(\{(c, [0.7]), (d, [0.6]), (e, [0.4])\})$ = $\{(c, [0.7]), (d, [0.6]), (e, [0.4]), (f, [0.4])\}$,
$\overline{T}^3_{3,2}$=$\overline{T}_{3,2}(\{(c, [0.7]), (d, [0.6]), (e, [0.4]), (f, [0.4])\})$=$\{(c, [0.7]), (d, [0.6]), (e, [0.4]), (f, [0.4])\}$.

Thus, $E^2 = \{(a, [0.9]), (b, [0.8]), (c, [0.7]), (d, [0.6]), (e, [0.4]), (f, [0.4])\}$.

Therefore, the possibilistic grounded equilibrium of $\overline{M}$ is
$\overline{S} = (\{(a, [0.9])\}, \{(b, [0.8])\}, \{(c, [0.7]), (d, [0.6]), (e, [0.4]), (f, [0.4])\})$.

Proposition 7. *Let $M = (\overline{C}_1, \ldots, \overline{C}_n)$ be a definite poss-MCS and $S = (S_1, \ldots, S_n)$ be a belief state. Then for each i ($1 \leq i \leq n$) and a cardinal t, $\overline{T}_{S_i,t}$ is monotonic, i. e. for all sets $\overline{A}$ and $\overline{B}$ of possibilistic atoms with $\overline{A} \sqsubseteq \overline{B}$, it holds that*

$$\overline{T}_{S_i,t}(A) \sqsubseteq \overline{T}_{S_i,t}(B).$$

Proof. For any $\overline{A} \sqsubseteq \overline{B}$, $\forall a \in head(\overline{K}), App(\overline{K}, \overline{A}, t) \subseteq App(\overline{K}, \overline{B}, t)$. And relies on the *max* operator, $A \sqsubseteq B \Rightarrow \overline{T}_{S_i,t}(A) \sqsubseteq \overline{T}_{S_i,t}(B)$. Thus $\overline{T}_{S_i,t}$ is monotonic.

By Taski's fixpoint theorem, we can state the following result.

Proposition 8. *The operator $\overline{T}_{S_i,t}$ has a least fixedpoint when $\overline{S_i}$ is a definite poss-program. We denote $\overline{T}^{\infty}_{S_i,t} = \overline{S}_i$ then the $\overline{S} = (\overline{S}_1, \ldots, \overline{S}_n)$ is the equilibrium of $\overline{M}$ and we denote it as $\Pi\overline{GE}(\overline{M})$.*

We can now show the relationship between the semantical approach and fixed point approach:

Theorem 1. *Let $\overline{M}$ be an definite poss-MCS, then $\overline{GE}(\overline{M}) = \overline{MD}(\overline{M})$.*

The proof is similar to that in [11].

4 Normal Poss-MCS

Having introduced the semantics for definite poss-MCS, we are ready to define the semantics for normal poss-MCS with default negation. The idea is similar to the definition of answer sets, we will reduce a poss-MCS with default negation to a definite poss-MCS. Based on the definition of rule reduct in Equation (2), we can define the reduct for normal poss-MCS.

Definition 16. *Let $\overline{M} = (\overline{C}_1, \ldots, \overline{C}_n)$ be a normal poss-MCS and $S = (S_1, \ldots, S_n)$ be a belief state. The possibilistic reduct of $\overline{M}$ w. r. t. S is the poss-MCS*

$$\overline{M}^S = (\overline{C}_1^S, \ldots, \overline{C}_N^S). \tag{16}$$

where $\overline{C}_i^S = (\Sigma_i, \overline{K}_i^S, \overline{B}_i^S)$.

We note that the reduct of $\overline{K}_i$ relies only on S_i while the reduct of $\overline{B}_i$ depends on the whole belief state S. This is another role difference of $\overline{K}_i$ from $\overline{B}_i$.

Given the notion of reduct for normal poss-MCS, the equilibrium semantics of normal poss-MCS can be defined easily.

