Chapter 4
Knowledge Accessibility, Economic Growth and the Haavelmo Paradox

Åke E. Andersson

Abstract Economic growth has conventionally been modelled for space-less economies. Econometrically, growth models have mostly been estimated on time series of national economies with minimal distinctions between economies as large as Japan or the USA and as small as the smallest economies of Asia and Europe. This approach to the analysis of economic growth is especially dangerous when the impact of scientific and technological knowledge is important for the process of growth. Creative activities and the formation of knowledge are highly clustered in space. Thus, the spatial distribution of accessibility to knowledge capital and investments determines economic growth of nations and other spatial aggregates.

The Haavelmo paradox contrasts chaos as the generic property of non-linear dynamic models with the fact that most statistics on macroeconomic growth processes tend towards persistent constant positive rates of growth. The paradox can be resolved if the non-linear dynamic model is subdivided into fast, private variables and very slow, public variables. Modelling spatial accessibility of knowledge as a slow, public variable and machinery and similar material capital as a relatively faster, private variable ensures stable growth, at least in the short and medium terms of the economic growth processes.

Keywords Economic growth • Chaos • Multiple time scales • Synergetics • Adiabatic approximation

Å.E. Andersson (✉)
Jönköping International Business School, Jönköping, Sweden
e-mail: Ake.Andersson@jibs.hj.se

J. Klaesson et al. (eds.), *Metropolitan Regions*, Advances in Spatial Science, 73
DOI 10.1007/978-3-642-32141-2_4, © Springer-Verlag Berlin Heidelberg 2013

1 Background

Macroeconomic theory was traditionally focused on comparative static and business cycle analysis. The problem of economic growth was not properly addressed before 1936, when the seminal paper by John von Neumann appeared in German (von Neumann 1935–1936, 1937, 1945). In this paper, von Neumann proved the existence of a general equilibrium rate of growth, product proportions and relative prices with the growth rate equal to the equilibrium rate of interest. This growing economy was modelled with interdependencies, joint production, possibilities of substitution and constant returns to scale. The analysis was quite abstract mathematically, and was accordingly not observed by the English speaking world until 1945, when a translated and commented version of the paper was published in *Journal of Economic Theory* (von Neumann 1935–1936, 1937, 1945). The paper triggered an interest in the problem of economic growth, resulting in a flow of books and papers on the subject (most notably by Harrod (1939), Domar (1957), and Leontief (1951)).

These analysts showed that the rate of growth of a macro economy would be determined as the product of the net savings-output-ratio and the productivity of capital, under the provision of unlimited supply of labour (possibly measured as efficiency units of labour).

In two papers by Solow (1956) and Swan (1956), the assumption of unlimited growth of the supply of labour was questioned. Reformulating the theory in terms of product and capital per unit of labour and assuming constant returns to scale they concluded that the macro economy would eventually converge towards a constant capital–labour ratio, at which there would be no growth in consumption per unit of labour. The only expansion of standard of living, as reflected in income and consumption per capita, would be by an increase in the total factor productivity, defined as A in the production function $Af(K,L)$. Assuming A to shift upwards over time is then interpreted as exogenous changes of technology (or improved economic organization).

However, by the mid-1960s Uzawa (1965) and Shell (1966) suggested that the changes of A were actually endogenous and determined by investments in research and technology.

2 Knowledge Accumulation and Growth: The Accounting Approach

In the early 1960s Denison (1967) initiated an extensive discussion about the relative role of different factors of production in determining the rate of growth at different national economies. One of his starting points was the observation by Robert Solow (1956) that the long-term rate of growth of per capita income in the USA could only to a very limited extent be explained by the growth of the stock of material capital and the increases of quantitative labour supply. In an econometric estimation of the rate of growth of the national product per capita, Solow estimated the contributions by growing capital stock and labour inputs to account for no more than one third of the total rate of growth. The remaining two thirds of the rate of growth of the USA had to be some 'residual factor', interpreted as technological change. A number of economists beside Dennison proceeded to explain the different components of the residual factor of economic growth accounting, among these Kuznets (1966) should be mentioned, especially because of his development of an outstanding data base for a large number of countries.

In important contributions by Angus Maddison (1982, 1995) the macroeconomic accounting data assembled by Kuznets was extended to cover a very long historical record for a large number of industrialized nations. These data bases have increased our possibilities to understand the relative importance of the growth of different inputs contributing to the rate of growth of real national products.

If we assume that the national (or regional) product Y could be explained by the use of different inputs according to a linearly homogenous Cobb–Douglas macro production function (corresponding to the idea that the net output of the economy would be determined by a weighted geometric average of different inputs) we have the following growth accounting equation:

$$\frac{\Delta Y}{Y} = \alpha_1 \frac{\Delta R}{R} + \alpha_2 \frac{\Delta K}{K} + \alpha_3 \frac{\Delta H}{H} + \alpha_4 \frac{\Delta L}{L} + \alpha_0 \tag{4.1}$$

where

$\frac{\Delta Y}{Y}$ = rate of growth of GDP,

$\frac{\Delta R}{R}$ = rate of growth of knowledge, in which ΔR = investments in industrial research and development (R&D) and R = the stock of technological knowledge (equal to the accumulated investments in industrial R&D),

$\frac{\Delta K}{K}$ = rate of growth of the material capital stock,

$\frac{\Delta H}{H}$ = rate of growth of human capital,

$\frac{\Delta L}{L}$ = rate of growth of the number of working hours.

