A Formalization of Topological Relations Between Simple Spatial Objects

Gutemberg Guerra-Filho, Claudia Bauzer Medeiros and Pedro J. de Rezende

Abstract This paper presents a new framework for modeling topological relations among objects of type point, line, and region. The main contributions are in two directions: First, the formalism proposed allows specifying all possible relations by means of symmetric matrices (whereas the usual formulation of such relations does not have this property). Symmetric matrices enable the efficient and automatic verification of valid matrices associated only with the possible topological relations. Second, it allows the specification of cases where two objects are spatially related in more than one way (*e.g.*, a line that crosses a given region in one part and is adjacent to the same region region in another part). This increases the flexibility that users are offered to model queries on spatial databases using topological relations.

Keywords Topological relations · 3-axis-Intersection · Conceptual neighborhood diagram

1 Introduction

Spatial relations play a central role in their (GIS) description into database query and spatial constructs, down to the query processing level (Clementini et al. 1992; Clementini et al. 1994). Most spatial query languages provide facilities and functions for expressing different spatial predicates, usually referring to topological and metric

Claudia Bauzer Medeiros is partially supported by grants from CNPq, CAPES and FAPESP. Pedro J. de Rezende is partially supported by CNPq grants 473867/2010-9, FAPESP (Fundação de Amparo à Pesquisa do Estado de São Paulo) grant 07/52015-0.

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S. Timpf and P. Laube (eds.), *Advances in Spatial Data Handling*,
Advances in Geographic Information Science, DOI: 10.1007/978-3-642-32316-4_5,

relations (Egenhofer 1991; Frank 1982; Herring et al. 1988; Raper and Bundock 1991; Roussopoulos et al. 1988). In order to support these operations, different formal frameworks have been introduced. Topological relations are the ones that have so far been studied the most. Direction relations have also been studied, and a recent formal model thereof can be found in Papadias et al. (1996).

In this paper, we present a new formal model for expressing topological relations among geometric objects. In particular, we focus on intersection matrices representations, which describe spatial relations in terms of the intersections of *interiors*, *boundaries*, and *exteriors* of objects. Existing matrix representations only allow for either the mere detection of empty intersection (*i.e.*, disjoint objects) or the enumeration of the dimension of intersections.

The various possible types of intersections between interiors, boundaries, and exteriors of geometric objects, once identified, are much more elucidative than simply knowing that an intersection exists or the set of intersection components' dimensions. In this sense, better models are needed to obtain a greater level of detail that allows for better distinction of seemingly identical topological relations making it possible for the user to focus on specific situations. However, this generality makes it necessary to define groups of relations that can be used to guide the user of the query system among the exceedingly large number of possibilities. Once such groups are well defined, one can take full advantage of the exact definitions of relations and consider all possible combinations of intersections. In this sense, Alboody et al. (2009) proposed detailed descriptions for four topological relations between regions in the intersection and difference model (Deng et al. 2007) using the separation number.

In this paper, we present a new formalism for topological relations where the (flat) intersection matrix representation is expanded into a (3-*d*) cube. The third and new dimension of this (3-*d*) cube, named the *3-axis-intersection model*, is the means for expressing and counting the ways in which two geometric objects are spatially related. A major contribution of this new framework is allowing the formal description of complicated scenarios. In these complicated scenarios, users are required to express when two objects maintain multiple different spatial relations at the same time (*e.g.*, a line that crosses a given region in one part and is adjacent to the same region in another part). Existing formalisms only describe a single topological relation at a time. In addition to that, our model allows to discriminate when two objects maintain the same topological relation multiple times by counting the number of such instances. We show that, by using our framework, one can determine beforehand an upper bound for all possible types of topological relations composed of combinations of multiple relations.

Our formalism uses definitions of topological parts (*i.e.*, interior, boundary, and exterior) for simple objects based on the topology of metric spaces. These definitions are slightly different from the ones used by previous models. Our approach leads to automatic methods to quantitatively compare different relations (*i.e.*, a topological distance). Based on this topological distance and on the ability to validate matrices, our approach allows the automatic computation of a conceptual neighborhood diagram.

