Transportation under Nasty Side Constraints

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Abstract. The talk discusses planning problems where a set of items has to be transported from location A to location B subject to certain collision and/or resource constraints. We analyze the behavior of these problems, discuss their history, and derive some of their combinatorial and algorithmic properties.

The first transportation problem. Let V be a set of items/vertices, and let $G = (V, E)$ be a graph. We consider a scenario where the items in V have to be transported from point A to point B. There is a transportation device with enough capacity to carry $b \geq 1$ of the items, and there is a single driver. If two items are connected by an edge in E, they are *conflicting* and thus cannot be left alone together without human supervision. A *feasible schedule* is a finite sequence of triples (A_1, T_1, B_1), (A_2, T_2, B_2), $\ldots$, (A_s, T_s, B_s) of subsets of the item set V that satisfies the following conditions (FS1)–(FS3). The odd integer s is called the *length* of the schedule.

- (FS1) For every k, the sets A_k, T_k, B_k form a partition of V. The sets A_k and B_k form stable sets in G. The set T_k contains at most b elements.
- (FS2) The sequence starts with $A_1 \cup T_1 = V$ and $B_1 = \emptyset$, and the sequence ends with $A_s = \emptyset$ and $T_s \cup B_s = V$.
- (FS3) For even $k \geq 2$, we have $T_k \cup B_k = T_{k-1} \cup B_{k-1}$ and $A_k = A_{k-1}$. For odd $k \geq 3$, we have $A_k \cup T_k = A_{k-1} \cup T_{k-1}$ and $B_k = B_{k-1}$.

Intuitively speaking, the kth triple encodes the kth trip: A_k contains the items currently in point A, T_k the items that are currently transported, and B_k the items in point B. Odd indices correspond to forward trips, and even indices correspond to backward trips. Condition (FS1) states that the (unsupervised) sets A_k and B_k must not contain conflicting item pairs, and that set T_k must fit into the transportation device. Condition (FS2) concerns the first trip and the final trip. Condition (FS3) says that whenever the man reaches point A or B, he may arbitrarily re-divide the set of available items.

We are interested in the smallest possible capacity of a transportation device for which a given graph $G = (V, E)$ possesses a feasible schedule. For instance for the path P_3 on three vertices, it can be seen that a capacity $b = 1$ is sufficient. We discuss a variety of combinatorial and algorithmical results on these concepts; in particular we show that the smallest possible capacity has an NP-certificate.

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The second transportation problem. Let I be a set of items, and let $w(i)$ be the positive integer weight of item $i \in I$. For $J \subseteq I$ we throughout denote $w(J) = \sum_{j \in J} w(j)$, and as usual we let $w(\emptyset) = 0$. We consider a scenario where the items have to be transported from a point A at the top of a building to a point B at the bottom of the building. The transporation is done with the help of a pulley with a rope around it, and a basket fastened to each end of the rope of equal weight. One basket coming down would naturally draw the other basket up. To keep the system stable, the weights of the item sets in the two baskets must not differ by more than a given threshold Δ.

A *state* of the underlying discrete system is specified by the item set $J \subseteq I$ that currently is at point A, and with the remaining items in $I - J$ located at point B. The system can move directly from state $J \subseteq I$ to state $K \subseteq I$ if

$$|w(J \cap (I - K)) - w(K \cap (I - J))| \;\le\; \Delta,$$

where the positive integer bound Δ specifies the maximum allowed weight difference between the two exchanged subsets in the baskets. A state K is *reachable* from state J, if there is a sequence of moves that transforms J into K. It is easy to see that reachability is a symmetric relation.

We are interested in the following question: Given an item set I with weights $w(i)$, a positive integer bound Δ, an initial state I_0, and a final state I_1. Is the goal state I_1 reachable from the initial state I_0? We discuss a number of results on the algorithmic and combinatorial behavior of this motion planning problem. In particular, we show that it is Π_2^p-complete. The special case where the item weights are encoded in unary is (trivially) solvable in pseudo-polynomial time. The special case where the number of moves is bounded by a number encoded in unary is NP-complete. Some other natural (hevaily structured) special cases turn out to be solvable in polynomial time.

References

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