The parameters α could be estimated by the factor shares in GDP, if $\alpha_i > 0$ and $\sum \alpha_i = 1$, corresponding to an assumption of constant returns to scale. If material investments are constrained by savings, Eq. 4.1 could be rewritten as Eq. 4.2. Thus

$$\Delta K = S = (s_H + s_M + n + t - g)Y = s_T Y \tag{4.2}$$

where

S = savings,

s_H = household savings ratio,

s_M = firm savings ratio for material investment,

n = net import surplus ratio,

t–g = net government taxation surplus ratio,

s_T = total net savings ratio.

Further $\alpha_2 = \frac{\partial Y}{\partial K} \cdot \frac{K}{Y}$. Thus

$$\alpha_2 \frac{\Delta K}{K} = [s_H + s_M + n + (t - g)] Y \frac{\partial Y}{\partial K} \frac{K}{Y} \Big/ K. = s_T \frac{\partial Y}{\partial K} \tag{4.3}$$

where
$\frac{\partial Y}{\partial K}$ = marginal productivity of material capital,
ΔR = research and development investments
= $s_R \cdot Y$

$$\alpha_1 \frac{\Delta R}{R} = s_R \frac{Y}{R} \alpha_1 = s_R \frac{Y}{R} \frac{\partial Y}{\partial R} \frac{R}{Y} = s_R \frac{\partial Y}{\partial R} \tag{4.4}$$

where $\partial Y/\partial R$ = the marginal productivity of research and development investments.

The accumulation of human capital is primarily determined by government spending and foregone earnings.

The growth of labour supply, measured in working hours, is close to zero in many advanced economies. Equation 4.1 can then be rewritten as Eq. 4.5.

The most important explanatory variables accounted for in Maddison are capital investments, knowledge accumulated by an increased number of school years and the role of international trade in the supply of material capital investments. In addition to the national supply of savings from household income and profit retention ratios of firms, the economy can also support a high rate of capital accumulation by an import surplus (equalling international credits). This is reflected by Eq. 4.5.

$$\frac{\Delta Y}{Y} = s_R \frac{\partial Y}{\partial R} + s_T \frac{\partial Y}{\partial K} + \alpha_3 \frac{\Delta H}{H} + \alpha_4 \frac{\Delta L}{L} + \alpha_5 \tag{4.5}$$

Assume s_R to be in the range of 3%, $\partial Y/\partial R$ to be in the order of 0.25, s_T to be in the order of 0.20, $\partial Y/\partial K$ equal to 0.10, K_3 equal to 0.3, $\Delta H/H$ equal to 0.015 and α_S and $\Delta L/L$ to be zero. Then the rate of growth of the economy would be in the order of 3.0% per year, which would also be the rate of growth of income per working hour.

As the rate of national growth of labour supply is often close to zero in mature economies the main factors determining growth are:

- Marginal productivities of material and knowledge capital,
- Propensity to save of households, firms and government,
- Rate of growth of industrial knowledge capital, and
- Rate of growth of human capital by investments in education.

The marginal productivities of material and knowledge capital are to a large extent determined by the spatial allocation. It is well known that the public nature of ideas and other knowledge capital tends to be more clustered in space than material capital and other mainly private resources. It is one intention of this chapter to show that the dynamics of such clustering can be endogenously determined in the long term economic growth process.

Table 4.1 Annual hours of work per capita of the labour force in different OECD countries 1870–1979

	1870	1900	1929	1950	1979
Australia	2,945	2,688	2,139	1,838	1,619
Belgium	2,964	2,707	2,272	2,283	1,747
Canada	2,964	2,789	2,399	1,967	1,730
Denmark	2,945	2,688	2,279	2,283	1,721
France	2,945	2,688	2,297	1,989	1,727
Germany	2,941	2,684	2,284	2,316	1,719
Netherlands	2,964	2,707	2,260	2,208	1,679
Norway	2,945	2,688	2,283	2,101	1,559
Sweden	2,945	2,688	2,283	1,951	1,461
United Kingdom	2,984	2,725	2,286	1,958	1,617
United States	2,964	2,707	2,342	1,867	1,607

Sources: different sources as given in Maddison (1982)

The growth of human capital by education has been substantial in all of the industrialized economies. According to Maddison (1982) the average level of education of the labour force of most OECD countries was close to 3 years of formal education around 1900. Currently the average level of formal education of the labour forces of Europe, USA and Japan is close to 12 years. This corresponds to an average increase of the supply of educational capital per capita of 1.5% per year since 1900.

