Workspace Improvement of Two-Link Cable-Driven Mechanisms with Spring Cable

Amir Taghavi, Saeed Behzadipour, Navid Khalilinasab and Hassen Zohoor

Abstract The idea of multi-body cable-driven mechanisms is an extension of the original cable robots where the moving platform is replaced by a multi-body. Cables with variable lengths are attached between the fixed base and the links of the multi-body to provide the motion. There are possible applications for such mechanisms where complex motions as well as low moving inertia are required. One of the main challenges with such mechanisms is the high chance of interference between the cables or between the cables and the links of the multi-body mechanism. This can further reduce the usable workspace. In this article, the idea of adding passive cables in series with springs (spring cable) to improve the workspace is investigated. The spring cables can be added between the multi-body and ground or between the links. The idea is applied to a two-link planar multi-body cable-driven mechanism. The wrench feasible workspace (WFW) is found using the interval analysis. The WFW is shown to improve both in shape and volume.

Keywords Cable-driven · Multi-body · Spring · Workspace improvement

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T. Bruckmann and A. Pott (eds.), *Cable-Driven Parallel Robots*,
Mechanisms and Machine Science 12, DOI: 10.1007/978-3-642-31988-4_13,

1 Introduction

Cable driven robots are mechanisms in which the end-effector is moved by controlling the lengths of the cables connected to it. The cable robots are appealing because of their structural simplicity, high stiffness, and high exerted wrench-to-weight ratio and easiness of reconfiguration. Their main drawback is their small workspace and interference of cables, so one of their key issues is their optimal design for a desired workspace and given constraints [1].

A cable driven parallel manipulator, according to its number of cables (m) and the degrees of freedom of the end-effector (n), are classified as follows [2]:

IRPMs: Incompletely Restrained Positioning Mechanisms, in which the number of cables is less than or equal to the number of the DOFs, namely,

$$m \leq n$$

IRPMs robots rely on the presence of gravity or another ballast force to determine the resulting pose of the end-effector.

CRPMs: Completely Restrained Positioning Mechanisms, in which there is an extra cable, i.e.:

$$m = n + 1$$

RRPMs: Redundantly Restrained Positioning Mechanisms, in which there are more than one extra cable:

$$m > n + 1$$

Since IRPMs use less number of cables and actuators, the probability of cable interference as well as the production cost is lowered. However, in these robots, the volume of workspace and the magnitude of the externally applied wrench of the robot are limited by the ballast force. In contrast with IRPMs, RRPMs have larger workspace but the interference of cables and production cost become more challenging.

With such classification, a number of different definitions for the workspace of such robots are introduced and studied in the literature. One of the early works is for the one of the NIST ROBOCRANE [3], Which is a realization of a Gough–Stewart platform parallel manipulator while prismatic actuators are replaced by cables. Verhoeven and Hiller [4] used “controllable workspace” defined as “the set of poses in which the robot can maintain equilibrium against all external wrenches”. The statically reachable workspace is defined by Agrawal and coworkers [5] as the set of poses of the mobile platform for which the cables can balance the weight of both the platform and the payload with tension forces only. This is of particular interest for IRPMs, which rely on gravity to keep the cables taut. Dynamic workspace analysis has been introduced by Gosselin and Barrette [6] in which the motion of a moving platform is incorporated into a set of wrenches called a pseudopyramid. A more practical workspace definition is wrench feasible workspace (WFW) which is the set

