

Texture Feature Extraction Based on Fractional Mask Convolution with Cesáro Means for Content-Based Image Retrieval

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Abstract. This paper introduces a texture features extraction technique for content-based image retrieval using fractional differential operator mask convolution with Cesáro means. We propose one general fractional differential mask on eight directions for texture features extraction. Image retrieval based on texture features is getting unusual concentration because texture is an important feature of natural images. Experiments show that, the capability of texture features extraction by fractional differential-based approach appears efficient to find the best combination of relevant retrieved images for different resolutions. To compare the performance of image retrieval method, average precision and recall are computed for query image. The results showed an improved performance (higher precision and recall values) compared with the performance using other methods of texture extraction.

Keywords: Fractional calculus, fractional differential, Cesáro means, fractional mask, texture segmentation, content based image retrieval.

1 Introduction

A typical content-based image retrieval (CBIR) system consists of two main tasks, feature extraction and similarity measurement. The key to a successful retrieval system is choosing the right features to accurately represent the images and the size of the feature vector. Commonly the features used in CBIR are include color, shape, texture or any combination of them.

Texture is an important feature of natural images a variety of image texture applications and has been a subject of intense study by many researchers [1]. Image texture, defined as a function of the spatial variation in pixel intensities (gray values). Therefore, texture retrieval is relevant to CBIR since texture characteristics are powerful in discriminating between images [2]. A wide variety of techniques for texture have been proposed. Few of the techniques used global color and texture features [3-5].

The objective of this paper is to develop a technique which captures texture features in an image using fractional differential mask convolution with Cesáro means, which is the main contribution of this work. Each pixel of the image is convolved with the fractional differential mask on eight directions. The features computed on these masks serve as local descriptors of texture. Compare to co-occurrence matrix and Gabor filters for texture features extraction methods, the proposed method performs well with very less computational time. The outline of the paper is as follows: Fractional calculus is presented in Section 2. The construction of fractional differential mask scheme is presented in Section 3. Experimental results and conclusion are shown in Sections 4 and 5, respectively.

2 Fractional Calculus

Fractional calculus and its applications (that is the theory of derivatives and integrals of any arbitrary real or complex order) has importance in several widely diverse areas of mathematical physical and engineering sciences. It generalized the ideas of integer order differentiation and n-fold integration. Fractional derivatives introduce an excellent instrument for the description of general properties of various materials and processes. This is the main advantage of fractional derivatives in comparison with classical integer-order models, in which such effects are in fact neglected. The advantages of fractional derivatives become apparent in modeling mechanical and electrical properties of real materials, as well as in the description of properties of gases, liquids and rocks, and in many other fields.

Nowadays, fractional calculus (integral and differential operators) arises in signal and image processing. The fractional calculation is able to enhance the quality of images, with interesting possibilities in edge detection and image restoration, to reveal faint objects in astronomical images and devoted to astronomical images analysis [6,7]. Furthermore, fractional calculus is employed in texture segmentation [8], design problems of variables [9] and image denoising [10]. Finally, the fractional calculus (differential operators) is used in different applications in engineering [11].

3 Construction of Fractional Differential Mask

This section briefly describes the mathematical background for the fractional differential mask (φ) that has been used by the proposed algorithm. We have proceeded to construct the generalized fractional mask using the following generalized fractional differential operator [12] :

$$D_z^{\alpha,\mu} f(z) := \frac{(\mu+1)^\alpha}{\Gamma(1-\alpha)} \frac{d}{dz} \int_0^z \frac{\zeta^\mu f(\zeta)}{(\zeta^{\mu+1} - z^{\mu+1})^\alpha} d\zeta; 0 < \alpha \leq 1, \tag{1}$$

where the function $f(z)$ is analytic function.

Proposition 3.1. The generalized derivative of the function $f(z) = z^\nu, \nu \in \mathbf{R}$ is given by

$$D_z^{\alpha,\mu} f(z) = \frac{(\mu+1)^{\alpha-1} \Gamma(\frac{\nu}{\mu+1} + 1)}{\Gamma(\frac{\nu}{\mu+1} + 1 - \alpha)} z^{(1-\alpha)(\mu+1)+\nu-1}.$$