Definition 17. *Let $\overline{M}$ be a normal poss-MCS and $\overline{S}$ be a possibilistic belief state. $\overline{S}$ is a possibilistic equilibrium of $\overline{M}$ if $\overline{S} = \overline{GE}(\overline{M}^S)$.*

Example 3. *Let $\overline{M} = (\overline{C}_1, \overline{C}_2, \overline{C}_3)$ be a definite poss-MCS with 3 contexts:*
- $\overline{K}_1 = \{(a, [0.9])\}$, $\overline{B}_1 = \{(b \leftarrow not\ 3 : e, [0.8])\}$;
- $\overline{K}_2 = \{(d \leftarrow not\ c, [0.7])\}$, $\overline{B}_2 = \{(c \leftarrow 1 : a, [0.6])\}$;
- $\overline{K}_3 = \emptyset$, $\overline{B}_3 = \{(e \leftarrow not\ 1 : b, [0.5])\}$

We have $S_1 = (\{a\}, \{c\}, \{e\})$ and thus $\overline{M}^{S_1}$ is obtained as

$$\overline{M}^{S_1} = \begin{cases} \overline{K}_1 = \{(a, [0.9)\}, & \overline{B}_1 = \emptyset \\ \overline{K}_2 = \emptyset, & \overline{B}_2 = \{(c \leftarrow 1 : a, [0.6])\} \\ \overline{K}_3 = \emptyset, & \overline{B}_3 = \{(e, [0.5])\} \end{cases} \tag{17}$$

Following Definition 15, we can get $\overline{S}_1 = \{(a, [0.9]\}, \{c, [0.6]\}, \{e, [0.5]\})$.

And also $S_2 = (\{a, b\}, \{c\}, \{\emptyset\})$, then $\overline{M}^{S_2}$ is as follows:

$$\overline{M}^{S_2} = \begin{cases} \overline{K}_1 = \{(a, [0.9)\}, & \overline{B}_1 = \{(b, [0.8])\} \\ \overline{K}_2 = \emptyset, & \overline{B}_2 = \{(c \leftarrow 1 : a, [0.6])\} \\ \overline{K}_3 = \emptyset, & \overline{B}_3 = \emptyset \end{cases} \tag{18}$$

So we have $\overline{S}_2 = (\{(a, [0.9]), (b, [0.8])\}, \{(c, 0.6)\}, (\{\emptyset\}))$.

The following proposition shows that a possibilistic equilibrium is actually determined by its classical counterpart and the necessity function, and vice versa.

Proposition 9. *Let $\overline{M}$ be a poss-MCS.*
1. If $\overline{S}$ is a possibilistic equilibrium of $\overline{M}$ and $a_i \in \Sigma_i$, then $(a_i, \alpha) \in \overline{S_i}$ iff $\alpha = N_{M^S}(a_i)$.
2. If S is an equilibrium of M, then $\overline{S} = (\overline{S}_1, \ldots, \overline{S}_n)$ where $\overline{S}_i = \{(a_i, N_{\underline{M}^S}(a_i)) \mid N_{M^S}(a_i) > 0$ and $a_i \in \underline{\Sigma}_i\}$ $(i = 1, \ldots, n)$.
3. If $\overline{S}$ be a possibilistic equilibrium of $\overline{M}$, then S is an equilibrium of M.

Let us introduce the possibility distribution for normal poss-MCS.

Definition 18. *Let $\overline{M}$ be a normal poss-MCS and S be an belief state. Then the possibility distribution, denoted $\tilde{\pi}_{\overline{M}}$, is defined by:*

$$\forall S, \tilde{\pi}_{\overline{M}}(S) = \pi_{\overline{M}^S}(S). \tag{19}$$

The possibility degree for a normal poss-MCS $\overline{M}$ and an equilibrium of the classical projection M of $\overline{M}$ has the following connection.

Proposition 10. *Let $\overline{M}$ be a poss-MCS and S be a belief state. Then $\tilde{\pi}_{\overline{M}}(S) = 1$ iff S is an equilibrium of M.*

Proof. If $\tilde{\pi}_{\overline{M}}(S) = 1$ then $\pi_{\overline{M}^S}(S) = 1$, thus $S = GE(M^S)$. So S is an equilibrium of M. And if S is an equilibrium of M, then $S = GE(M^S)$, thus $\pi_{\overline{M}^S}(S) = 1$ then $\tilde{\pi}_{\overline{M}}(S) = 1$.