The contribution to the growth of the national product from material capital is determined by the product of the savings rate (as influenced by national savings and import surpluses) and the marginal productivity of investments. The marginal productivity of investments can essentially be influenced by two factors only. The first factor is the capacity to reallocate capital from inefficient to efficient firms or regions, and the second factor is the capacity to develop and use new technologies of production. The development of new technologies of production is closely related to the accumulation of knowledge by research and development investments.

The data on the long-term growth of the supply of labour as provided by Maddison show that the total number of working hours per unit of labour is steadily decreasing with the rate of growth of income per capita. This indicates that leisure time is a complement to consumption of goods. Table 4.1 summarizes the development of the per capita working hours.

Estimating the dependence on working hours upon real income indicates an income elasticity of working hours per capita of approximately −0.2 to −0.3 for the different OECD countries.

The effect of increasing number of persons employed and the decline of working hours per employed has implied a fairly constant supply of labour in most developed market economies. This implies that growth is primarily dependent upon *three forms of capital accumulation*:

1. In material capital,
2. In human capital by education investments and
3. In other knowledge capital by research and development investments,
4. And the productivity of such investments

3 The Productivity and Returns of Educational Knowledge Capital

Education is primarily a private capital and measurement of the returns to investment in education ranges from 5 % to 10 % for advanced economies, when measured with the Mincer–Becker estimation equation. This type of measurement separates the effect of years of education and years of work experience into account, but abstains from analysing the effects of occupational and regional mobility. Such mobility is in fact decided upon as part of the decision to take on some education.

A large number of studies of the empirical relation between education and income have been performed, all supporting a strong positive relation between personal income and the level of education (ceteris paribus). Relating the gender, age and years of education with income gives the following estimate for Sweden, 1990. The estimation is based on census data after grouping. The result of the regression is given by the following table.

There is no self-evident way of deciding on a best functional form and therefore a number of different forms have been tested, including the log-linear form proposed by Becker and Mincer. The advantage of the equation estimated as in Table 4.2 is the non-linearity of returns to years of education. In this equation the rate of return declines from around 10 % at the level of junior high school towards approximately 5 % return on post-graduate university education. These levels of rates of return are quite low, especially when compared to the USA, where returns to higher education tend to be as high as 10 %. Some reasons for this discrepancy are presumably the difference in the size of the labour market, the much larger mobility of the Americans and the government subsidies of higher education in Sweden.

Much of the advantages of education can only be realized after a proper relocation of the household. In Sweden and most other European countries young people tend to either relocate to a region with long-run growth potential before they embark upon a higher education or immediately after graduation. After the formation of a family most tend to stay in the region chosen earlier.

The human (or educational) capital thus tends to be clustered in certain regions as is illustrated by the following map of Sweden (Fig. 4.1).

The university graduates tend to finally locate in the southern metropolitan commuting regions of Stockholm, Göteborg and Malmö or close to university towns elsewhere. Concentration of human capital tends to reinforce and be reinforced by the concentration in space of scientific and industrial knowledge capital.

Table 4.2 The econometric cross-section relation between the logarithm of personal income, age, and years of education in Sweden 1990 with gender as a dummy variable

Variable	Coefficient	Standard error of estimate	t-value
Intercept	2.31		
Gender (female = 1, male = 0)	−0.25	0.026	9.6
ln of age (experience)	0.32	0.05	6.4
ln of years of education	0.73	0.05	14.7

Source: Swedish central bureau of statistics, census 1990

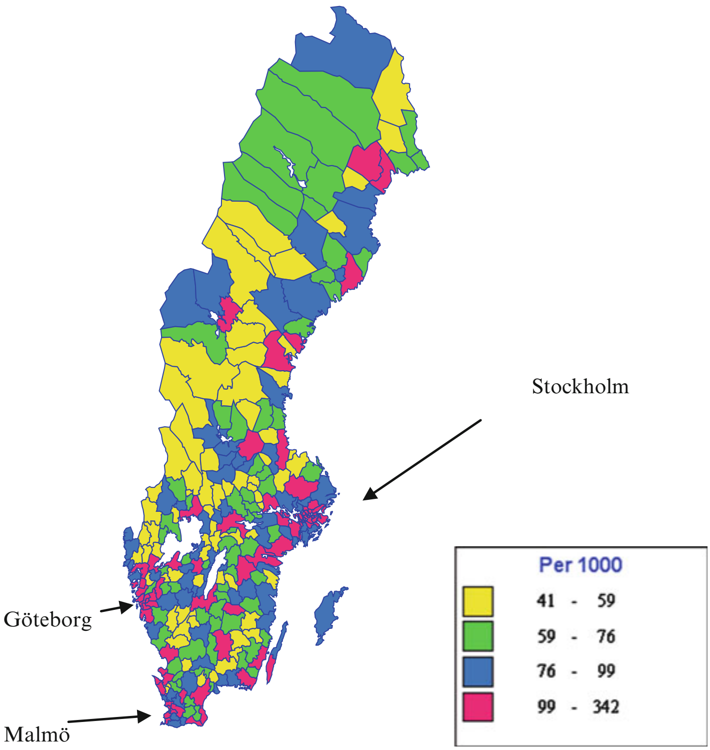

Fig. 4.1 Location of university graduates per 1,000 of inhabitants of regions. Locality quartiles

4 Knowledge as Public Goods and Spatial Discounting of Knowledge Value

Knowledge in the form of research findings is different from educational capital. Education is clearly private in terms of effects, while research findings are at least potentially public. Early measurements by Mansfield clearly show that the industry wide returns to investment in industrial R&D can be as large as three times the level of private returns. From such micro-economic studies of knowledge as a public good some macroeconomists (e.g. Romer (1986, 1990)) have concluded that total research capital would exhibit increasing returns to scale at the level of national macroeconomies.