Proof. We let $\eta := (\frac{\zeta}{z})^{\mu+1}$ then we have

$$\begin{aligned} D_z^{\alpha,\mu} z^\nu &= \frac{(\mu+1)^\alpha}{\Gamma(1-\alpha)} \frac{d}{dz} \int_0^z \frac{\zeta^{\mu+\nu}}{(z^{\mu+1} - \zeta^{\mu+1})^\alpha} d\zeta \\ &= \frac{(\mu+1)^{\alpha-1}}{\Gamma(1-\alpha)} \frac{d}{dz} z^{(1-\alpha)(\mu+1)+\nu} \int_0^1 \eta^{\frac{\nu+\mu+1}{\mu+1}-1} (1-\eta)^{(1-\alpha)-1} d\eta \\ &= \frac{(\mu+1)^{\alpha-1} \Gamma(\frac{\nu}{\mu+1} + 1)}{\Gamma(\frac{\nu}{\mu+1} + 1 - \alpha)} z^{(1-\alpha)(\mu+1)+\nu-1}. \end{aligned}$$

Assume that the uniformly sampled signal satisfying the n -degree polynomial

$$s_n(z) = \sum_{k=0}^n z^k$$

which is used to fit the given signal $z = 1, 2, \dots, I$. Now in view of proposition 3.1, we have

$$D_z^{\alpha,\mu} s_n(z) = \sum_{k=0}^n D^{\alpha,\mu} z^k = \sum_{k=0}^n \phi_k z^{(1-\alpha)(\mu+1)+k-1}. \tag{2}$$

with the following condiments

$$\begin{aligned} \phi_0 &= \frac{(\mu+1)^{\alpha-1}}{\Gamma(1-\alpha)} \\ \phi_1 &= \frac{(\mu+1)^{\alpha-1} \Gamma(\frac{1}{\mu+1} + 1)}{\Gamma(\frac{1}{\mu+1} + 1 - \alpha)} \\ &\vdots \\ \phi_{n-1} &= \frac{(\mu+1)^{\alpha-1} \Gamma(\frac{n-1}{\mu+1} + 1)}{\Gamma(\frac{n-1}{\mu+1} + 1 - \alpha)}. \end{aligned} \tag{3}$$

Note that when $\mu = 0$, (3) reduces to the case of the Riemann-Liouville fractional operator. However, in the context of image processing Eq.(3) applies uniformly in the whole digital image and therefore should be in two directions of z and w . Now for two variables function like images, the negative direction of z and w coordinates, can be expressed as

$$D_z^{\alpha,\mu} s(z_-, w) = \phi_0 s(z, w) + \sum_{k=1}^n \phi_k s(z - k, w) \tag{4}$$

and

$$D_z^{\alpha,\mu} s(z, w_-) = \phi_0 s(z, w) + \sum_{k=1}^n \phi_k s(z, w - k). \tag{5}$$

While for two variables on the positive direction of z and w coordinates, we have

$$D_z^{\alpha,\mu} s(z_+, w) = \phi_0 s(z, w) - \sum_{k=1}^n \phi_k s(z + k, w) \tag{6}$$

and

$$D_z^{\alpha,\mu} s(z, w_+) = \phi_0 s(z, w) - \sum_{k=1}^n \phi_k s(z, w + k). \tag{7}$$

For digital images, two dimensional fractional differential mask coefficients can be obtained in eight directions of 180°, 90°, 0°, 270°, 45°, 135°, 315°, 225°, as shown in Fig.1.

The output of each image block is eight values, which are representing the texture information in each image. Then the final texture value for each image block is calculated by using Cesáro means of the following equation [13]:

$$g_k = \sum_{n=1}^k \left(\frac{k-n+1}{k} \right) z^n \tag{8}$$

All texture image block values are combined to produce one texture feature for each image. In order to create the texture features vector, in the beginning each image is divided into non- overlapped image blocks. The size of each image block is equal to the size of the fractional differential mask (4x4). The number of image block is chosen to achieve the requirements of the image detail. The main objective of applying Cesáro means is to reduce the size of the texture feature vector to one value instead of 16 for each image block without compromising their discriminating ability. The similarity measurement of a new texture feature vector will be possible by searching the most similar vector into the database, where the retrieved images are ranked by their Euclidean distances $D(i,j)$ to their respective query image which is calculated as follows[1].

$$D_{(i,j)} = \sqrt{\sum_{n=1}^m (x_i - x_j)^2 + (y_i - y_j)^2} \tag{9}$$

where $D_{i,j}$ is the distance of two images x_i, y_i, x_j, y_j and n, m is the image size.