The possibility distribution for normal poss-MCS defines two measures.

Definition 19. *The two dual possibility and necessity measures for each atom in a normal poss-MCS are defined by*

- $\tilde{\Pi}_{\overline{M}}(a_i) = max\{\tilde{\pi}_{\overline{M}}(S) \mid a_i \in S_i\}$
- $\tilde{N}_{\overline{M}}(a_i) = 1 - max\{\tilde{\pi}_{\overline{M}}(S) \mid a_i \notin S_i\}$

5 Conclusion

In this paper we have established the first framework for possibilistic reasoning and nonmonotonic reasoning in multi-context systems, called possibilistic multi-context systems (poss-MCS). In our framework, a context is represented as a possibilistic logic programs and the semantics for a poss-MCS is defined by its equilibria that are based on the concepts of possibilistic answer sets and possibility distributions. We have studied several properties of poss-MCS and in particular, developed a fixpoint theory for poss-MCS, which provides a natural connection between the declarative semantics and the computation of the equilibria. As a result, algorithms for poss-MCS are also provided. Needless to say, this is just the first and preliminary attempt in this direction. There are several interesting issues for future study. First, as we have seen in the last two sections, the possibilistic equilibriua of a poss-MCS are computed using a procedure based double iterations. Such an algorithm can be inefficient in some cases. So it would be useful to develop efficient algorithm for computing possibilistic equilibriua. Another important issue is to apply poss-MCS in some semantic web applications.

Acknowledgement. This work was supported by the Australia Research Council (ARC) Discovery grants DP1093652 and DP110101042.

References

1. Benferhat, S., Sossai, C.: Reasoning with multiple-source information in a possibilistic logic framework. Information Fusion 7, 80–96 (2006)
2. Bettini, C., Brdiczka, O., Henricksen, K., Indulska, J., Nicklas, D., Ranganathan, A., Riboni, D.: A survey of context modelling and reasoning techniques. Pervasive and Mobile Computing 6(2), 161–180 (2010)
3. Bikakis, A., Patkos, T., Antoniou, G., Plexousakis, D.: A Survey of Semantics-Based Approaches for Context Reasoning in Ambient Intelligence. In: Mühlhäuser, M., Ferscha, A., Aitenbichler, E. (eds.) AmI 2007 Workshops. CCIS, vol. 11, pp. 14–23. Springer, Heidelberg (2008)
4. Brewka, G., Eiter, T.: Equilibria in heterogeneous nonmonotonic multi-context systems. In: Proc. AAAI, pp. 385–390 (2007)
5. Brewka, G., Roelofsen, F., Serafini, L.: Contextual default reasoning. In: Proc. IJCAI, pp. 268–273 (2007)
6. Dubois, D., Lang, J., Prade, H.: Possibilistic logic. In: Handbook of Logic in Artificial Intelligence and Logic Programming, vol. 3, pp. 439–513 (1995)
7. Serafini, L., Giunchiglia, F.: Multilanguage hierarchical logics, or: how we can do without modal logics. In: Artificial Intelligence, pp. 29–70 (1994)
8. Gelfond, M., Lifschitz, V.: The stable model semantics for logic programming. In: Proc. 5th ICLP, pp. 1070–1080 (1988)
9. Lloyd, J.W.: Foundations of Logic Programming, 2nd edn. Springer, New York (1987)
10. McCarthy, J.: Notes on formalizing context. In: Proc. IJCAI, pp. 555–560 (1993)
11. Nicolas, P., Garcia, L., Stéphan, I., Lefèvre, C.: Possibilistic uncertainty handling for answer set programming. In: Annals of Mathematics and Artificial Intelligence, pp. 139–181 (2006)
12. Roelofsen, F., Serafini, L.: Minimal and absent information in contexts. In: Proc. IJCAI, pp. 558–563 (2005)
13. Yager, R.R.: An introduction to applications of possibility theory. Human Syst. Manag., 246–269 (1983)
14. Zadeh L.A.: Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets and Systems, 3–28 (1978)