
Analysis on the Time-Dependent Rotational Displacement of Retaining Wall During the Process of Earthquake

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H. Q. Yang, D. Huang, X. P. Zhou and Y. Chen

Abstract

A simple pseudo-dynamic method to predict the seismic rotational displacement of retaining wall is developed. The proposed method, which soil-structure interaction is considered, is a combination of the free-field seismic response of soil and pseudo-dynamic method. It is supposed that soil and structure is connected by a series of springs, the dynamic earth pressure is determined by the deformation of springs. So both active and passive conditions can be taken into account by means of different movement direction of retaining wall, no need to know whether active or passive earth pressure happens before analysis. A significant difference between this analysis and published method is that in the present analysis the time dependent process of earth pressure and displacement is obtained by iterative calculation. Although present analysis is limit to elastic state, comparisons with Mononobe–Okabe method show satisfactory agreement in the value of resultant forces acting on retaining wall. Moreover, it is revealed by numerical examples that the height of the resultant force from the base of the wall is underestimated by the Mononobe–Okabe method, this may cause unsafe factors. Furthermore, the effect of wide range of parameters like time, height of retaining wall, wall friction, horizontal and vertical seismic coefficients are taken into account to evaluate the seismic response of retaining wall. Apart from its intrinsic theoretical interest, the

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proposed analysis can be used for the assessment of the safety of retaining wall under seismic condition.

Keywords

Seismic · Retaining wall · Rotational displacement · Earth pressure · Soil-structure interaction

1 Introduction

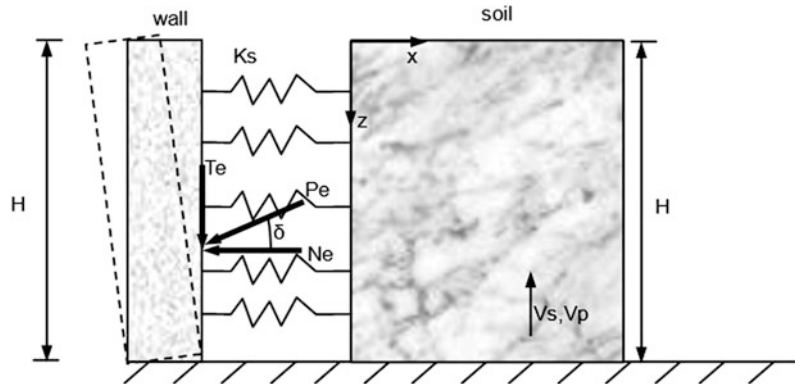
Seismic damage of highway system caused in seismic world wide indicated that modern highway system is still vulnerable, it may be damaged extensively and shut down the local traffic, which always makes big troubles for rescue in a past earthquake situation. The earthquake damage of highway system shows that the failure of retaining structures is one of the main causes of road damage. So it is key important to assess the reliability of retaining structure under seismic condition.

The seismic response of retaining structures is a complex soil-structure interaction problem. Wall movements and dynamic earth pressures depend on the response of the soil underlying the wall, the response of the backfill, the inertial and movement of the wall itself, and the nature of the input motions. Many efforts have been made in the field of seismic response of retaining wall, but it is not still sufficient. Most of the methods employed for the design of a retaining wall subjected to seismic loading are limit equilibrium. The pioneering work on earthquake induced lateral earth pressure under active and passive conditions acting on a retaining wall were published by Okabe (1926) and Mononobe and Matsuo (1929). The pseudo-static approach which is known as Mononobe–Okabe method, is widely used to calculate the dynamic earth pressure (either active or passive) on the back of retaining wall. The limit equilibrium method considers the response of the retaining wall as isolated from that of the soil. In other words, neither the interaction between the backfill and the retaining wall nor the displacement of retaining wall are considered in traditional limit equilibrium method. In order to take the soil-