The proposed texture features extraction includes the following steps:

- i. Resize the images to $128 \times 128 \times 3$ and converted to grayscale.
- ii. Divide each image into a specific number of blocks (32x32). The size of each image block is equal to the size of the fractional differential mask (4x4).
- iii. Set the mask window size and the values of the fractional powers α and μ .
- iv. Apply fractional differential mask convolution on eight directions with the gray value of corresponding image pixels, and adding all product terms to obtain weighting sum on eight directions.
- v. Find the arithmetic mean of the weighting sum value on the eight directions as approximate value of fractional differential for image pixel.
- vi. Apply Cesáro means to reduce the size of the texture feature vector.
- vii. Repeat steps iii to vi for whole image pixels.
- viii. Store the complete texture vector for each image.

0	0	0	0	0
0	0	0	0	0
φ_{n-1}	φ_1	φ_0
0	0	0	0	0
0	0	0	0	0

(a)

0	0	φ_{n-1}	0	0
0	0	...	0	0
0	0	...	0	0
0	0	φ_1	0	0
0	0	φ_0	0	0

(b)

0	0	0	0	0
0	0	0	0	0
φ_0	φ_1	φ_{n-1}
0	0	0	0	0
0	0	0	0	0

(c)

0	0	φ_0	0	0
0	0	φ_1	0	0
0	0	...	0	0
0	0	...	0	0
0	0	φ_{n-1}	0	0

(d)

0	0	0	0	φ_{n-1}
0	0	0	...	0
0	0	...	0	0
0	φ_1	0	0	0
φ_0	0	0	0	0

(e)

φ_{n-1}	0	0	0	0
0	...	0	0	0
0	0	...	0	0
0	0	0	φ_1	0
0	0	0	0	φ_0

(f)

φ_0	0	0	0	0
0	φ_1	0	0	0
0	0	...	0	0
0	0	0	...	0
0	0	0	0	φ_{n-1}

(g)

0	0	0	0	φ_0
0	0	0	φ_1	0
0	0	...	0	0
0	...	0	0	0
φ_{n-1}	0	0	0	0

(h)

Fig. 1. Fractional differential masks on directions of 180°, 90°, 0°, 270°, 45°, 135°, 315°, 225°

4 Experimental Results

Image retrieval is the process of finding similar images from a large image database with the help of some key attributes related to the images or features contained in the images [1]. Texture features for all images are extracted and stored in a database for comparison with the texture feature value of query image. Each row in the feature vectors represents the feature vector of an image, and each column represents the result vector for each mask windows (1×1024) as the complete texture vector for each image to be used for image retrieval. The main objective of applying Cesáro means is to reduce the size of the texture feature vector to one value instead of 16 for each mask windows (4×4). The measurement of a new feature vector will be possible by searching the most similar vector into the database, where the retrieved images are ranked by their Euclidean distances to their respective query image.

Performance tests for the system proposed by this paper were implemented using Matlab 2010a on Intel(R) Core i7 at 2.2GHz, 4GB DDR3 Memory, system type 64-bit, Window 7. The method is tested using the VisTex database which is a collection of texture images with images spread across classes containing 16 images each. Fig. 2, illustrates some of these images.

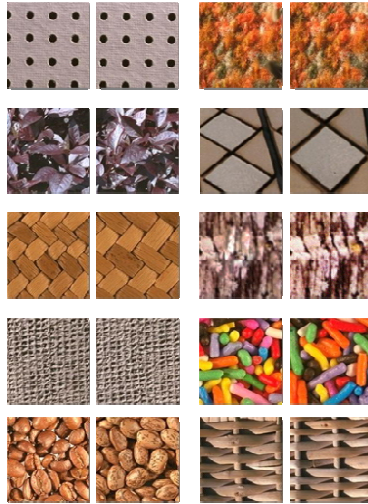


Fig. 2. Images from the database

To evaluate the retrieval performance, we have used the precision-recall crossover point. Precision-Recall, are calculated as follows:

$$\text{Precision} = \frac{\text{Number of relevant images retrieved}}{\text{Total number of images retrieved}}$$

$$\text{Recall} = \frac{\text{Number of relevant images retrieved}}{\text{Total number of relevant images in database}}$$



Fig. 3. Query Image

To discuss the performance of the algorithm we have used one class image, which is shown in Fig. 3 considered as an example. The algorithm is applied on the database of 304 images to generate texture feature vector for each image in the database and the query image and then calculate the Euclidian distance to find the relevant images. The proposed algorithm exhibited good results as shown Fig. 4, where the first 9 retrieved images are shown. The number of relevant images from same class are 7, and a few images from different classes retrieved. The precision-recall crossover plot for the same is shown in the Fig. 5, where both the precision and recall curves intersect is called crossover point in precision and recall. Crossover point can be used in a way to measure how correct the proposed algorithm is, higher the crossover point, better is the performance of the method [14]. The average precision-recall crossover of the CBIR method acts as one of the important parameters to judge its performance. A comparison with co-occurrence matrix and Gabor filters as standard methods for texture extraction is presented in this paper.