However, econometric studies have shown that such claims are unwarranted (see e.g. Braconier 1998). The spillovers are within segments of the economy, where the segments could be product groups, occupations or regions.

Affinity of or distance between scientists is thus of importance in determining spillovers.

Knowledge available at distances is less valuable as an input than knowledge within certain borders (of the firm, occupation or region). The problem of assessing the value of public knowledge available elsewhere is analogous to assessment of returns accruing at distant instances of time. To resolve the issue of valuing over time economists use discounting of future returns. A similar procedure can be used in spatial discounting. The idea is then to estimate the value of knowledge available in different locations in the following way:

$$V_i = \sum_j \exp\left(-\beta d_{ij}\right) R_j \tag{4.6}$$

where

V = knowledge value

d = distance

R = stock of knowledge

i,j = locations

β = constant rate of spatial discounting

The advantage of spatial exponential discounting is the unit upper bound, which indicates that the full value of knowledge can only occur in the 'home' location, while the value of knowledge as an input approaches zero at infinite distance (in geographical, occupational or industrial space). The value equation is conventionally called *accessibility of knowledge*.

5 The Haavelmo Paradox and the Fast and Slow Processes of Economic Development

In a conversation with the Nobel laureate T. Haavelmo, I asked him how he reacted to the new mathematics of complex dynamic systems. According to the theorems of complex dynamic systems the mathematically most probable outcome is chaos or total un-predictability. He then reacted in a way that I would call the Haavelmo Paradox.

> **The Haavelmo Paradox**: It is true that in non-linear economic dynamics outcomes are generically unpredictable if models are general. Realistic models obviously tend to be non-linear. However, national statistical yearbooks report similar relative economic data, year by year indicating surprisingly persistent, stable equilibrium growth.

Evidently, some mechanisms are there to generate conditions leading to the mathematically improbable stable equilibrium results (at least in the short and medium run).

Synergetics has been proposed by Haken (1978, 1982) which could provide a method to resolve the Haavelmo paradox.

The synergetics modelling strategy implies primarily a subdivision of variables, according to relevant time scales.

It has e.g. been successfully applied to experiment oriented modelling of:

- Lasers
- Cognition and pattern formation.
- Biochemical processes.
- Neural nets.
- Physiological phenomena.

Applying the synergetic approach to modelling of non-linear interdependencies in economic growth and development requires:

1. Careful separation of time scales.
2. Careful separation of variables according to their individual (or private) versus collective (or public) effects.

The following table shows such a subdivision of the different goods for a synergetic analysis of the dynamic economic system (Fig. 4.2).

Each group of goods should, according to the principles of synergetics, be modelled so as to represent the differences in time scales and degree of collectivity of impacts.

Individual goods would then be represented by the following equation:

$$dp/dt = \mathrm{f}(p, k, y, A) \tag{4.7}$$

where p is a vector of prices of ordinary market goods (including factor services), k = a vector of capital or investment goods, y = information, and A = infrastructure as represented by accessibility.

Investment or capital accumulation would be represented by the following equation:

$$s(k)dk/dt = \mathrm{g}(p, k, y, A) \tag{4.8}$$

where $s(k)$ = a constant representing the time scale conversion between ordinary market goods and capital goods, i.e. $s(k) = t/T(k)$. If t is equal to 1 year and $T(k)$ is equal to 10 years we would have $s(k) = 0.1$.

Information is modelled as:

$$s(y)dy/dt = \mathrm{h}(p, k, y, A) \tag{4.9}$$

where $s(y)$ is larger than or equal to 1, signifying a rapid time scale.

EFFECTS\Rate of change	Fast	Slow
INDIVIDUAL (PRIVATE)	*ORDINARY MARKET GOODS*	*INVESTMENT GOODS*
COLLECTIVE (PUBLIC)	*INFORMATION*	*INFRASTRUCTURE*

Fig. 4.2 Different goods, classified by rate of change and scope of effects

Finally, the development of infrastructure (as represented by accessibility to fundamental knowledge) can be represented by the equation:

$$s(A)dA/dt = \mathrm{m}(p, k, y, A) \tag{4.10}$$

where $s(A)=t/T(A)$. $T(A)$ is a very slow (although positive) time scale, indicating that $s(A)$ is a very small, positive number, possibly in the order of 0.01 or lower. This implies that in the time frame of the other variables of this system dA/dt can be set approximately equal to zero, most of the time.