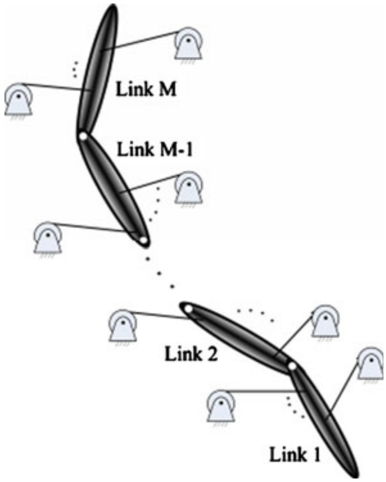

Fig. 1 Schematic of multi-body cable-driven robot

of all poses in which a specified range of external wrenches can be generated using a limited range of cable tensions [7, 8]. Wrench Closure Workspace (WCW) is a special case of WFW when both cable tension and the wrench sets are unbounded [9, 10]. The force closure workspace is the very special case of a WFW whose required set of wrenches is the whole space of wrenches and the only constraint on the cable tensions is nonnegativity [11]. Another definition that is in the literature is tensionable workspace. A pose of cable-driven mechanism belongs to tensionable workspace when it can generate any arbitrary external force/moment while maintaining tensile forces in all cables [12]. One can see that WCW, tensionable workspace, controllable workspace and force closure workspace are equivalent. They are merely dependent on the kinematics of the manipulator rather than the external loading, static or dynamic equilibrium or cable properties.

In another classification of cable robots, they are classified as single-body and multi-body cable-driven robots. In single-body cable-driven robots, all cables are attached to a rigid end-effector while in multi-body cable-driven robots; cables are attached to different links of a multi-body. An example is shown in Fig. 1, where a typical serial multi-body is driven by cables.

A possible application for multi-body cable robots is a reconfigurable robotic cell to be used for physical rehabilitation purposes. Using this concept, the human limbs are considered as multi-body systems which will be driven by cables attached to them using proper brace and shells. The cell can be easily reconfigured by changing the cable locations to provide the desired motion for the intended body part.

Determination of the workspace of cable-driven multi-body systems, due to the existence of inter-link constraints, is a problem of higher complexity. As a result, the literature on this subject is yet to be developed. Yang and coworkers proposed a kinematic design of a 7-DOF cable-driven humanoid arm with 14 cables [13]. They used force-closure method in multi-finger grasping to investigate the workspace.

Recently, two systematic approaches have been reported to determine the WCW of multi-body cable-driven mechanisms. One of them is based on Lagrange's approach in equilibrium analysis of multibody system [14] and the other one uses reciprocal screw theory [15]. They used the notion of generalized forces and Lagrange's method to eliminate the constraint forces/moments from the equilibrium equations.

In multi-body cable driven mechanisms the higher probability of interference between the cables or between the cables and the links of the multi-body and therefore smaller usable workspace is a major challenge. For example, in using cable robots in physical rehabilitation, the interference of the cables with each other or the patient's body significantly reduces the usable workspace of the robot. As a result, solutions need to be developed to improve the quality and size of their workspace before these mechanisms can find real applications.

One possible solution which is investigated in this paper is adding springs in between the links. Intuitively, it is expected that such springs help in keeping cables taut resulting in larger workspace. Also they are not expected to cause much interference with the cables as they stay close to the links of the multi-body.

In this paper the conceptual solution of adding springs to improve the WFW of a two-link cable-driven mechanism is investigated. In the following sections, a mathematical framework is developed to incorporate the springs and formulate the equilibrium of the mechanism for any number of links, cables, and spring cable. The determination of the WFW is then performed using interval analysis.

2 Kinetostatic Modeling of Cable Robots Without Spring

It is known that the workspace of cable robots is obtained from kinematics and equilibrium due to the cable tension condition. In this section, we review the kinetostatic modeling of single-body cable robot and its extension to multi-body systems.

