

Inventory Rationing Based on Different Shortage Cost: A Stable Inventory Demand Case

Qifan Chen and Jie Xu

Abstract This paper discusses an inventory rationing system with a threshold in a relatively stable inventory demand environment. There are two different demand classes in this inventory system, prior demand and secondary demand, distinguished by different shortage cost. The inventory system functions as follow. When the inventory level is above the threshold, demands come from both classes are filled. Once the inventory level falls below the threshold, all demands come from secondary demand class are backordered, only prior demand would be served. Heuristic approach was used to solve the model in this paper, and the optimal order quantity, single inventory cycle time and inventory threshold were given. Compare between inventory system with and without inventory threshold shows the cost advantage of the first system. Sensitivity analysis was performed to give a better understanding of two crucial parameter, inventory threshold and cost saving rate.

Keywords Inventory replenishment • Inventory rationing • Shortage cost

1 Introduction

As an important component of the cost of a company, inventory cost has long been the mainly focus of the management. Both business and academic world have come up with many different practice and ideas in inventory management field in decades. Among them, inventory rationing, which differentiated replenishment for different customers based on their own attributions, has brought in increasing attention. Differentiating replenishment refers to issuing the inventory to satisfy some customers, while rejecting or delaying other customers' needs.

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In the modern marketing, the practice of providing different standards of service to different customers according to their special needs is a common business strategy. Industries like airline industry, hotel industry and car rental industry have already provided differentiated service (for example, airline companies provides first class, business class and economic class seats), and they mainly depend on price that charged from different customers, as well as possible rationing levels such as booking limits to indentify different customer priorities [1]. It is worth mentioning that, from the perspective of inventory, services provided by above industries can be seen as timely inventory resources with limited supply, while no need to consider the inventory replenishment strategy. Thus, it is a special inventory management case.

As for single product, multiple demand classes inventory rationing research, Veinott [2] is known to be one of the first person to investigate in this field. Under a multi-period, non-stationary inventory environment, he got a solution of the replenishment strategy of the inventory system (when to replenish and order quantity). Based on Veinott's work, Topkis [3] further discussed how to allocate inventory among two different demand classes within one single inventory circle in a periodic review inventory system. Topkis' method of distinguishing demand classes by their different shortage cost is widely adopted in many research, Vinayak et al. [4] also use different shortage cost to describe demand classes that requested 90% fulfillment rate and 95% fulfillment rate in the research of U.S. military spare part logistics system. Other criteria to separate demand class such as Elliot [5], when he developed an inventory rationing strategy for a company that possesses both traditional physical distribution and internet distribution channel, he distinguished different classes by assuming the company would choose to satisfy the physical distribution channel first.

The assumption of inventory demand distribution is another key point in inventory management research. Assuming inventory demand subject to Poisson distribution, Nahimas and Demmy [6] discussed the fulfillment rate of two demand classes for given rationing and replenishment levels in a continuous review (Q, r) system. Other examples of Poisson demand assumption include Ha [7]'s research of a single product, make to stock inventory system, and Erhan and Mohit [8]'s research of a time-based service target, two-echelon service part distribution system. Anteneh et al. [9] assumed inventory demand to be triangle distributed, then discussed an internet retailer inventory system under such assumption. The sensitivity analysis they did also indicated best inventory threshold values and optimal profits in different cases.

The interest of this paper stemmed from a company's inventory system. This company holds inventory to provide materials for both the railway system (prior demand) and to the open market (secondary demand). The railway system is a major client for the company. As a result, fails to supply would cost larger shortage cost that includes penalty, loss of creditability and profit, even social and political effects. While the shortage of secondary inventory demand contents much less penalty and creditability loss, also loss of profit. Hence, inventory rationing by different demand class should be a good way for the company to cope with its inventory management problem.

2 Basic Model

2.1 Model Assumption

Following most research on inventory rationing, this paper also adopts shortage cost as the criteria to separate different demand classes. The shortage cost per time unit of prior and secondary demand classes are denoted by c_1 and c_2 respectively, therefore, the weighted shortage cost per time is denoted by $c = (R_1c_1 + R_2c_2)/(R_1 + R_2)$. The prior demand comes from railway system is relatively stable, so we assume this demand is linear distributed with demand rate R_1 per time unit. Compare to Poisson demand assumption, linear demand is more reasonable when meeting with more stable demand such as supplying for projects with well planned time schedule, or long term contract supply. The company has both long term major clients with stable demand and temporary clients that create demand much more like Poisson distributed in the open market. In order to simplify our discussion, we also assume secondary demand is linear distributed with demand rate R_2 per time unit. Therefore, total demand rate per time unit is $R = R_1 + R_2$. Other parameters are explained as follow:

s : Reorder point;

h : Inventory holding cost per time unit;

c_3 : Fixed setup cost;

Q : Order quantity;

L : Lead time;

K : Threshold, only prior demand would be served once inventory level falls below K , secondary demand would be backordered.