A *conceptual neighborhood diagram* (Egenhofer and Mark 1995; Freksa 1992) is a graph where every node is a different topological relation and two nodes are linked if their topological distance is minimum. The quantitative comparison between different topological relations allows the automatic construction of conceptual neighborhood diagrams. Therefore, using our formalism, one can automatically establish a detailed conceptual neighborhood diagram. This lowest-level diagram can be partitioned into groups of similar topological relations. This process results into a generalized hierarchy of conceptual neighborhood diagrams, thereby helping users express their view of reality in more abstract ways and at different levels or granularities.

All existing formalizations are based on modelling topological relations between simple spatial objects. The applications of our approach also extends to consider relations among *complex* spatial objects—*e.g.*, a region composed of disjoint regions. The relationships between two complex objects are described in terms of the relations among their individual components.

Ultimately, our framework contributes to provide formal methods to naive geography, where users' intuitive descriptions can be mathematically modelled in a formal and compact notation. This notation enables the specification of queries according to topological relations and allows the search and retrieval of spatial and geographical entities in a database.

In summary, the main contributions of our new formalism for topological relations are (1) allowing the formal description of scenarios when two spatial objects have multiple relations at the same time, (2) enabling the automatic construction of conceptual neighborhood diagrams at different levels, (3) generalizing to complex spatial objects by considering the relations of individual components, and (4) providing formal methods to naive geography and to the specification of spatial queries.

The rest of this paper is organized as follows. Section 2 gives a brief overview of the main formal models that have been proposed to describe binary topological relations. Section 3 shows how these notions can be re-stated by using mathematical topology definitions. Section 4 extends this formalism by considering that two objects can simultaneously relate in several ways. Section 5 introduces the topological distance between relations and the construction of conceptual neighborhood diagrams. Finally, Sect. 6 presents conclusions and future work.

2 Related Work

In this section, we give an overview of the theoretical basis for defining binary topological relations and summarize matrix-based representations reported in the literature.

The *4-intersection* model is a widely accepted means for the representation of topological relations between region objects (Egenhofer and Herring 1990), in which the definition of relations between objects A and B is based on the four intersection sets of their interiors ($^\circ$) and boundaries (∂). The intersection sets are denoted by $S_{i,j}$, where i, j indicate the operands of the intersections as follows:

$$
\begin{aligned}
S_{0,0} &= A^{\circ} \cap B^{\circ}, \\
S_{0,1} &= A^{\circ} \cap \partial B, \\
S_{1,0} &= \partial A \cap B^{\circ}, \\
S_{1,1} &= \partial A \cap \partial B.
\end{aligned}
$$

A 2 × 2-matrix $\Im_4$ concisely represents these criteria (Eq. 1). These four intersection sets form a topological invariant of the relation between A and B. The set of values that represents the content of the intersection sets is denoted by *domain* $Dom(S)$, where S is an intersection set. The 4-intersection method regards only the values empty and non-empty as domain ($Dom_4(S) = \{\emptyset, \neg\emptyset\}$).

$$
\Im_4(A, B) = \begin{pmatrix} S_{0,0} & S_{0,1} \\ S_{1,0} & S_{1,1} \end{pmatrix} \tag{1}
$$

Egenhofer and Herring (1991a,b) introduced the *9-intersection* model extending the 4-intersection model to account for intersections between pairs of objects other than (2-d) regions, such as pairs of lines, or a line and a region. The 9-intersection model describes binary topological relations based on the intersections of the interiors, boundaries, and exteriors ($^-$) of two given spatial objects A and B:

$$
\begin{aligned}
S_{0,2} &= A^{\circ} \cap B^{-}, \\
S_{1,2} &= \partial A \cap B^{-}, \\
S_{2,0} &= A^{-} \cap B^{\circ}, \\
S_{2,1} &= A^{-} \cap \partial B, \\
S_{2,2} &= A^{-} \cap B^{-}.
\end{aligned}
$$