A popular formulation of the equilibrium in single-body cable robots has the following form:

$$\mathbf{A}\boldsymbol{\tau} = \mathbf{b} \tag{1}$$

where $\boldsymbol{\tau}$ is a column vector containing the cable tensions, $\mathbf{A}$ is the structure matrix in the form of:

$$\mathbf{A}_{m\times n} = \begin{bmatrix} \mathbf{u}_1 & \dots\ \mathbf{u}_m \\ \mathbf{r}_1 \times \mathbf{u}_1 & \dots\ \mathbf{r}_m \times \mathbf{u}_m \end{bmatrix}$$

where $\mathbf{u}_i$ and $\mathbf{r}_i$ are unit direction vectors of the i^{th} cable and the corresponding moment arm on the end-effector, respectively. Column vector $\mathbf{b}$ consists of external wrenches and inertia terms exerted on the end-effector. A given configuration of the robot ($\mathbf{A}$) and loading ($\mathbf{b}$) will satisfy the equilibrium and can be realized only if there is a solution for $\boldsymbol{\tau}$ in which all cable tensions are nonnegative and remain in the permissible range.

In order to extend this formulation to multi-body cable robots, we need to handle the internal joint reaction forces properly. In Newtonian method for instance, the size of matrix **A** becomes very large which is due to presence of all internal reaction forces/moments. In Lagrange's method, on the other hand, as long as the multi-body is a serial chain, the internal reaction forces/moments are eliminated and hence **A** will have the minimum size.

The general form of Lagrange's equation, if the Lagrangian can be expressed in terms of a minimal set of generalized coordinates, is:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i, i = 1, ..., n \tag{2}$$

where L is the Lagrangian, n is the degrees of freedom of the multi-body system, and q_i, Q_i are the generalized coordinates and generalized forces, respectively.

In a multi-body cable-driven mechanism, the contribution of cables to the dynamics is modeled as point forces applied to the links (i.e. the inertia and elastic stiffness of the cables are neglected). Therefore, Q_i's in Eq. (2) are divided into two parts: $Q_i = Q_i^c + Q_i^r$, where Q_i^c is the part pertaining to the cable forces, and Q_i^r includes all other generalized external forces/moments. The latter part together with the terms in the left hand side of Lagrange's equation can be incorporated in a vector named $\mathbf{B}_L$:

$$\mathbf{B}_L = \begin{bmatrix} \frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_1}) - \frac{\partial L}{\partial q_1} - Q_1^r \\ \vdots \\ \frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_{DOF}}) - \frac{\partial L}{\partial q_{DOF}} - Q_{DOF}^r \end{bmatrix} \tag{3}$$

In order to use the Lagrange's formulation, the cable forces need to be presented in generalized coordinates. Suppose that r_j is the position vector of the connection point of the jth cable to the multi-body, expressed in the fixed Cartesian frame. According to Lagrange's method, one can express Q_i^c in terms of the cable forces as:

$$Q_i^c = \sum_{j=1}^{m} (t_j \mathbf{u}_j \cdot \frac{\partial \mathbf{r}_j}{\partial q_i}) \tag{4}$$

Which can be then arranged in a matrix form as:

$$Q_i^c = \begin{bmatrix} \mathbf{u}_1 \cdot \frac{\partial \mathbf{r}_1}{\partial q_1} & \dots & \mathbf{u}_n \cdot \frac{\partial \mathbf{r}_m}{\partial q_1} \\ \vdots & \ddots & \vdots \\ \mathbf{u}_1 \cdot \frac{\partial \mathbf{r}_1}{\partial q_{dof}} & \dots & \mathbf{u}_n \cdot \frac{\partial \mathbf{r}_m}{\partial q_{dof}} \end{bmatrix} \begin{bmatrix} t_1 \\ \vdots \\ t_m \end{bmatrix} \tag{5}$$

Now, $\mathbf{A}_L$ and τ_L are defined according to Eq. (5) as:

$$\mathbf{A}_L = \begin{bmatrix} \mathbf{u}_1 \cdot \frac{\partial \mathbf{r}_1}{\partial q_1} & \dots & \mathbf{u}_n \cdot \frac{\partial \mathbf{r}_m}{\partial q_1} \\ \vdots & \ddots & \vdots \\ \mathbf{u}_1 \cdot \frac{\partial \mathbf{r}_1}{\partial q_{dof}} & \dots & \mathbf{u}_n \cdot \frac{\partial \mathbf{r}_m}{\partial q_{dof}} \end{bmatrix} \tag{6}$$