2.2 Inventory Model Without Rationing

Suppose the company doesn't differentiate the two demand classes, then the system issues stocks to both classes equally, despite the inventory level (Fig. 1). This inventory system performs as Fig. 2.

The inventory holding cost and backorder cost are:

$$H_1 = \frac{1}{2}h\frac{Q^2}{R} \quad (1)$$

$$B_1 = \frac{1}{2}cR\left(t - \frac{Q}{R}\right)^2 \quad (2)$$

The inventory cost per time unit of this inventory system is:

$$C_1 = \frac{H_1 + B_1 + c_3}{t} \quad (3)$$

Fig. 1 Inventory system without rationing

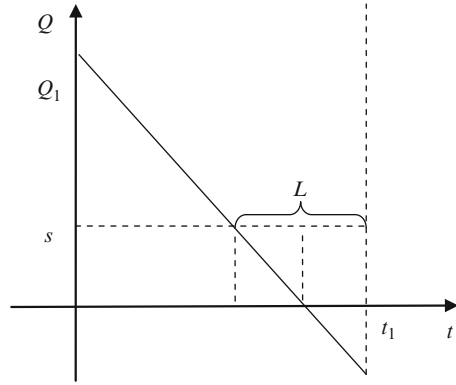
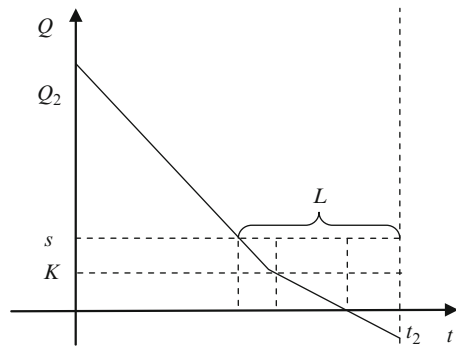


Fig. 2 Inventory system with rationing policy



The first order differential conditions to minimize C_1 with respect to Q and t are:

$$\frac{\partial C_1}{\partial Q} = \frac{1}{t} \left[h \frac{Q}{R} - c \left(t - \frac{Q}{R} \right) \right] = 0 \tag{4}$$

$$\frac{\partial C_1}{\partial t} = -\frac{1}{t^2} \left[h \frac{Q}{R} - c \left(t - \frac{Q}{R} \right) \right] + \frac{1}{t} cR \left(t - \frac{Q}{R} \right) = 0 \tag{5}$$

Because $t \neq 0$, we solve Eqs. (4) and (5), the optimal order quantity and single inventory circle time

$$t_1^* = \sqrt{\frac{hQ^2 + cQ^2 + 2c_3R}{cR^2}} \tag{6}$$

$$Q_1^* = \sqrt{\frac{2cc_3R}{h(h+c)}} \tag{7}$$

Note, by using lead time L , Eqs. (6) and (7), we can easily calculate two parameters that much more practice, reorder point and reorder time.

3 Rationing Model

Our rationing model separates two demand classes by their different shortage costs. The rationing policy functions like this: when the inventory level is above threshold K , stocks would be issued for both demand classes; once the inventory level drops below threshold K , stocks would be issued just for prior demand class, requests come from secondary demand class would not be responded until replenishment. The rationing inventory system works as Fig. 2.