The nine intersections provide a formal description of the topological relations between the objects, which can be concisely represented by a 3 × 3-matrix $\Im_9$ (Eq. 2). This model applies the same domain of 4-intersection: $Dom_9(S) = Dom_4(S)$.

$$
\Im_9(A, B) = \begin{pmatrix} S_{0,0} & S_{0,1} & S_{0,2} \\ S_{1,0} & S_{1,1} & S_{1,2} \\ S_{2,0} & S_{2,1} & S_{2,2} \end{pmatrix} \tag{2}
$$

In the *dimension extended method* (Clementini and Felice 1995; Clementini et al. 1992), Clementini et al. take into account the highest dimension of the intersection, instead of only distinguishing the content (emptiness or non-emptiness) of the intersection. This method is also an extension of the 4-intersection model, where the intersection set S can now be either ∅ , 0-d, 1-d, or 2-d: $Dom_{dim}(S) = \{\emptyset, 0{-}d, 1{-}d, 2{-}d\}$. For instance, the 4 intersections between a line and a region result in the following possible cases:

Table 1 The number of relations for all relation groups

Relation groups	4-int	9-int	ext-dim	ref-dim
Region/region	8	8	9	16
Region/line		19	17	43
Line/line		33	18	61
Region/point			3	3
Line/point			3	3
Point/point			2	2

$$
\begin{aligned}
Dom_{dim}(S_{0,0}) &= \{\emptyset, 1\text{-}d\}, \\
Dom_{dim}(S_{0,1}) &= \{\emptyset, 0\text{-}d\}, \\
Dom_{dim}(S_{1,0}) &= \{\emptyset, 0\text{-}d, 1\text{-}d\}, \\
Dom_{dim}(S_{1,1}) &= \{\emptyset, 0\text{-}d\}.
\end{aligned}
$$

Based on the dimension extended model, Clementini et al. also show that, from the users' point of view, all binary relations can be expressed in terms of 5 operators (cross, in, overlap, disjoint, touch) and two boundary functions. This means that, in order to formulate queries or to describe a scenario, users can precisely express what they want using only this vocabulary.

McKenney et al. (2005) proposed the 9-*intersection dimension matrix* based on the dimension of an intersection as the topological invariant. In their model, the refined dimension of a given point set is the union of dimensions of its maximal connected components. Hence, the dimension matrix actually considers all possible dimension combinations of points, lines, and regions: $\{\bot$, 0-d, 1-d, 2-d, 01-d, 02-d, 12-d, 012-$d\}$, where $\bot$ is the undefined dimension of an empty set, 0 is the dimension of single points, 1 is the dimension of single lines, and so on. Besides considering all possible combinations of dimensions in a more explicit manner where each unique dimension has its own entry in the matrix, our approach counts the number of components per intersection and for each dimension. This enables a higher level of discriminative power to represent more details in terms of topological relations. Furthermore, the basic predicates defining topological parts (*i.e.*, interior, boundary, exterior) are fundamentally different in our model. We follow a more strict topological definition.

Each matrix-based model allows representing a number of feasible binary topological relations among objects. These numbers are shown in Table 1. The table shows, for instance, that the 9-intersection model allows representing up to 8 different cases of relations among regions, and up to 33 cases of relations among lines.

The 9^+-*intersection model* (Kurata and Egenhofer 2007) considers the intersections of topological primitives of spatial objects. A topological primitive is a self-connected and mutually-disjoint subset of a topological part of the spatial object. Therefore, the 9^+-intersection model describes topological relations between complex objects that may consist of multiple disjoint subparts.

The *intersection and difference model* (Deng et al. 2007) uses only the interior and the boundary of regions. The model describes topological relations according to intersection sets ($A^\circ \cap B^\circ$ and $\partial A \cap \partial B$) and difference sets ($A - B$ and $B - A$) of two given spatial objects A and B.