And:

$$\tau_L = \begin{bmatrix} t_1 \\ \vdots \\ t_m \end{bmatrix}$$

Consequently, the general equilibrium equations of the system given in Eq. (2), can be written in the following form:

$$\mathbf{A}_L \tau_L = \mathbf{B}_L \tag{7}$$

where $\mathbf{B}_L$ was defined in Eq. (15) and includes all the external forces (other than cables) as well as the inertia effects. Note that the left hand of Eq. (7) is a linear combination of the columns of $\mathbf{A}_L$ by the cable tensions. The columns of $\mathbf{A}_L$, according to Eq. (6), can be perceived as the cable wrenches expressed in the space of generalized coordinates.

3 Kinetostatic Modeling of Cable Robots with Spring

Spring cable in this work refers to cables that act similar to a spring. Therefore they provide a tensile force proportional to their displacement.

In multi-body cable-driven robots, spring cables can be attached between the fixed ground and one of the links. They can also connect one link to another. As mentioned above, the idea here is to investigate if they can provide an affordable solution for workspace improvement without adding redundant actuators. Using cable springs between links also decreases the probability of interference of cables with each other and the environment.

In order to model spring cables, we consider them as linear axial springs with a stiffness constant K. The generated force will then become:

$$\mathbf{F}_s = K\mathbf{u}_s \tag{8}$$

where $\mathbf{F}_s$ is the force vector of spring cable and $\mathbf{u}_s$ is the elongation vector of the spring defined as:

$$\mathbf{u}_s = (l - l_0)\mathbf{u} \tag{9}$$

where $\mathbf{u}$ is the unit direction vector of the spring cable. l and l_0 are the current and initial lengths of the spring cable, respectively.

Note that the wrenches of spring cables on cable-driven robots are determined by the configuration of the robot. Hence they are treated here as external wrench. Therefore in Newtonian approach the external wrench matrix is defined as follows:

$$\mathbf{b} = \left[\mathbf{W}_{req}\right] - \begin{bmatrix} \mathbf{F}_s \\ \mathbf{r}_s \times \mathbf{F}_s \end{bmatrix} \tag{10}$$

In order to incorporate spring cables in our Lagrangian formulation, one needs to present the potential energy of the springs in terms of the generalized coordinates. Using linear axial spring model, the potential energy of a spring cable is:

$$V = \frac{1}{2} k \mathbf{u}_s^{\mathbf{T}} \mathbf{u}_s \tag{11}$$

where k is the stiffness coefficient of the spring. And:

$$L = T - V \tag{12}$$

As mentioned above in Eq. (3), the wrench matrix, $\mathbf{B}_L$, is dependent on the derivatives of the Lagrangian with respect to the generalized coordinate, so the elongation vector of spring cables must be expressed in terms of the generalized coordinates. This is possible since the end point of the spring cables are either on the ground and hence known or belong to the multi-body which can be determined using the forward kinematics of the multi-body.

It is clear that adding spring cables to a cable-driven mechanism does not change its WCW since the spring cables provide a bounded force. However they do change the WFW by providing bounded cable wrenches through the springs. Therefore, their impact on the WFW can be modified and optimized through the geometry and the spring coefficient which will be investigated in the following.

4 Method

As mentioned above, we need to use WFW definition. There are various analytical and numerical methods for determination of WFW in the literature. The numerical methods suffer from the discretization error as they can only handle a meshed workspace. The analytical methods are appropriate for particular types of robots and kinematics. The interval analysis method, on the other hand, provides a solution which is general and applicable to any kinematics and addresses the discretization problem as well. In this method an n-dimensional vector x is considered that denotes the pose of the end-effector. If we replace any real component of this vector with an interval, then we have a box denoted by [x]. Two sufficient conditions are then