structure interaction into account, elastic analysis of the seismic response of retaining structures is used by Scott (1973). Ortigosa and Musante (1991) proposed a simplified kinematic method to calculate the seismic earth pressure against retaining wall with restrained displacement. Futhermore, Richards (Richards et al. 1999) established an alternate analysis model, in which the soil supported by the retaining wall is modeled by springs in parallel. The fundamental solution to the free-field seismic response considering nonlinear, plastic behavior of soil is included in the retaining analysis. In previous approach, the dynamic loading induced by earthquake is considered as time-independent, which ultimately assumes that the magnitude and phase acceleration is uniform throughout the backfill. To overcome this constraint, Steedman and Zeng (1990) developed the pseudo-dynamic method to predict the seismic active earth pressure behind retaining wall. Later Munwar Basha and Sivakumar Babu (2010) extended pseudo-dynamic method to compute the rotational displacements of gravity retaining walls under passive condition when subjected to seismic loads. Similar alternatives have been made by Syed Mohd Ahmad and Deepankar Choudhury (2010) and Choudhury and Nimbalkar (2008). For pseudo-dynamic method, equilibrium equations of force or moment are still needed, and the soil structure interaction is ignored.

In light of the previous methods, it appears that the development of a new method for assessing seismically-induced earth pressure and rotational displacement would be desirable. The purpose of this paper is trying to combining the pseudo-dynamic method and free field solution, and then calculating the earth pressure and the rotational displacement of retaining wall under seismic loads. In this paper, not only the soil-

Fig. 91.1 Sketch of soil structure interaction model



structure interaction is taken into account, but also the time and phase difference due to finite shear wave velocity is considered. One of the main features of this paper is the adoption of a new iterative program to evaluate the time-dependent seismic earth pressure and rotational displacement of retaining wall. The analysis of present is limit to cohesionless soil in elastic state.

2 Method of Analysis

The basic system studied is a semi-infinite layer of cohesionless soil of unit weight γ that is free at its upper surface, is bonded to a rigid base at bottom, and is retained on its vertical boundary by a rigid retaining wall. The soil is approximately considered as elastic body, the backfill is supposed to be never yield or fail under seismic load. In general it is not appropriate to simulate the seismic response of a retaining structure using elastic models because such models neither capture the static or cyclic elasto-plastic behavior of the soil, nor any failure state that may be reached. However, elastic models provide a first approximation to wall and soil response and, in some cases, they may be even acceptable. As shown in Fig. 91.1, the height of the wall and the stratum is considered to be the same and is denoted by H .

The free-field stress and deformation solutions are applied in the analysis of seismic response of retaining structures. The dynamic response of the soil-wall system can be analyzed using superposition for the particular case of

rotation about the base, soil in the free field has the horizontal displacement u_f and the wall has the horizontal displacement u_w . The response of this soil-wall system is the sum of two cases: (1) the wall has the same deformation as the free field under the inertia body force and (2) the wall is pushed back with some horizontal displacement Δu equal to the difference between the horizontal displacement of the free field u_f and horizontal displacement of wall. This superposition approach is applicable no matter how the wall moves (Huang et al. 1999; Huang 1996).

The pseudo-dynamic analysis, which considers shear and primary wave velocity, can be developed by assuming that the shear modulus G is constant with depth through the backfill and the phase not the magnitude of acceleration varies. It is assumed that the base of the retaining wall is subjected to harmonic horizontal and vertical earthquake accelerations with amplitude $k_h g$ and $k_v g$, the acceleration at any depth z below the ground surface and time t can be expressed as

$$a_h(z, t) = k_{hd}(z, t)g, k_{hd}(z, t) = k_h \sin \left[\omega \left(t - \frac{H - z}{V_s} \right) \right] \tag{91.1}$$

$$a_v(z, t) = k_{vd}(z, t)g, k_{vd}(z, t) = k_v \sin \left[\omega \left(t - \frac{H - z}{V_p} \right) \right] \tag{91.2}$$

where, ω is the angular frequency, V_s is the shear wave velocity, V_p is the primary wave velocity. k_h and k_v are horizontal and vertical seismic amplitude coefficients respectively.