We thus have a dynamic system:

$$\begin{aligned} dp/dt &= \mathrm{f}(p, k, y, A^*) \\ s(y)dy/dt &= \mathrm{h}(p, k, y, A^*) \\ s(k)dk/dt &= \mathrm{g}(p, k, y, A^*) \end{aligned} \tag{4.11}$$

to be solved subject to the constraint:

$$\mathrm{m}(p, k, y, A^*) = 0 \tag{4.12}$$

For systems of this kind we can apply Tikhonov's theorem (Sugakov 1998):

Assume a dynamic system of N ordinary differential equations, which can be divided into two groups of equations. The first group consists of m fast equations, the second group consists of $m+1, \ldots, N$ slow equations. Tikhonov's theorem states that the system

$$dx_i/dt = \mathrm{f}_i(x, g); \quad i = 1, \ldots, m \text{ (fast equations)}$$

$$\mathrm{f}_j(x, g) = 0; \quad j = m+1, \ldots, N \text{ (slow equations)}$$

has a solution if the following conditions are satisfied:

1. The values x_j are isolated roots,
2. The solutions x_j constitute a stable stationary point of the system of $f_j = 0$ for any x_i and the initial conditions are in the domain of attraction of this point.

For each position of the slow subsystem the fast subsystem has plenty of time to stabilize. Such an approximation is called adiabatic (Sugakov 1998).

In the very short run, a market equilibrium could be established as the fixed point solution of the first two Eqs. (4.7) and (4.8), i.e. $f^*(p,y) = 0$ and $h^*(p,y) = 0$, keeping the approximate values of $dk/dt = 0$ and $dA/dt = 0$. This solution corresponds to a conventional 'general equilibrium' of the Walras type.

In the medium term we would have an expansion of capital, implying that $s(k)dk/dt = h^{**}(k)$, where the double star indicates that A is approximately constant, and x and y are kept at their equilibrium values (mutatis mutandis).

The solution is thus a fixed point solution of $dk/dt/k = g$, where g is the balanced rate of growth, which is ensured to exist as long as $h^{**}(k)$ fulfils the requirements of the Nikaido theorem, given below. This solution could either correspond to Solowian or von Neumann steady state solutions.

In the very long run dA/dt cannot be assumed to be zero and the system as a whole would then cease to be as well behaved as in the short and medium terms of dynamics. The system would in the very long term have all the bifurcation properties, typical of non-linear, interactive dynamic systems. Between periods of change of the economic structure, there would be periods of stable growth equilibrium.

We can thus conclude that the Haavelmo paradox can be resolved if we admit the possibility of separation of time scales and degree of collectivity (publicness) of different economically important variables. This implies that general equilibrium theory as conventionally formulated by Arrow, Debreu and others are not general enough to be expandable into dynamic systems (or combined spatial and dynamic systems).

The representation of the economy along these lines is not unknown in economics although the mathematics needed for dynamic modelling has not been available to these theoretical economists.

Important examples are:

- Classical population–economy interaction theory (Wicksell 1901; Hotelling in Puu 1997).
- Theories of interactions between transport infrastructure and economic development (von Thünen 1875; Pirenne 1939; Braudel 1982).
- Theories of interactions between institutions and economic development (Adam Smith 1776, 2001; Eli F. Heckscher 1955; Douglass North 1991).
- Theories of cultural and economic interactive development (Weber 1930; Morishima 1982).
- Schumpeterian theories of interaction between knowledge, entrepreneurial activity, political processes and economic development (Schumpeter 1912; Zhang 1991).

- Recent modelling of interactions between networks, knowledge and economic development (Andersson and Beckmann 2009).
- Emerging theories of dynamic ecology–economy interaction (Rosser 2008).

6 Modeling Accessibility to Knowledge Resources and Economic Growth

I now assume that scientific knowledge is measured in accumulated units of research output. For simplicity, we will assume that in each point of space there will be an aggregate of such knowledge capital, called R_i. We will further assume that the productivity of the knowledge capital, available elsewhere is declining monotonously with an increasing distance, from a user of knowledge j to a holder of knowledge. The maximal productivity is reached if $i=j$. As argued above, a reasonable and yet simple candidate of an accessibility function of i with respect to j is the spatial discounting function:

$$A_{ia} = \sum e^{-d_{ij}} R_j \tag{4.13}$$

where A = accessibility and R = scientific knowledge capital. The production function of each node i of the network can be formulated as:

$$Q_i = \mathrm{F}(K_i, A_i, M_i) \tag{4.14}$$

where

Q_i = production in node i

A_i = accessibility of scientific knowledge capital

M_i = economically useful area of node i

K_i = private capital, available in node i

Accumulation of knowledge capital of node i is determined by the equation:

$$s(R)dR_i/dt = H(Q_i, A_i, M_i) \tag{4.15}$$

$$s(A)dA_i/dt = L(Q, A), \tag{4.16}$$

where Q and A are vectors giving the production and accessibility of every node. s(R) and $s(A)$ are both assumed to be positive, very small numbers representing the speed of adjustment.