The inventory holding cost and backorder cost of rationing model are:

$$H_2 = \frac{1}{2}h\frac{(Q-K)^2}{R} + \frac{1}{2}hK\left[\frac{2(Q-K)}{R} + \frac{K}{R_1}\right] \quad (8)$$

$$B_2 = \frac{1}{2}c_1R_1\left(t - \frac{Q-K}{R} - \frac{K}{R_1}\right)^2 + \frac{1}{2}c_2R_2\left(t - \frac{Q-K}{R}\right)^2 \quad (9)$$

The inventory cost per time unit of this rationing inventory system is:

$$C_2 = \frac{H_2 + B_2 + c_3}{t} \quad (10)$$

The first order differential conditions to minimize C_2 with respect to Q is:

$$\frac{\partial C_2}{\partial Q} = \frac{1}{Rt} \{ [h(Q-K) + hK] - [c_1R_1\left(t - \frac{Q-K}{R} - \frac{K}{R_1}\right) + c_2R_2\left(t - \frac{Q-K}{R}\right)] \} = 0 \quad (11)$$

Because $t \neq 0$, we can get an equation of t and Q :

$$t = \left[\frac{hR + c_1R_1 + c_2R_2}{R(c_1R_1 + c_2R_2)} \right] Q - \frac{c_1R_1K + c_2R_2K - c_1RK}{R(c_1R_1 + c_2R_2)} \quad (12)$$

The first order differential conditions for C_2 with respect to t is not very easy to get. Even if we got it, the accurate value for the three variables, Q , K , t is still unknown because there would be only to equations. Here, we use a heuristic approach to solve this problem.

Note that when the inventory level falls below threshold K , demand comes from secondary demand classes was backordered in order to be sure more demands come from prior demand class would be satisfied, this inventory system is expected to have a lower backorder cost. Thus, it is reasonable to conclude that $Q_2 \leq Q_1^*$, as the rationing system doesn't need to prepare as much inventory as Q_1^* to lower the risk of backorder because of threshold K .

Based on this conclusion, we set up an approach as follow:

Step 1: let $Q_2 = Q_2' = Q_1^*$, $K = 0$, we get $C_2 = C_1$ as the initial value;

Step 2: while $K' = 0, 1, 2 \dots Q_2'$, calculate t_2' and C_2' , if $C_2' < C_2$, then $C_2 = C_2'$, $Q_2 = Q_2'$, $K = K'$, $t_2 = t_2'$;

Step 3: let $Q_2' = Q_2' - 1$, if $Q_2' > 0$, repeat step 2, else, stop the approach.

Now, we let $c_1 = 50$, $c_2 = 10$, $R_1 = 30$, $R_2 = 20$, $h = 5$, $L = 1.5$ as a numerical example. By our approach, results for the basic model are minimum inventory cost $C_1 = 660.23$ (correct to two decimal), optimal order quantity $Q_1^* = 132$ (correct to the nearest integer), optimal inventory cycle time $t_1^* = 3.03$ (correct to two decimal), reorder point $s_1 = 6$ (correct to the nearest integer); results for the rationing model are $C_2 = 637.23$ (correct to two decimal), optimal order quantity $Q_2^* = 127$ (correct to the nearest integer), optimal inventory cycle time $t_2^* = 4.27$ (correct to two decimal), reorder point $s_2 = 2$ (correct to the nearest integer), inventory threshold $K = 23$.

4 Sensitivity Analysis

In order to further discuss the inventory model we built, we performed a sensitivity analysis for the key parameter, thresholds K , and a critical indicator, inventory cost saving rate r ($r = \frac{C_1 - C_2}{C_1} \times 100\%$). One of the major focuses of this paper is how to set up the inventory threshold K , thus, the sensitivity analysis for K are showed in Figs. 3, 4, 5 and 6 below (parameters do not show in the figures are the same as previous section).

From Fig. 3, inventory threshold K increase as the increase of prior demand class shortage cost per time unit c_1 , which is also very reasonable that if the shortage cost of prior demand increase, more inventory should be kept to ensure the supply. Figure 4 shows that, when fix setup cost c_3 increase, the threshold K increase correspondently. We believe the reason for K to increase is that setup cost extended the inventory period, therefore, more inventory should be kept for the prior demand class. In Fig. 5, when inventory holding cost h is relatively small, K increase as h increase. This is because the inventory threshold functions as that the additional inventory holding cost is less than the reduced shortage cost, when h increases, more shortage cost should be reduced in order to “offset” the increase of holding cost. However, when h is relatively big, K decrease as h increase, the reason is that optimal order quantity Q gets smaller with h increase, which then “compress” K . In Fig. 6, K increase as prior class demand rate per time unit R_1 increase, indicating a strong positive correlation between these two parameters.

Another focus in this section is inventory cost saving rate for the rationing model we set up, which is also a major motivation for companies to adopt new inventory management policy. Sensitivity analysis for r are showing as follows (parameters do not show in the figures are the same as previous section).