Table 1 Parameters of single point cable-driven mechanism

τ_{min}	τ_{max}	$[b^T]$	K	L_0
1N	900 N	([−20,20]N,[−20,20]N)	200 N/m	0.1 m

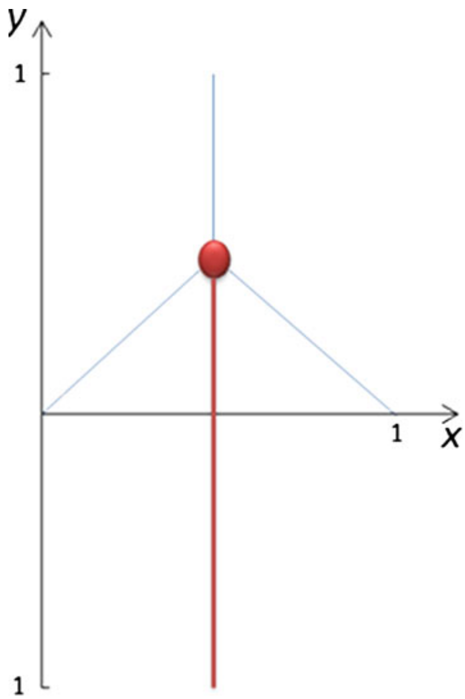

Fig. 2 A point driven by cables. A spring cable (*red*) is added to compare the workspaces

evaluated: a sufficient condition for a box of poses to be fully inside the WFW and a sufficient condition for a box of poses to be fully outside the WFW. If these two sufficient conditions aren't satisfied, the box is bisected [16]. The interval analysis method as documented well in the literature, eliminates the need for discrete meshing and therefore provide a sufficiently accurate determinations of the workspace borders. It has been also used for the design of the cable robots to fulfill a desired workspace [17, 18].

5 Results

In this section, the above formulation is implemented on a two-link planar cable-driven mechanism to show the impact of spring cable on the workspace. In our implantation, we used interval arithmetic of the INTLAB. The computation times have been obtained on a DELL XPS PC (Core 2 Duo CPU T9300, 2.50 GHz).

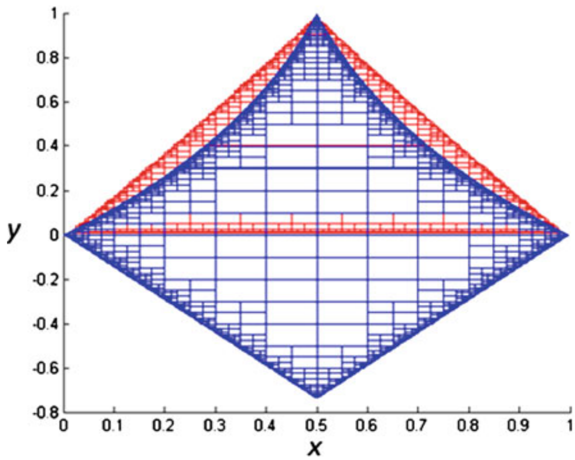

Fig. 3 Workspace of a point driven by cables with (*blue*) and without (*red*) a spring cable

Table 2 Parameter values of multi-body mechanism

Length of link 1	**Location of winch 1**	**Location of winch 2**	**Location of winch 3**	**Location of spring on ground**	
1 m	(0,3)	(1.5,-3)	(1.5,3)	(0,3)	
d_1	d_2	d_3	d_{s1}	d_{s2}	K
0.6m	0.3 m	0.8 m	0.5 m	0.5 m	100 N/m

The effects of a spring cable are intuitively understood in simple systems such as a point driven by cables on a plane. Such a point needs three cables to be fully constrained (Fig. 2). Adding a single spring cable as shown in red in the same figure has a significant impact on the workspace. For typical parameters shown in Table 1, the WFW of the mechanism is shown with and without cables in Fig. 3.

As seen in Fig. 3, the spring cable almost doubles the WFW of the mechanism as expected.