In order to illustrate some possible properties of such a differential equation assume that the production function can be decomposed into three factors: (1) local impact of capital, (2) spatial congestion, and (3) accessibility of knowledge.

$$\dot{K}_i = s_i K_i^{\alpha} \left(\frac{\bar{M}_i}{K_i} \right)^{\beta} \left[\sum_{i \mp j} e^{-\gamma d_{ij}} R_j \right]^{\lambda} \quad (4.17)$$

$$\dot{K}_i = s_i K_i^{\alpha-\beta} \bar{M}_i^{\beta} \left[\sum_{i \mp j} e^{-\gamma d_{ij}} R_j \right]^{\lambda}$$

Because of the slow speed of change of scientific knowledge and transport capacities Eqs. 4.9 and 4.10 are approximately equal to zero. Thus, the equation system (4.11) will be solved for an equilibrium rate of growth, subject to Eqs. 4.9 and 4.10.

The following theorem (Nikaido 1968) ensures the existence of a growth equilibrium:

$$\dot{x}_i = M(x) = \lambda x_i; \ \mathrm{i} = 1, \ldots, n \quad (4.18)$$

where $M(x)$ is a semi-positive mapping from x to $\dot{x}$. For such a system a theorem by Nikaido (1968) can be applied.

Assumptions.

(a) $M(x) = [M_i(x)]$ is defined for all $x \geq 0$.
(b) $M(x)$ is continuous as a mapping M: $R_+^n \rightarrow R_+^n$, except possibly at $x = 0$.
(c) $M(x)$ is positively homogenous of order m, $0 \leq m \leq 1$ in the sense that $M(x) \geq 0$ and $x \geq 0$.

Theorem. Let $\Lambda = \{M(x) = \lambda x\}$ *for* $x \in p_n$

where

$$p_n = \left\{ x \middle| x \geq 0, \sum_{i=1}^{n} x_i = 1 \right\}$$

is the standard simplex. Then $M(x)$ contains a maximum characteristic value which is denoted $\lambda(M)$. Furthermore, if $M(x)$ is homogenous of degree 1, i.e. if $m = 1$ as a special case of assumption (c) then $\lambda(M)$ is the largest of all the eigen-values of $M(x)$.

Proof. Nikaido (1968).

In the vicinity of an equilibrium $\lambda(M)$ can be locally linearized as $M[x(t)] = M(x^*)$, where x^* is x on the equilibrium trajectory. We then have the eigen-equation

$$\gamma z = M(x^*) z \quad (4.19)$$

with the equilibrium growth rate γ^* $[M(x^*)]$.

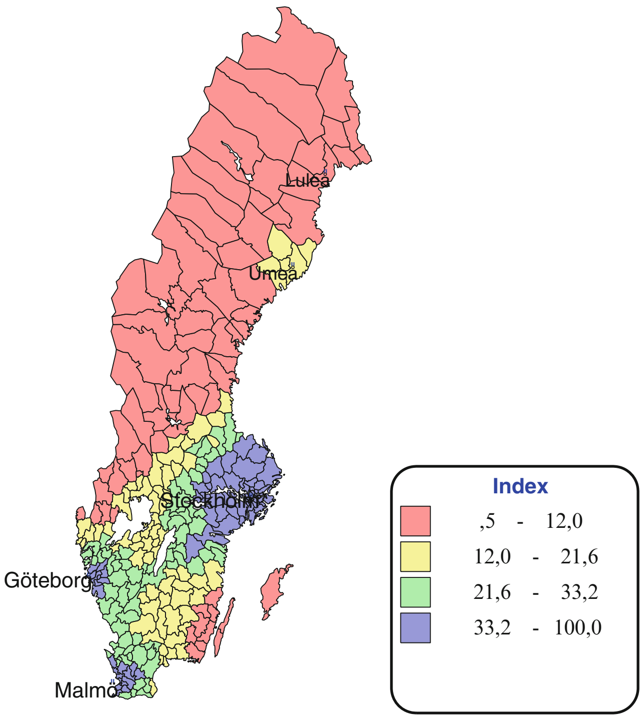

Fig. 4.3 Accessibility of university based research in Sweden 2001, local quartiles

7 Accessibility of Knowledge and Regional Economic Growth: An Empirical Example

The accessibility of fundamental knowledge varies systematically in all countries with peaks in the vicinity of university high-tech-industry locations. A typical empirical example is the knowledge accessibility landscape of Sweden as pictured in the following maps.

As can be seen from the map in Fig. 4.3, Sweden has a spatially very uneven distribution of accessibility to university based research activities and thus presumably also to scientific knowledge inputs.

The map in Fig. 4.4 illustrates the spatial distribution of accessibility of industrial research and development. There is a remarkable similarity between the scientific and industrial research maps.

The most accessible regions will in the long run be regions of relative growth of material and human capital as well as real income.

