Computation of Least Fixed Points

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Many problems from different areas can be formulated as problems of computing a fixed point of a suitable function. Classical examples include the computation of equilibria for games, price equilibria for markets, and many others. There has been significant progress in understanding better the computational nature of such problems and characterizing their complexity in terms of classes like FIXP, which captures the complexity of the computation of fixed points for general (nonlinear) algebraic functions, with the 3-player Nash equilibrium problem as a prototypical example, and PPAD for the computation of fixed points for piecewise linear functions, with the 2-player Nash equilibrium problem as a prototypical example.

For many problems, the function in the fixed point formulation is *monotone*, and the objects we want to compute are given by a specific fixed point, namely the *least fixed point* of the function. Many models and problems from a broad variety of areas can be thus expressed as least fixed point computation problems, including in particular the analysis of various probabilistic models, problems in stochastic optimization, and games. Examples include the analysis of multitype branching processes; stochastic context-free grammars; recursive Markov chains; quasi-birth-death processes; (recursive) Markov decision processes; stochastic games, whether turn-based or concurrent, flat finite-state or recursive. The objects of interest in all these problems can be computed by a natural iterative algorithm based on the least fixed point formulation, however, the algorithm takes in all these cases exponential time to converge. The question is whether the desired objects can be computed in polynomial time by alternative methods. In recent years there has been significant progress in answering this question for several of these problems. It turns out that for some classes of functions we can compute the least fixed point in polynomial time and thus we can analyze efficiently several of these models, while for others there are strong indications that they cannot be solved in polynomial time, and for yet others the question remains open. In this talk we will discuss progress in this area. The talk is based on a series of works with Kousha Etessami and Alistair Stewart.

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