Co-occurrence matrix is a statistical method to describe textures in an image is achieved primarily by modeling texture as a two-dimensional gray level variation[15]. Gabor filter (or Gabor wavelet), widely adopted to extract texture features from images for image retrieval. Many proposed retrieval techniques adopt the Gabor wavelet as a useful texture descriptor[1].

As the comparison is done on the bases of these two algorithms, the relevancy of image decreases are shown in Figs. 6 - 8 for co-occurrence matrix and Gabor filter respectively. The average precision-recall crossover for co-occurrence matrix and Gabor filter are shown in Fig.7 and 9 respectively. Table 1, shows the comparison of experimental results of proposed method with co-occurrence matrix and Gabor filter.

The proposed technique for CBIR system provides satisfactory results, extracting quite relevant images from the same class. The higher crossover point of the proposed algorithm, acts as one of the important parameters to judge its performance.



Fig. 4. Retrieved images for query image of Fig. 2 (proposed algorithm)

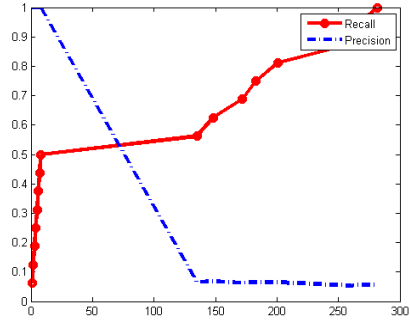


Fig. 5. Precision - recall for query image of Fig. 2 with the number of images retrieved (proposed algorithm)



Fig. 6. Retrieved images for query image of Fig. 2 (co-occurrence matrix)

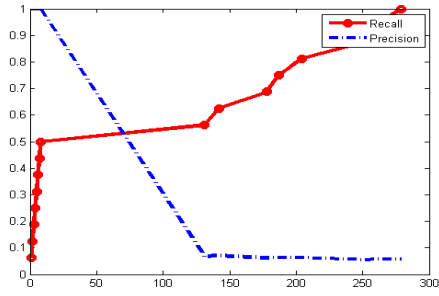


Fig. 7. Precision - recall for query image of Fig.2 with the number of images retrieved (co-occurrence matrix)

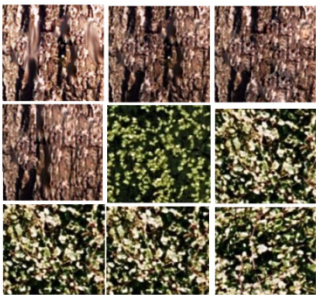


Fig. 8. Retrieved images for query image of Fig. 2 (Gabor Filters)

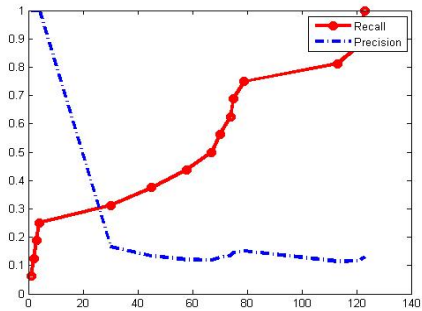


Fig. 9. Precision - recall for query image of Fig.2 with the number of images retrieved (Gabor Filters)

Table 1. Comparison of the experimental results with other standard methods

Algorithm	Average Precision	Average Recall	Cross over point
Proposed	0.5432	0.5401	0.53
Co-occurrence matrix	0.5207	0.5113	0.51
Gabor Filters	0.3491	0.5313	0.32

5 Conclusion

In the current paper, a texture features extraction technique, using fractional differential mask convolution with Cesáro means was used to retrieve desired images from their databases. Fractional differential mask convolution on eight directions with the gray value had been applied on eight directions. The experiment results means demonstrate the efficacy of this algorithm in comparison with the existing standard methods for texture extraction. Beside, the proposed algorithm exhibited better retrieval precision than the co-occurrence matrix and Gabor filters as standard methods for texture extraction.

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