Kurata (2009) proposed a method to build a conceptual neighborhood graph for a given set of topological relations according to the 9^+-intersection model. The graph links pairs of topological relations according to a smooth transformation that changes one relation into the other. Two topological relations r_a and r_b are *conceptual neighbors* when there exists a smooth transformation that changes the topological relation of two simple objects from r_a to r_b. Instead of using a smooth transformation, we construct conceptual neighborhood diagrams similarly to the snapshot model (Egenhofer and Al-Taha 1992). This approach considers similarity as the smallest number of different elements in a matrix-based representation. This allows the automatic computation of conceptual neighbors for a set of topological relations.

3 Simple Spatial Objects

The interior, boundary, and exterior are the *topological parts* of objects used in the literature to describe topological relations. Some of the previous representations are based on formal definitions for topological parts different from the pure mathematical theory (Alexandroff 1961). The difference lies in the definition of “boundary”. First, unlike mathematical topology, points have no boundaries. Second, the boundary of a line is defined as being composed of two nodes at which exactly one 1-cell ends; its interior is the union of all interior nodes and all connections between the nodes.

Our definitions strictly follow the theory of general topology with a single definition of topological parts for all spatial objects. As some other previous models (Egenhofer and Franzosa 1991), our approach uses definitions of interior, boundary, and exterior for a simple object based on the standard general topology. These definitions are slightly different from the ones used by previous models. Here, we stress the differences between our work and the related literature with regards to the definitions concerning topological parts of simple spatial objects.

3.1 Definitions

To review the main definitions, we briefly recap a couple of them from topology of metric spaces. Let S be a subset of $\mathbb{R}^2$.

- A point $p \in S$ is an *interior point* of S if there is an open disc[1] centered at p totally contained in S. The set of all interior points of S is the *interior* of S—denoted S°.

[1] An open disc centered at a point p is the set $\{q \in \mathbb{R}^2 | d(p,q) < r\}$ for some positive radius r, where d is the Euclidean distance.

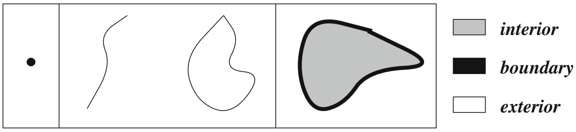

Fig. 1 Topological parts of simple objects

- A point $p \in \mathbb{R}^2$ is a *boundary point* of S if all discs of positive radius centered at p intersect both S and its complement (or *exterior*) $S^- = \mathbb{R}^2 - S$. The set of all boundary points of S is the *boundary* of S—denoted ∂S.
- S is said to be (topologically) *closed* if it contains its boundary.

From now on, the notions of interior, boundary, and exterior are to be regarded as those from traditional topology (Alexandroff 1961) whereby they are to be viewed in relation to the whole embedding topological space ($\mathbb{R}^2$), and not to a subspace of it.

We consider here any spatial objects in $\mathbb{R}^2$ of three possible dimensions, namely, (0-*d*) points, (1-*d*) curves, and (2-*d*) regions, provided that the last two satisfy certain conditions. A curve (which we often refer to as a *line*) must be a topologically closed arc of a simple Jordan curve of finite length. This means that it may have either two endpoints (included in the curve, in which case it is homeomorphic to a closed interval), or no endpoints (in which case it is homeomorphic to a circle—and is alluded to as a *cycle*). A region must be bounded and homeomorphic to a closed disc. Therefore, a line is connected, without self intersection, of finite length; and a region is topologically closed, connected, bounded, and simply-connected (without holes).

The interior of a point and that of a line are empty, while each of these is equal to its own boundary[2] (see Fig. 1). The interior of a region is homeomorphic to an open disc and its boundary is a line.

We will use the term *feature* to represent any spatial object of the types above. A *simple spatial object* obeys two properties:

a) it is (topologically) closed; and
b) it is connected, that is, it is not the union of two separated features.

4 A New Formalization for Spatial Relations

As shown in Sect. 2, previous formalizations of spatial relations between two objects are based on the specification of one relation at a time. This precludes the possibility of describing cases where two objects are related to each other in multiple ways. Figure 2 shows one such example: the line L both crosses and touches the region R.