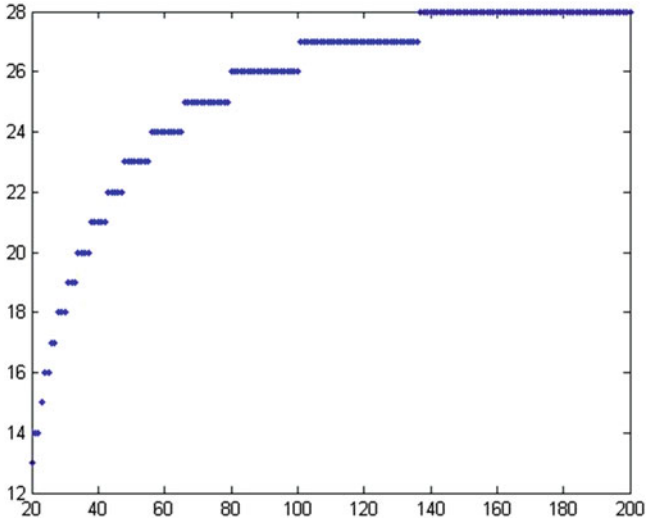


Fig. 3 C_1 increased from 20 to 200 with 1 per step

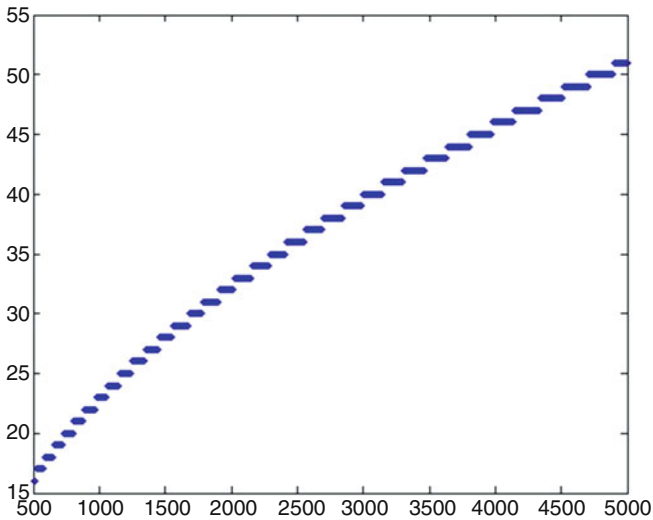


Fig. 4 C_3 increased from 500 to 5,000 with 10 per step

From Fig. 7, the inventory cost saving rate r increase as prior demand shortage cost increase, however, after $r > 6\%$, the increasing rate becomes very slowly. We further point out that as c_1 becomes infinitely great (which is similar to not shortage is allowed for prior demand), $r \approx 7\%$, indicating that the maximum inventory cost