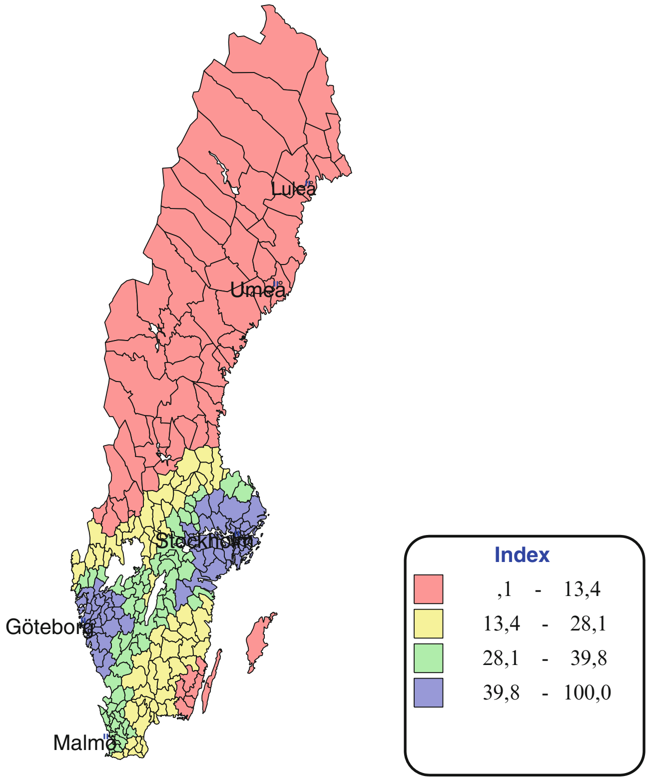

Fig. 4.4 Industrial R&D accessibility in Sweden 2001, local quartiles

There are substantial differences in growth rates of Swedish localities, as given by the Sharpe-ratios (defined as the average growth rates divided by the standard deviations of the growth rates).

The map in Fig. 4.5 clearly shows that the regional movements of people, which essentially mirror the movements of human capital, are towards the areas of superior scientific and technological accessibility of knowledge as indicated by the maps in Figs. 4.3, 4.4, and 4.5.

In post-industrial, knowledge oriented economies like the Swedish, we should expect a long run, almost complete, exodus from the regions of low accessibility to knowledge and a corresponding growth of population, human capital and income per capita in regions of high knowledge accessibility.

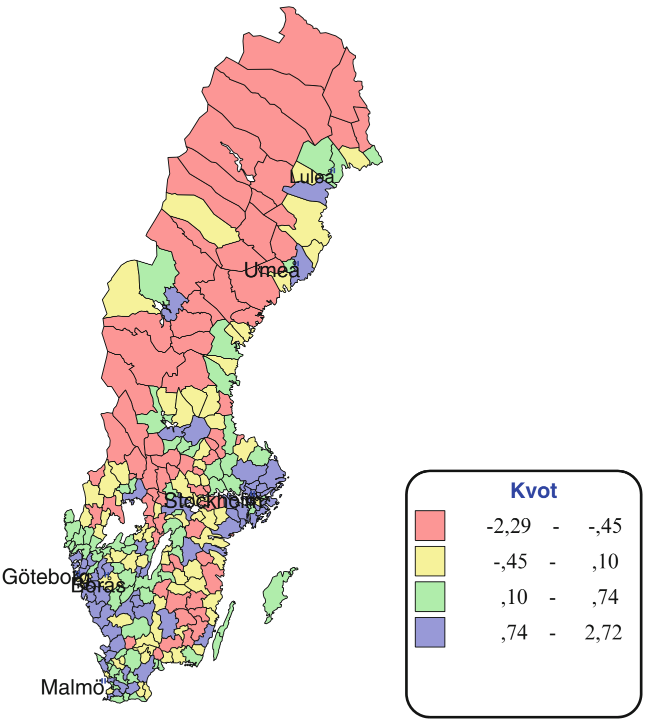

Fig. 4.5 Sharpe-ratios of persistent regional population growth patterns in Sweden, locality quartiles

8 Conclusions

The theory of economic growth was originally developed for a non-spatial macroeconomy. This was a reasonable approach as long as linearity (including log-linearity) could be assumed. However, as soon as public goods are introduced, non-linearity is a necessary aspect of the dynamic economy. Knowledge is the most important public good of a growing economic system. With the growing importance of scientific and industrial research and technological development it has become necessary to formulate the dynamic economic theory with an explicit inclusion of knowledge as an endogenous variable. This was done already in the 1960s by Uzawa and Shell, assuming constant returns and employing optimal control theory to achieve closed form solutions to their problem formulation.

In the 1970s and 1980s some important steps were taken to include publicness of knowledge into the growth models. Unfortunately, these approaches implied increasing returns and other non-linear features of the models proposed. Examples are the growth models by Paul Romer, featuring increasing returns at the level of the macro-economy. It can easily be shown (see Andersson and Beckmann 2009) that such a model diverges rapidly. Thus, such economies are not feasible.