The total horizontal stress σ_{xw} acting on the wall is the sum of horizontal stress σ_{xf} in the free field and the stress increment $\Delta\sigma_x$ due to the relative displacement between the wall and soil in the free field

$$\sigma_{xw} = \sigma_{xf} + \Delta\sigma_x \quad (91.3)$$

The horizontal normal stress increment, $\Delta\sigma_x$, can be expressed as

$$\Delta\sigma_x = K_s[u_s(t) - u_w(t)] \quad (91.4)$$

where, K_s is subgrade modulus of the backfill soil.

Since the springs in Fig. 91.1 can be thought of as bars with length proportional to the height of the wall (Scott 1973), their stiffness, defined as the subgrade modulus in Eq. (91.4) can be written as

$$K_s = C_2 G/H \quad (91.5)$$

In most cases, a value of $C_2 = 1.35$ seems appropriate based on finite-element analysis (Huang 1996).

3 Free Field Deformation and Stress Solutions of Backfill Without Retaining Structures

The two dimensional differential equations of equilibrium can be written as

$$\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{xz}}{\partial z} = k_{hd}\gamma \quad (91.6)$$

$$\frac{\partial\tau_{xz}}{\partial x} + \frac{\partial\sigma_z}{\partial z} = k_{vd}\gamma \quad (91.7)$$

Since we are dealing with the half-space, all stress, strain and displacement components of backfill soil are independent on the x coordinate. Thus, this free-field problem is actually a one-dimensional problem, Eqs. (91.6) and (91.7) becomes

$$\frac{\partial\tau_{xz}}{\partial z} = k_{hd}\gamma \quad (91.8)$$

$$\frac{\partial\sigma_z}{\partial z} = k_{vd}\gamma \quad (91.9)$$

Stress components can be solved from Eq. (91.8), with the condition of when $z = 0$ and $t = 0$, $\tau_{xz} = 0$, $\sigma_z = 0$.

$$\tau_{xz} = \frac{2k_h v_s \gamma}{\omega} \left\{ \sin\left[\frac{\omega(2tv_s + z - 2H)}{2v_s}\right] \sin\frac{z\omega}{2v_s} \right\} \quad (91.10)$$

$$\sigma_z = \frac{2k_v v_p \gamma}{\omega} \left\{ \sin\left[\frac{\omega(2tv_p + z - 2H)}{2v_p}\right] \sin\frac{z\omega}{2v_p} \right\} \quad (91.11)$$

The horizontal stress is usually written as a lateral earth pressure coefficient K times the vertical stress σ_z . That is

$$\sigma_{xf} = K\sigma_z \quad (91.12)$$

where, $K = K_0 = 1 - \sin\phi$ is the coefficient of lateral earth pressure in the elastic state.

For soil in the elastic state, the shear strain can be expressed as follow

$$\gamma_{xz} = \frac{\tau_{xz}}{G} \quad (91.13)$$

According to elastic deformation theory, the horizontal displacement of soil in the free field is

$$u_s = - \int \gamma_{xz} dz = \frac{k_h v_s \gamma}{G\omega^2} v_s \sin\left[\frac{\omega(tv_s + z - H)}{v_s}\right] > -z\omega \cos\left[\omega\left(t - \frac{H}{v_s}\right)\right] \quad (91.14)$$

It is simply assumed that the rotational component of displacement can be ignored. The horizontal stress acting on the retaining wall can be expressed as

$$\sigma_{xw} = \sigma_{xf} + K_s[u_s(t) - u_w(t)] \quad (91.15)$$

$$u_w(z, t) = \eta(H - z) \quad (91.16)$$

where, $u_w(t)$ represents the displacement of retaining wall, η represents the rotating angle.