[2] Some authors consider the boundary of a (non cycle) line as consisting of its two endpoints and its interior as the (non-empty) remaining arc.

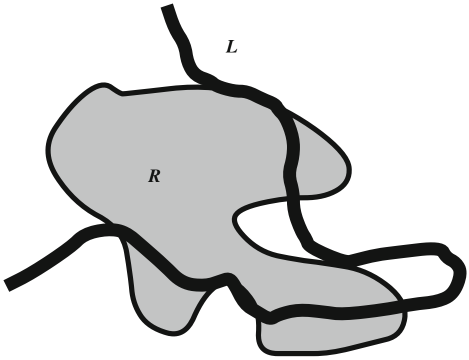

Fig. 2 A situation with two objects related in multiple ways

Users can still describe this, but the 4- or 9-intersection models are no longer able to represent this.

If one uses the 4- or 9-intersection models, these multiple relations would require a set of matrices, each of which describing one such relation. Furthermore, L touches R three times and also crosses it three times. Again, this cannot be expressed in previous formalizations. We now present a model, named the 3-axis-intersection model, that can be used to express these multiple relations.

Let us now build upon the formalism described in the previous section. We introduce the mechanisms needed to define the binary topological relations in terms of intersections of the topological parts of spatial objects. This new formalism considers the dimension of the intersection. More specifically, we use the 9-intersection matrices for the 0-d, 1-d, and 2-d components as different sets. These components are represented by the three 3×3-matrices, $\Im_{3-axis_{0-d}}$, $\Im_{3-axis_{1-d}}$, $\Im_{3-axis_{2-d}}$, which we call the *3-axis-intersections*:

$$\Im_{3-axis_{0-d}}(A, B) = \begin{pmatrix} S_{0,0,0} & S_{0,1,0} & S_{0,2,0} \\ S_{1,0,0} & S_{1,1,0} & S_{1,2,0} \\ S_{2,0,0} & S_{2,1,0} & S_{2,2,0} \end{pmatrix},$$

$$\Im_{3-axis_{1-d}}(A, B) = \begin{pmatrix} S_{0,0,1} & S_{0,1,1} & S_{0,2,1} \\ S_{1,0,1} & S_{1,1,1} & S_{1,2,1} \\ S_{2,0,1} & S_{2,1,1} & S_{2,2,1} \end{pmatrix},$$

$$\Im_{3-axis_{2-d}}(A, B) = \begin{pmatrix} S_{0,0,2} & S_{0,1,2} & S_{0,2,2} \\ S_{1,0,2} & S_{1,1,2} & S_{1,2,2} \\ S_{2,0,2} & S_{2,1,2} & S_{2,2,2} \end{pmatrix}.$$

Each of these three matrices corresponds to nine intersection sets. Each of these usual intersection sets is generated as the union of connected intersection components in a particular dimension (0-d, 1-d, and 2-d), amounting to a total of 27 possible sets $S_{i,j,k}$. The first index (i) indicates the topological part of the first object, the second index (j) specifies the topological part of the second spatial object, while the last index (k) indicates the dimension of the connected components in the set. For example,

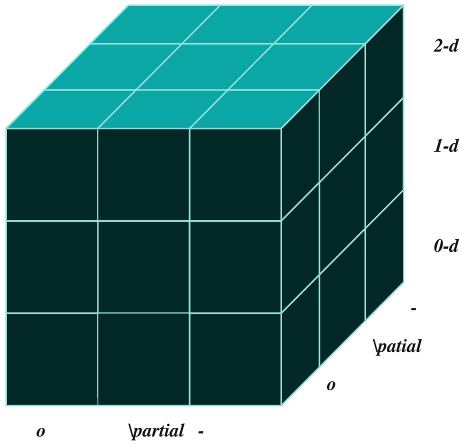

Fig. 3 The 3-axis-intersection model

$S_{1,2,1}$ identifies the intersection set of the boundary of the first object with the exterior of the second object including only 1-dimensional connected components.