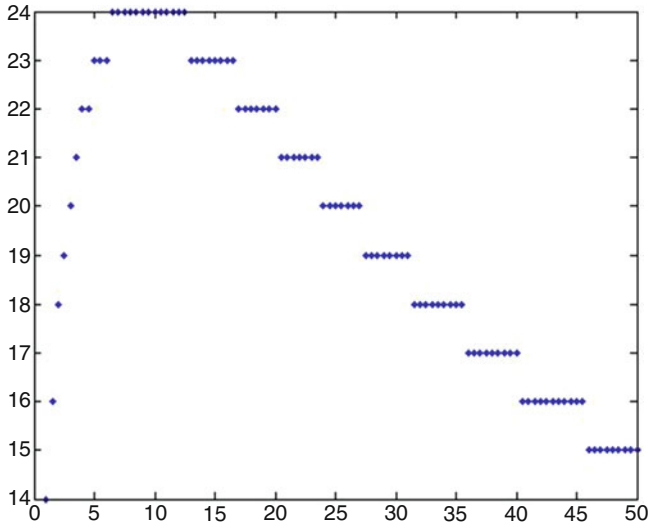


Fig. 5 h increased from 1 to 50 with 0.5 per step

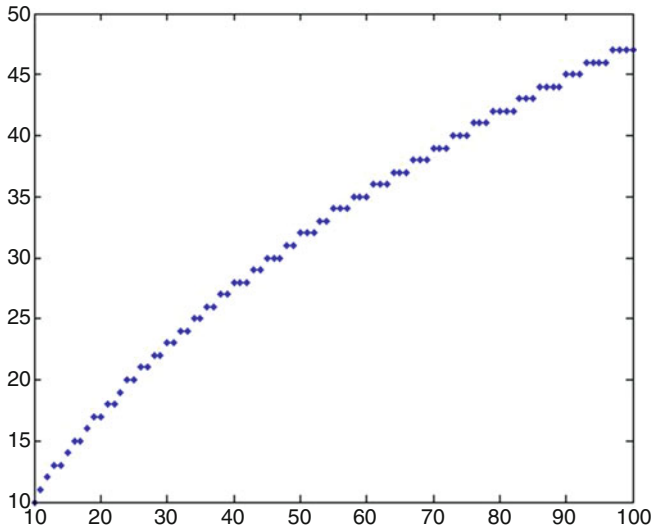


Fig. 6 R_1 increased from 10 to 100 with 1 per step

saving rate is 7%. The relation between inventory saving rate and inventory holding cost per time unit is showed in Fig. 8, the highest saving rate is obtained when inventory holding cost per time unit is 20.

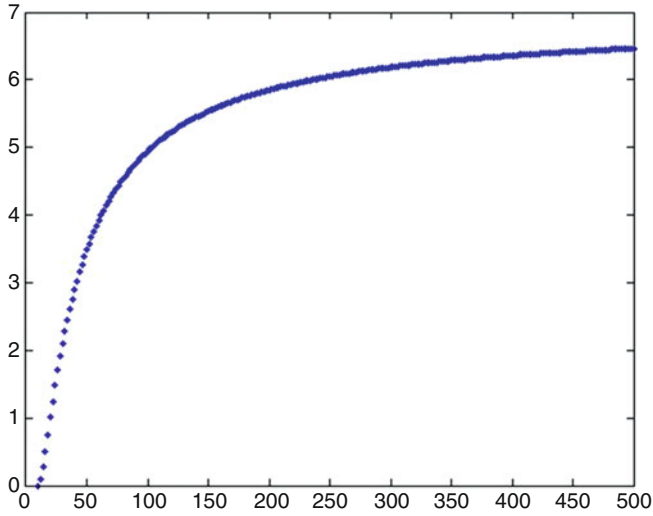


Fig. 7 C_1 increased from 10 to 500 with 2 per step

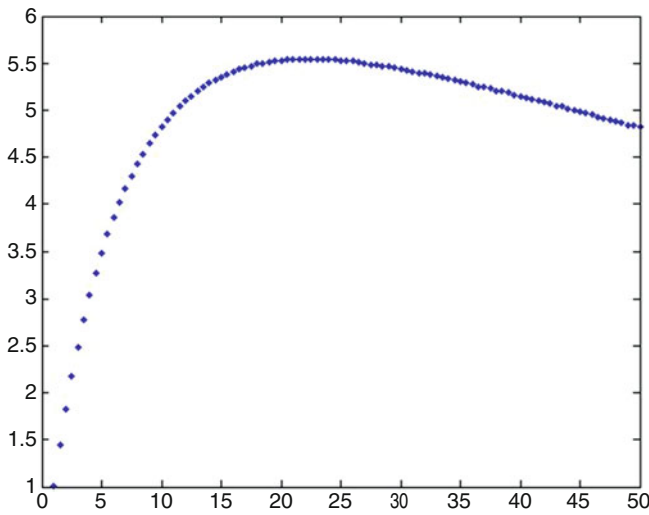


Fig. 8 h increased from 1 to 50 with 0.1 per step

5 Conclusions and Extensions

In this paper, we discussed an inventory rationing system with two demand classes in a relatively stable demand rate environment. The two demand classes were distinguished by different shortage cost. The rationing policy operates under the following rules. Demands come from both classes are filled when the inventory

level is above threshold K , once the inventory level drops below K , all remaining stocks would be reserved for prior demand class, secondary demand is backordered. Heuristic approach used in this paper gave the optimal order quantity, single inventory circle time and threshold. Results showed that compare to inventory system without rationing, our rationing system with threshold is fairly cost efficient. Besides, the approach we designed for the model is easy to be utilized and solved. Sensitivity analysis performed in this paper provided us a better understanding of our two major interests, inventory threshold K and inventory saving rate r .

One shortcoming of our model is that we assume the secondary demand rate to be fixed for simplification. In reality, this demand rate is more like a random number, which would be better described by random distribution such as Poisson distribution. Our target function could also be extended to calculate profit rather than just inventory cost, which is the price of stocks multiplies quantity, then minus total inventory cost.

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