The total horizontal thrust acting on the retaining wall can be calculated by integrating the horizontal stress along the height of the retaining wall

$$\begin{aligned}
N_e &= \int_0^H \sigma_{xw} dz = \frac{1}{2G\omega^3} \{-GH^2 K_s \eta \omega^3 \\
&\quad - 2k_h K_s v_s^3 \gamma \cos(\omega t) + k_h K_s v_s \gamma (2v_s^2 - H^2 \omega^2) \\
&\quad \times \cos m_1 + 2GKk_v v_p \gamma \omega [H\omega_2 \\
&\quad + v_p (\sin m_2 - \sin \omega t)]\}
\end{aligned} \tag{91.17}$$

where, $m_1 = \omega t - \frac{H\omega}{v_s}$; $m_2 = \omega t - \frac{H\omega}{v_p}$.

The vertical force acting on the retaining wall is determined with knowledge of the wall frictional angle

$$T_e = N_e \tan \delta \tag{91.18}$$

The total force acting on the retaining wall is expressed as

$$P_e = \sqrt{T_e^2 + N_e^2} \tag{91.19}$$

The moment of the acting earth pressure about the toe of the retaining wall is

$$\begin{aligned}
M_e &= \int_0^H (H-z) \sigma_{xw} dz \\
&= \frac{1}{6G\omega^4} \{-2GH^3 K_s \eta \omega^4 \\
&\quad + 6\gamma GKk_v v_p^3 \omega \cos \omega t + 3\gamma GKk_v v_p \omega \\
&\quad \times (H^2 \omega^2 - 2v_p^2) \cos m_3 \\
&\quad - 6\gamma GHKk_v v_p^2 \omega^2 \sin m_3 + \gamma k_h K_s v_s [H\omega \\
&\quad \times (6v_s^2 - H^2 \omega^2) \cos m_1 \\
&\quad + 6v_s^3 (\sin m_1 - \sin \omega t)]\}
\end{aligned} \tag{91.20}$$

where, $m_3 = \frac{H\omega - \omega v_p}{v_p}$.

So the height of the resultant forces P_{ac} from the base of the wall is

$$h = \frac{M_e}{N_e} \tag{91.21}$$

4 Rotational Displacement of Retaining Wall

The following equations of motion are presented to estimate the rotational displacements of gravity retaining wall, similar to equations reported in Zeng (Zeng and Steedman 2000) and Choudhury (Choudhury and Nimbalkar 2007).

The moment of the acting earth pressure about the toe of the retaining wall is

$$\sum M_0 = M_e + W_w x_c \tag{91.22}$$

where, W_w is the weight of the gravity wall, x_c is the horizontal distance from the center to toe of retaining wall.

Again, the motion equation of the retaining wall about its toe can be written as (Choudhury and Nimbalkar 2008)

$$\sum M_0 = -y_c (W_w/g)(a_c)_x + x_c (W_w/g)(a_c)_y + I_c A \tag{91.23}$$

$$(a_c)_x = a_g - A y_c - \beta^2 x_c \tag{91.24}$$

$$(a_c)_y = A x_c - \beta^2 y_c \tag{91.25}$$

where, $a_g = k_{hd}g$, y_c is the vertical distance from the center to toe of retaining wall, I_c is polar moment of inertia of the about the centroid, A is angular acceleration.

Substituting Eqs. (91.24) and (91.25) into Eq.(91.23), we have (Choudhury and Nimbalkar 2008)

$$\sum M_0 = (W_w/g)r_c^2 A - (W_w/g)a_g y_c + I_c A \tag{91.26}$$

where, $r_c^2 = x_c^2 + y_c^2$.