As for multi-body cable-driven mechanisms, there are two options for adding spring cables. One choice is to attach spring cables between the fixed ground and one of the links and the other one is that spring cable connects one link to another.

First, let us consider the case that the spring cable is attached between the fixed ground and link 2 as depicted in Fig. 4. Typical values were used for the parameters of mechanism as shown in Table 2.

The WFW of the mechanism with and without the spring cable are found through interval analysis and depicted in Figs. 5, 6 respectively. It is evident from the figures that the workspace is improved. The workspace has increased in terms of volume by 83 % compared to the one without the spring cable. Also it is seen that the workspace is more continuous, which is a critical aspect for robotic applications. The possibility

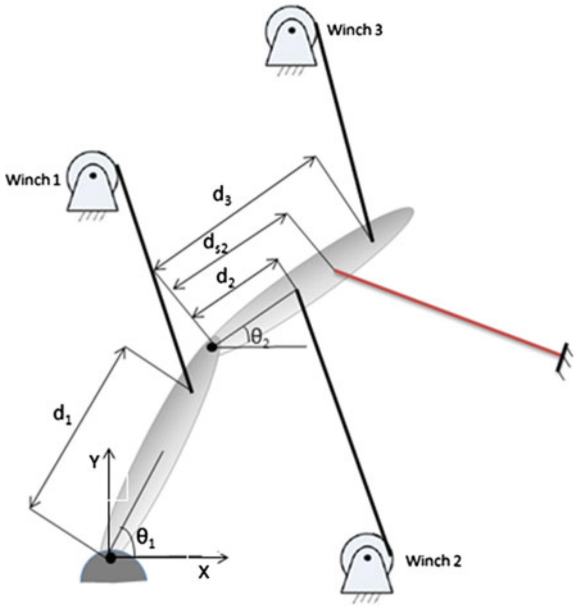

Fig. 4 Schematic of a two-link mechanism with a spring cable that is attached to ground

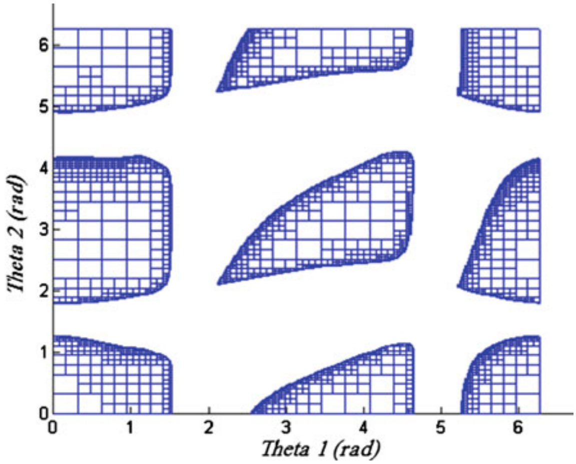

Fig. 5 Workspace of the two-link cable-driven mechanism without any spring cable

of interference, however, will increase when a spring cable is present between the mechanism and ground.

The interference problem becomes less apparent if the spring cable doesn't connect to the ground and instead goes from one link to another. A typical example is depicted in Fig. 7.

The WFW of this mechanism is seen in Fig. 8. In terms of the workspace volume, this mechanism shows an increase of 25 % with respect to the original mechanism.

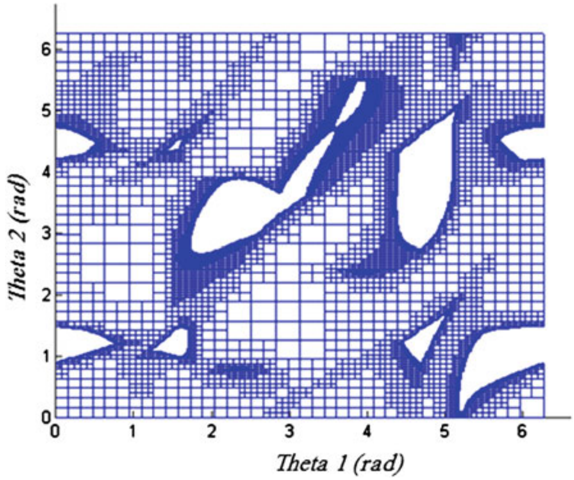

Fig. 6 Workspace of the two-link cable-driven mechanism with a spring cable attached to ground