The 27 elements can be graphically represented as a 3 × 3 × 3 cube composed of 27 unit cubes, each of which represents an intersection set (see Fig. 3). The cube is depicted in three layers of unit cubes, where the bottom layer corresponds to the 0-*d* matrix, the middle layer corresponds to the 1-*d* matrix, and the topmost layer corresponds to the 2-*d* matrix.

The domain of our formalism is the number of connected components per intersection ($Dom_{3-axis}(S) = \{0, 1, 2, \ldots\}$). Now, instead of just describing the absence of an intersection or its highest dimension, $S_{i,j,k}$ "counts" the number of connected components of an intersection. Figure 2 is now described using this formalism by means of three matrices:

$$\Im_{3-axis_{0-d}}(L, R) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\Im_{3-axis_{1-d}}(L, R) = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 2 & 4 \\ 0 & 6 & 0 \end{pmatrix},$$

$$\Im_{3-axis_{2-d}}(L, R) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 4 & 0 & 3 \end{pmatrix}.$$

For instance, $S_{1,1,1} = 2$ denotes that the boundaries of L and R intersect twice in terms of 1-*d* connected components. The 7 intersection sets with non zero values have their geometric interpretations shown in Fig. 4. For instance, consider $\Im_{3-axis_{2-d}}$, which describes the intersections of L and R with respect to 2-*d* connected

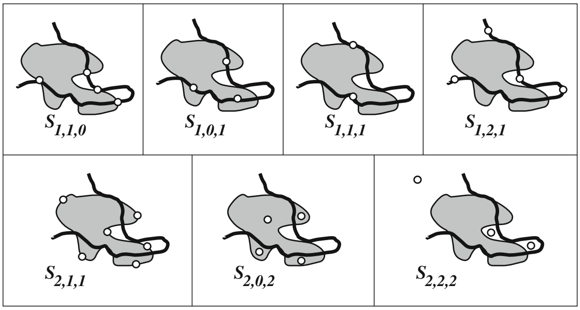

Fig. 4 The geometric interpretations of the set of intersections

components. $S_{2,0,2} = 4$ because there are 4 connected components in the intersection set $(L^{-} \cap R^{\circ})$.

It may appear discouraging that there might be $(n+1)^{27}$ possible relations between two objects, where n is the maximum number of connected components. However, a rather remarkable consequence of our restricting the spatial objects to be connected, is that most such combinations are impossible for objects embedded in the plane due to their topological properties (Egenhofer and Franzosa 1991; Egenhofer and Herring 1991a) and their codimensions (Egenhofer and Herring 1990; Herring 1991; Pigot 1991).

Since the dimension of the intersection cannot be higher than the lowest dimension of the object parts involved, some elements of the matrices are impossible (non-occurring), denoting an unfeasible relation. When a given matrix element is impossible, the corresponding unit cube does not exist. Figure 5 shows the cubes for each relation group after impossible elements are discarded. It shows, for instance, that there are at most 6 (point/point) intersection sets and at most 22 (region/region) possible intersection sets. The point/point and region/region relations determine the lower and upper bounds of the possible relations between two objects. Thus, we have $[(n+1)^6, (n+1)^{22}]$ possible relations, but the number is still unlimited.

Further simplification comes from the fact that the sets of intersections can only have connected components with the highest possible dimension. The only exception is in the boundary/boundary intersections that can have 0-d and 1-d components. Thus, the lower and upper bounds become $[(n+1)^4, (n+1)^{10}]$ possible relations.