Combining Eqs. (91.22) and (91.26), the rotating acceleration of retaining wall about its toe can be determined by

$$A = \frac{M_e + W_w x_c + (W_w/g)a_g y_c}{(W_w/g)r_c^2 + I_c} \tag{91.27}$$

The rotating angle velocity β is obtained by

Fig. 91.2 dynamic iterative strategy for the determination of rotating displacement

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Initial state:  $t=0, u_s= u_w=0.$ 
for each new time step  $n+1$ 
    compute horizontal thrust force  $N_e$  by eq(17)
    compute the moment  $M_e$  acting on retaining wall by eq(20)
    if  $N_e>0$  and  $M_e>0$ 
        compute the height of the resultant force from the base of the wall by eq(21)
        compute the rotating angle velocity by eq (28)
        compute the rotating displacement by eq(29)
    endif
end for
    
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$$\beta = \int Adt \tag{91.28}$$

The tilting angle can be derived as

$$\begin{aligned} \eta_{(t_f)} &= \int_0^{t_f} \beta dt \\ &= \frac{W_w x_c + (W_w/g) a_g y_c}{2[(W_w/g)r_c^2 + I_c]} t_f^2 \\ &\quad + \frac{1}{(W_w/g)r_c^2 + I_c} \left[-\frac{1}{6} H^3 K_s t_f^2 \eta_{(t_f-\Delta t)} \right. \\ &\quad - \frac{Kk_v v_p^3 \gamma}{\omega^5} (\cos \omega t_f - 1) \\ &\quad + \frac{Kk_v v_p^3 \gamma}{\omega^5} (\cos m_6 - \cos m_7) \\ &\quad + \frac{H^2 Kk_v v_p \gamma}{2\omega^3} (\cos m_7 - \cos m_6) \\ &\quad + \frac{H^2 K_s k_h v_s^3 \gamma}{G\omega^5} (\cos m_4 - \cos m_5) \\ &\quad + \frac{H^3 K_s k_h v_s \gamma}{6G\omega^3} (\cos m_5 - \cos m_4) \\ &\quad - \frac{K_s k_h v_s^4 \gamma}{G\omega^6} \sin \omega t_f \left. \right] \\ &\quad - \frac{HKk_v v_p^2 \gamma}{\omega^4} (\sin m_6 + \sin m_7) \\ &\quad + \frac{K_s k_h v_s^4 \gamma}{G\omega^6} (\sin m_4 + \sin m_5) \end{aligned} \tag{91.29}$$

where, η represents the cumulative rotating angle during the time from 0 to t_f . $m_4 = t_f \omega - \frac{H\omega}{v_s}$, $m_5 = \frac{H\omega}{v_s}$, $m_6 = t_f \omega - \frac{H\omega}{v_p}$, $m_7 = \frac{H\omega}{v_p}$.

It is revealed in Eq. (91.29) that the total rotational displacement is the summation of individual rotations during the entire earthquake motion. The above procedure is repeated for each cycle of vibration.

5 Implementation of Present Model

The algorithmic implementation of present model is based on an iterative strategy. The detailed iterative scheme is plotted in Fig. 91.2 .

Pseudo-dynamic method and free-field solution are coupled in this analysis. A simple iterative algorithmic is needed in present model and complex dynamic analysis as Newmark method is avoid. Although present method is still a pseudo-dynamic approximate one, the time-dependent seismic earth pressure and rotational displacement of retaining wall during the entire seismic process can be obtained conveniently by present model.

6 Results and Discussion

The results obtained from the numerical analyses, based on present method, are shown in this section. Comparisons have been made with the M–O

Fig. 91.3 Horizontal and vertical acceleration-time history response of backfill with $k_h = 0.1$, $\delta = 20^\circ$, at $z = 0$