Empirical studies have shown that the diffusion of knowledge tends to be constrained by space, industrial and occupational affinities. Macro models featuring whole economies are thus not suitable representations of endogenous growth processes. Accessibility of knowledge, based on spatial discounting is a reasonable way to represent the variable degree of publicness of knowledge. However, knowledge accessibility implies non-linearity of the growth models with severe consequences for the predictability of the dynamic trajectories of the model variables.

Subdivision of the different variables can provide a means to achieve predictability of the non-linear growth models. Such a synergetic procedure is chosen in this study. In the model proposed above, accessibility and knowledge are assumed to be on a slow time scale, allowing for an adiabatic approximation. This implies that in the faster time scales of ordinary, private goods (including private material capital) the solutions can be determined, subject to approximate constancy of accessibility of knowledge.

The Swedish regional economy fits such a theoretical approach. The accessibility of knowledge is relatively concentrated and stable. The pattern of growth of the regional economies is also quite stable and follows the pattern of accessibility of knowledge from the 1980s when the transformation into the postindustrial knowledge economy started.

References

Andersson ÅE, Beckmann MB (2009) Economics of knowledge – theory, models and measurements. Edward Elgar, Cheltenham/Northampton

Braconier H (1998) Essays on R&D and growth, vol 72, Lund economic studies. Department of Economics, Lund University, Lund

Braudel F (1982) The wheels of commerce civilization and capitalism 15th–18th century, vol II. Harper and Row, New York

Denison EF (1967) Why growth rates differ. The Brookings Institution, Washington

Domar ED (1957) Essays in the theory of economic growth. Oxford University Press, New York

Haken H (1978) Synergetics: an introduction. Nonequilibrium phase transitions and self-organization in physics chemistry and biology. Springer, Berlin

Haken H (1982) Synergetics: an introduction. Nonequilibrium phase transitions and self-organization in physics chemistry and biology. Springer, Berlin

Harrod RF (1939) An essay in dynamic theory. Econ J 49:14–33

Heckscher EF (1955) Mercantilism, 2 vols. Allen and Unwin, London

Kuznets S (1966) Modern economic growth: rate, structure and spread. Yale University Press, New Haven

Leontief WW (1951) Studies in the structure of the American economy 1919–1939. Oxford University Press, New York

Maddison A (1982) Phases of capitalist development. Oxford University Press, Oxford

Maddison A (1995) Monitoring the world economy: 1820–1992. Organization for Economic Cooperation and Development, Paris

Morishima M (1982) Why has Japan succeeded? Cambridge University Press, Cambridge

Nikaido H (1968) Convex structures and economic theory. Academic, New York

North D (1991) Institutions, institutional change and economic performance. Cambridge University Press, Cambridge

Pirenne H (1939) Medieval cities: their origins and the revival of trade. Princeton University Press, Princeton

Puu T (1997) Nonlinear economic dynamics, 4th edn. Springer, Berlin

Romer PM (1986a) Increasing returns and long-run growth. J Pol Econ 94:1002–1037

Romer PM (1990) Endogenous technological change. J Pol Econ 98:71–102

Rosser JB Jr (2008) Dynamic discontinuities in ecologic-economic systems. In: Allen CR, Holling CS (eds) Discontinuity theory in ecosystems and other complex systems. Columbia University Press, New York

Schumpeter JA (1912) Theorie der wirtschaftlichen Entwicklung. Duncker und Humblot, Berlin

Shell K (1966) Toward a theory of inventive activity and capital accumulation. Am Econ Rev 56(2):62–68

Smith A (1776, 2001) An inquiry into the nature and causes of the wealth of nations, Edinburgh (1776), edited and prefaced by Dr. Eamonn Butler. The Adam Smith Institute, London

Solow RM (1956) A contribution to the theory of economic growth. Quart J Econ 70:65–94

Sugakov VI (1998) Lectures in synergetics. World Scientific, Singapore

Swan W (1956) Economic growth and capital accumulation. Econ Record 32:334–361

Uzawa H (1965) Optimum technical change in an aggregative model of economic growth. Int Econ Rev 6:18–31

von Neumann (1935–1936, 1937, 1945) Uber ein ökonomisches Gleichungssystem und eine Verallgemeinerung des Brouwerschen Fixpunktsatzes. In: Ergebnisse eines Mathematischen Kolloquiums, 8, 1935–1936. Franz–Deuticke, pp. 73–83. (Translated: A model of general economic equilibrium. Rev Econ Stud 13:1–9, 1945)

von Thünen J-H (1875) Der isolierte Staat in Beziehung auf Landwirtschaft und Nationaloekonomie. Wiegandt, Hempel & Parey, Berlin

Weber M (1930) The protestant ethic and the spirit of capitalism. Allen and Unwin, London

Wicksell K (1901) Föreläsningar i nationalekonomi, första delen: Teoretisk nationalekonomi, Lund. Translated as lectures on political economy. vol 1. General theory. Routledge, London (1935). Reprint (1978), Kelley, New York

Zhang W-B (1991) Synergetic economics: time and change in non-linear economics. Springer Verlag, New York/Berlin