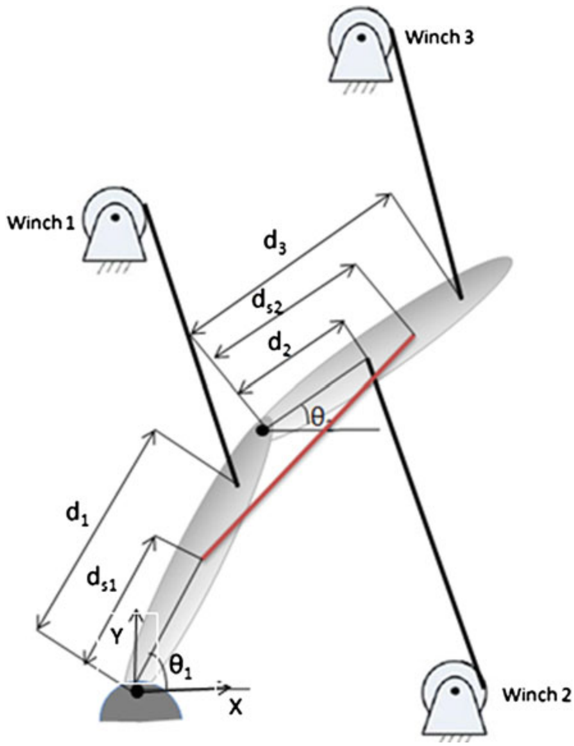

Fig. 7 Schematic of a two-link mechanism with a spring cable that is attached between the links

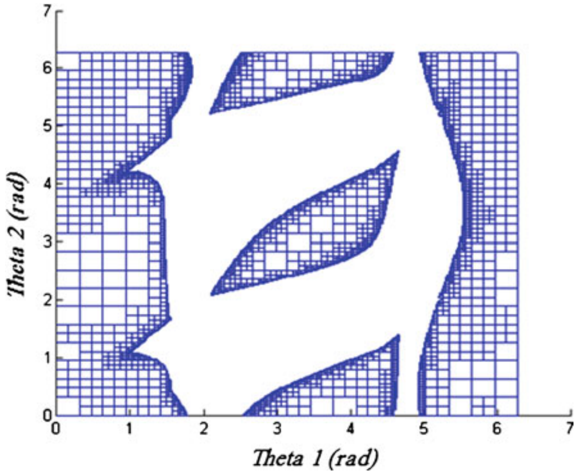

Fig. 8 Workspace of the two-link cable-driven mechanism with a spring cable attached between the links

However it is about 46 % lower than the one of the previous case where the spring cable connects to the ground, the probability of cable interference, on the other hand, is decreased from the previous case. It is also interesting to note that although the workspace has some discontinuity, it consists of larger continuous parts.

6 Conclusions

In this work, a formulation is presented to study the workspace of multi-body cable-driven mechanisms with spring cables. The method was then applied to a two-link serial mechanism to investigate the impact of the spring cable on the size and shape of the workspace.

Two cases were considered: in the first one, a spring cable was attached between a link and the ground, in the second case, the spring cable was added between the two links. It is apparent that the second case has a lower possibility of cable interference. It was shown that both cases provide a more continuous workspace which is favorable for robotic application. The first case provides a larger workspace volume however it seems that the actual workspace (considering the cable interference) becomes smaller.

Based on the early results of this study, it seems adding spring cables between the links in a multi-body cable-driven mechanism has higher potentials to improve the WFW of such mechanisms. In future works, the idea will be further developed for spatial and more general mechanisms.

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