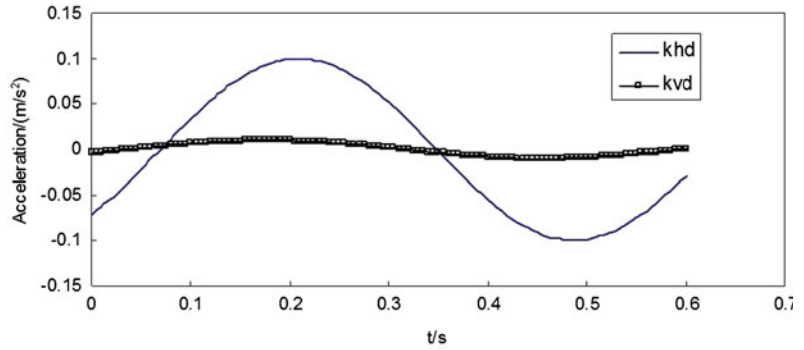
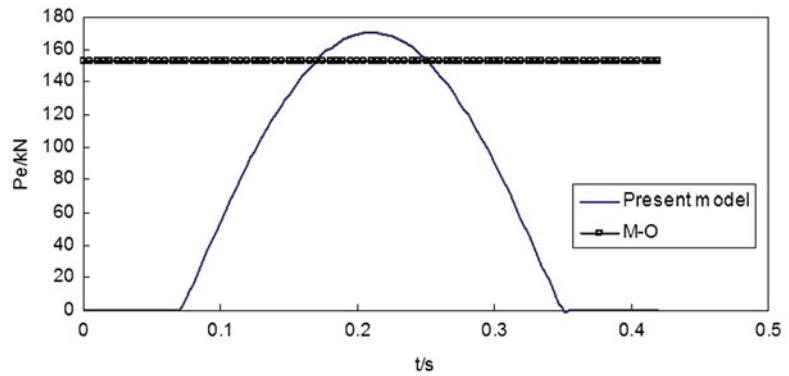


Fig. 91.4 Resultant force-time response which acting on the retaining wall with $k_h = 0.1$, $\delta = 20^\circ$



method. The retaining wall is 7.0 m in height, made of concrete with a unit weight of 25 kN/m^3 . Moreover, the backfill is made of cohesionless soil with a unit weight of 17 kN/m^3 . The following parameters are used in the calculation: $G = 350 \text{ MPa}$, $H = 7.0 \text{ m}$, $b = 1.2 \text{ m}$, $V_s = 100 \text{ m/s}$, $V_p = 187 \text{ m/s}$, $\phi = 25 \text{ Deg}$.

Figure 91.3 shows horizontal and vertical acceleration-time history response of backfill during the entire seismic process. The acceleration is time dependent, which is different from pseudo-static method. Moreover, the horizontal and vertical acceleration is independent, and the phase difference due to finite shear wave velocity is considered.

Resultant force-time response is plotted in Fig. 91.4. It is revealed that before $t = 0.22 \text{ s}$, as the time increases, the resultant force P_e acting on retaining wall also increases. But after $t = 0.22 \text{ s}$, the resultant force P_e acting on retaining wall decreases with increase in the time. Because, after $t = 0.22 \text{ s}$, the retaining

start to rotate about its toe and the relative displacement between retaining wall and backfill in the free-field decrease. Moreover, comparison between present and M-O method is carried out. It is obvious that M-O results are time-independent while present one is time-dependent. The maximum value of resultant force P_e is close to the results obtained by M-O method, with 6.25 % error.

The distribution of earth pressure is depicted in Fig. 91.5. It is revealed from Fig. 91.5 that non-linear seismic earth pressure distribution behind retaining wall in a more realistic manner compared to the pseudo-static method. It is also clear from Steedman and Zeng (1990) that the earth pressure distribution along the height is non-linear.

Figure 91.6 shows the time-dependent height of the resultant forces acting on retaining wall from the base of wall. It is well known that the height of the resultant forces obtained by pseudo-static method is time-independent, and is

Fig. 91.5 Distribution of earth pressure which varied with time

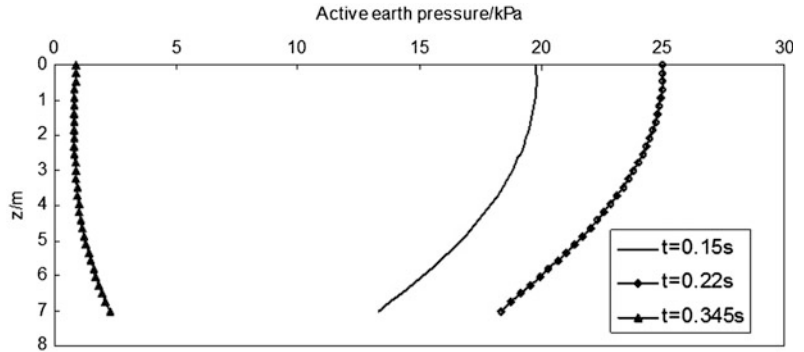


Fig. 91.6 The time-dependent height of the resultant forces acting on retaining wall from the base of wall