Note that the boundary/boundary intersections ($S_{1,1,0}$ and $S_{1,1,1}$) determine the maximum value, m, in the 3-axis intersection. Let c_1 and c_2 be integers such that

$$Dom_{3-axis}(S_{1,1,0}) = \{0, \ldots, c_1\}$$

and

$$Dom_{3-axis}(S_{1,1,1}) = \{0, \ldots, c_2\}.$$

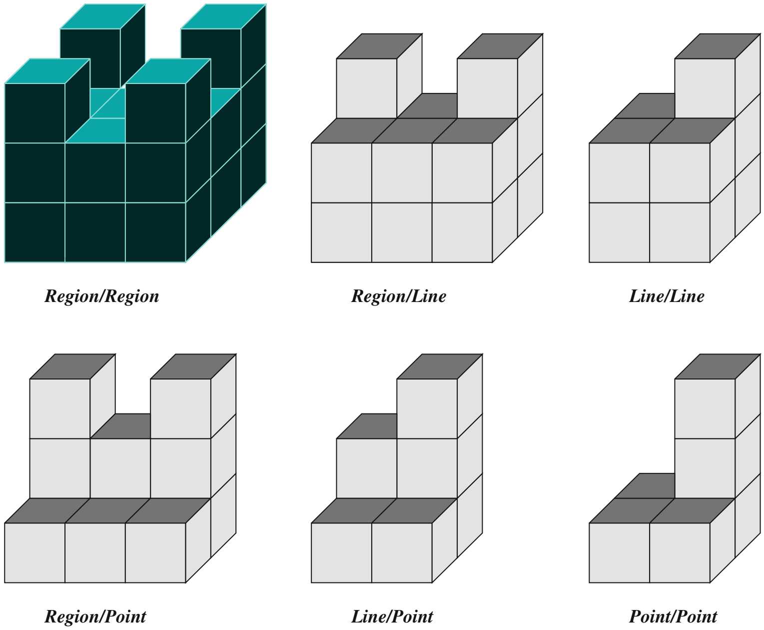

Fig. 5 The dimension of the intersection restricting the model

Table 2 The number of relations for all relation groups

Relation groups	3-axis-intersection
Region/region	10
Region/line	35
Line/line	42
Region/point	3
Line/point	5
Point/point	2

By restricting c_1 and c_2 to at most a certain constant value c, we are able to bound m to at most $(2 \times c) + 1$. By employing an appropriate value of c, we can control the maximum value in the 3-axis intersection and limit the number of possible relations to $[((2 \times c) + 2)^4, ((2 \times c) + 2)^{10}]$.

Hereafter, we assume $c = 1$, that is, $Dom_{3-axis}(S_{1,1,0}) = Dom_{3-axis}(S_{1,1,1}) = \{0, 1\}$. The number of feasible relations that can then be expressed is displayed in Table 2. This table shows, for instance, that one can define at most 35 different ways in which a line and a region can relate topologically simultaneously when $c = 1$ (*i.e.*, when there is at most one connected component per intersection).

The generalization of our approach to consider complex spatial objects (*e.g.*, a region composed of disjoint regions possibly including holes) is simple. Basically, the relationships between two complex objects are described in terms of the relations among their individual components. Formally, let O^1 and O^2 be two complex spatial objects $O^1 = \{o_1^1, \ldots, o_{n_1}^1\}$ and $O^2 = \{o_1^2, \ldots, o_{n_2}^2\}$, where o_i^k are the individual components of O^k for $k = 1, 2$ and $i = 1, \ldots, n_k$, and n_k is the number of individual components of O^k. Each individual component of a complex object is a connected simple object. We represent the topological relation $\Im(O^1, O^2)$ between O^1 and O^2 as the set of topological relations $\Im(o_i^1, o_j^2)$ between the simple objects corresponding to pairs (o_i^1, o_j^2) of individual components o_i^1 and o_j^2 of the respective complex objects O^1 and O^2 for $i = 1, \ldots, n_1$ and $j = 1, \ldots, n_2$.

5 Conceptual Neighbourhood Diagram

In order to help the user describe topological relations, they can be organized in a diagram based on the *snapshot model* (Egenhofer and Mark 1995; Freksa 1992). The snapshot model derives the conceptual neighborhoods among topological relations considering their degree of similarity. Conceptual neighborhood diagrams are used to schematize spatio-temporal relations. They enable spatio-temporal reasoning to infer properties of the relations, list possible transitions of a particular relation (*e.g.*, find a sequence of spatial configurations between two relations), and relax query constraints by including neighboring relations.

