A Toolkit for Proving Limitations of the Expressive Power of Logics

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Zero-one laws, Ehrenfeucht-Fraïssé games, locality results, and logical reductions belong to the, by now, standard methods of Finite Model Theory, used for showing non-expressibility in certain logics (cf., e.g., the textbooks [1,2] or the entries in the Encyclopedia of Database Systems [3]).

More recently, the close connections between logic and circuits, along with strong lower bound results obtained in circuit complexity, have led to new lower bounds on the expressiveness of logics (cf., e.g., [4,5,6,7]). In particular, [4] solved a long standing open question of Finite Finite Model Theory, asking about the strictness of the bounded variable hierarchy of first-order logic on finite ordered graphs.

To characterise the precise expressive power of logics on particularly well-behaved classes of finite structures, the following "algebraic" approach has recently been very successful (cf., e.g., [8,9,10]): The first step is to identify a number of closure properties that the classes of structures defined by sentences of the logic exhibit. The second step is to identify another logic that is characterised by these closure properties. An example of this methodology is a result of [8], stating that on successor-based strings, plain first-order logic (FO, for short) is as expressive as order-invariant FO. The proof of [8] proceeds by showing that languages definable in order-invariant FO are aperiodic and closed under swaps. Then, Beauquier and Pin's characterisation [11] of FO by aperiodicity and closure under swaps immediately leads to the desired result.

The aim of this talk is to give an overview of the above mentioned methods for proving limitations of the expressive power of logics.

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