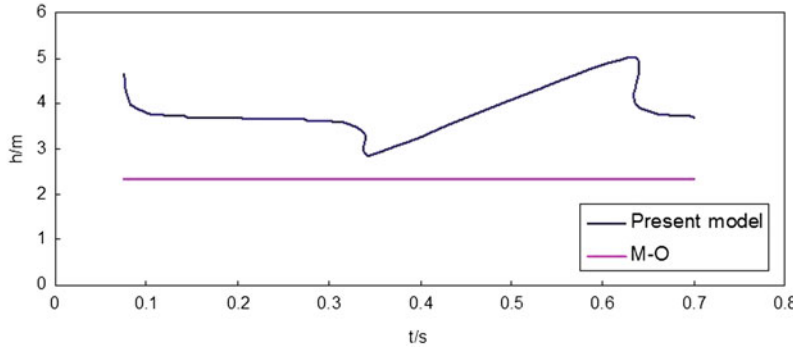
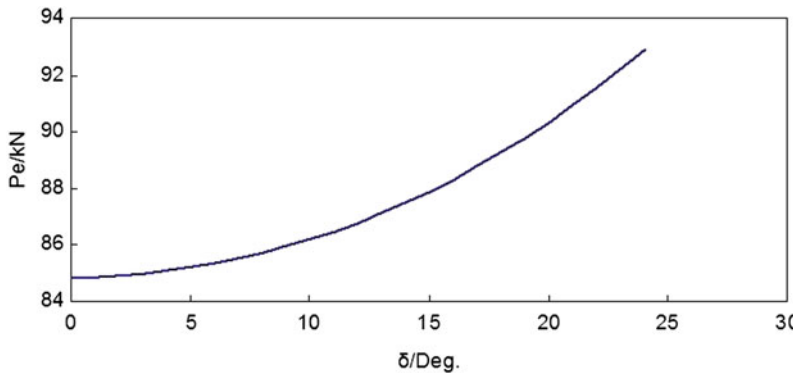


Fig. 91.7 Dependence of resultant force on the wall friction angle with $k_h = 0.1, \delta = 20^\circ$



supposed to be one-third of the height of retaining wall. However, the height obtained by present method is time dependent, ranges from 3.0 m to 5.0 m. It is higher than the pseudo-static method, so the pseudo-static method may cause some unsafe factors when the retaining wall rotating.

Figure 91.7 shows the dependence of resultant force on the wall friction angle. It is obvious that the resultant force increases with the wall friction angle increases.

7 Conclusions

In this paper, by considering the time effect and phase changes in shear and primary waves propagating in the backfill, a simple method to determine the dynamic earth pressure and rotating displacement is established. Both active and passive earth pressure are considered and it is found that earth pressure may transfer from active to passive earth pressure as the seismic

intensity reach the critical value. Moreover, the rotational displacement, distribution of earth pressure along the height of retaining wall, resultant force and its height from the base of wall are time-dependent. Little iterative calculation is needed, but the time dependent process of earth pressure and displacement is obtained. Although assumptions are adopted in this paper, the proposed analysis can be used for the assessment of the safety of retaining wall under seismic condition.

In the present model only one ideal case, rotation about the toe of retaining wall, has been studied. Nevertheless, the cases for combined failure modes such as sliding and rotation can be easily extended by modify the kinetic equation. Although the method developed here is primarily for cohesionless soil, analysis for the soil with cohesion using this approach is straightforward extensions of this work.

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