Conceptual neighborhood diagrams are constructed according to a particular smooth transformation or to a specific similarity measure. In this paper, we consider a topological distance (Egenhofer and Al-Taha 1992) based on the3-axis-intersection model. The *topological distance* $\tau(r_a, r_b)$ between two topological relations r_a and r_b is the sum of absolute values of the differences of corresponding elements in the 3-axis-intersection model:

$$\tau(r_a, r_b) = \sum_{i=0}^{2}\sum_{j=0}^{2}\sum_{k=0}^{2} |S_{i,j,k}^a - S_{i,j,k}^b|.$$

The shortest non-zero distance among all pairs of topological relations determines that two relations are considered conceptual neighbors. They are represented by graphs in which each relation is depicted as a node and conceptual neighbors are linked by edges.

Let us consider the 35 relations between a line and a region shown in Fig. 6. The 35 situations of region/line group can be presented in a conceptual neighborhood diagram. Each relation is a conceptual neighbor of at least one, and at most five other relations. The diagram is disconnected and has one subgraph G_1 with 15 nodes and another G_2 with 20 nodes. This diagram is disconnected because we only consider conceptual neighbors with a small (*i.e.*, close to the minimum) topological distance. This diagram has a particular symmetry with respect to the center, where on the left-hand side are all relations in which some parts of the line are inside the

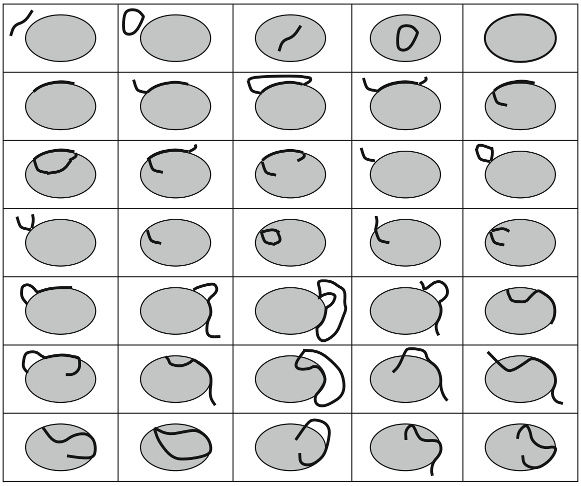

Fig. 6 The 35 region/line topological relations distinguished by the 3-axis-intersection

region, while on the right-hand side are the relations in which the corresponding parts of the line are outside. These diagrams allow partitioning groups of relations into more general cases, which in turn help users express the underlying spatial concepts.

6 Conclusions and Future Work

This paper presented a new framework for modeling topological relations among objects of type point, line, and region, which subsumes and extends previous work. It allows specification of complex scenarios where two objects are spatially related in more than one way. These results allow increasing the flexibility that users are offered to model their reality, thereby contributing to research in formal methods in naive geography.

In terms of expressiveness, our new approach displays a much higher discriminative power such that any relation represented by previous models will also have a unique 3-axis-intersection matrix. For example, even though all points in a line are considered boundary points, our model can differentiate between two open lines which cross at the middle and a line that terminates at its endpoint on another line. In the first situation, we have 5 components resulting from the intersection: the cross point and the four pieces of both lines (2 per line). In the second situation, we have only 4 components: the point where the lines touch, the entire second line (but the touch point), and the two 1-*d* pieces of the first line. The fact that the entire line is considered as a boundary does not decrease the representative power of our model. However, the fact that we count intersection components at each dimension for all 9 intersections, actually, increases the number of possible relations represented using our model. The intersection at end points of lines have a clear implication in the intersection sets of our model: the number of connected components differ as mentioned above. Therefore, the descriptive power of our model does not decrease due to our definition of topological parts (specifically boundary of lines). On the contrary, the fact that we count the number of components at each dimension for all possible 9 intersections increases significantly the number of relations that can be represented. By introducing the counting of components, the number of different relations described with our model is infinite. However, if we bound the maximum number of components per dimension per intersection, we bound the total number of feasible relations.

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