Chapter 1
Introduction

Abstract In this first chapter the content as well as the focus will be classified in various aspects. First, the development of the finite element method will be explained and considered from different perspectives.

1.1 The Finite Element Method at a Glance

Seen chronologically, the roots of the finite element method lie in the middle of the last century. Therefore, this method is a relatively young tool in comparison with other tools and aids for the dimensioning and laying of components. The development of the finite element method was configured in the 1950s. Scientists and users have brought in ideas from quite different fields and have therefore turned the method into a universal tool which has nowadays become an indispensable part in research and development and engineering application.
Initially rather basic questions were focused, for example on questions regarding the principle solvability. Regarding the software implementation, only rudimental resources were available—from today's point of view. The postprocessing consisted of punching of cards, which were fed in batches to a calculating machine. Mistakes during the programming were directly displayed with blinking lights. With progressive computer development, the programming environment has become more comfortable and algorithms could be tested and optimized on more challenging examples. From the point of view of engineering application, the problems, which were analyzed via the finite element method, were limited to simple examples. The computer capacities only allowed a quite rough modeling.
Nowadays, many basic questions have been clarified, the central issue of the problem rather lies on the user side. Finite element program packages are available in a large variety and are used in the most different forms. On the one hand there are program packages, which are primarily used in teaching. It is the goal to illustrate the systematic approach. Source codes are available for such programs. On the other

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hand, there are commercial program packages which are used to their full capacity regarding program technology as well as the content. Especially the program modules, which have been adapted to a computer platform or computer architecture (parallel computer) are quite efficient and make the processing of very comprehensive problems possible. Regarding the content, the authors venture to say that there is no physical discipline for which no finite element program exists.
In regard to the development of the finite element method, the focus is nowadays on the cooperation and integration with other development tools, as for example the point of intersection with engineering design. Both classical disciplines calculation and design become more and more connected and are partly already fused underneath by a common operation interface. Besides single finite element software packages there are also in a CAD system integrated solutions available on the market. From the view of the user an appropriate finite element preprocessing and postprocessing of *his/her* special problem is in the foreground. The time intensive process steps of the geometry preparation should not involve a considerable extra effort for the application of the finite element method. Calculation results are supposed to be integrated seamlessly in the according process chain.
Regarding the application areas, there are no limits for the application of the finite element method. The dimensioning and configuration of manufacturing elements, subsystems or complete machines surely is the focus in mechanical and plant engineering.
The application of the finite element method or in general of simulation tools in the product development is often seen as a competing tool to the experiment or test. The authors rather see an ideal complement at this point. Therefore, a single test stands test rig or complete test scenarios can be optimized ex ante via finite element simulation. In return, experimental results help to create more precise simulation models.

1.2 Foundations of Modeling

A model of a physical or technical problem represents the initial situation for the application of the finite element method. Part of the complete description of the problem are

- the geometry for the description of the domain,
- the field equations in the domain,
- the boundary conditions, and
- the initial conditions for time dependent problems.

Within this book solely *one-dimensional* elements will be regarded. The general procedure for two- and three-dimensional problems is similar. The mathematical demand, however, is much more complex.
Usually the problems can be described via the differential equation. Here, differential equations of second order are focused on. As an example, the differential equations

of a certain class of physical problems can in general be described as follows:

$$-\frac{\mathrm{d}}{\mathrm{d}x}\left[a\frac{\mathrm{d}u(x)}{\mathrm{d}x}\right]+cu(x)-f=0. \tag{1.1}$$

Depending on the physical problem a different meaning is assigned to the variables $u(x)$ and the parameters a, c, and f. The following table lists the meaning of the parameters for a few physical problems [1] (Table 1.1).

Table 1.1 Physical problems in the context of the differential equation. Adapted from [1]

	Field parameter	Coefficient		
Problem	$u(x)$	a	c	f
Heat conduction	Temperature	Heat conduction	Convection	Heat sources
	T	kA	$K\ \beta$	q
Pipe flow	Pressure	Pipe resistance		
	p	$1/\ R$		
Viscous flow	Velocity	Viscosity		Pressure gradient
	v_x	ν		$\frac{\mathrm{d}p}{\mathrm{d}x}$
Elastic bars	Displacement	Stiffness		Distributed loads
	u	EA		f
Elastic torsion	Rotation	Stiffness		Torsional moments
	φ	GI_{p}		m
Electrostatics	Electrical potential	Dielectricity		Charge density
	Φ	ϵ		ρ

To describe a problem completely, the statement about the boundary conditions is necessary besides the differential equation. The local boundary conditions can generally be divided into three groups:

- Boundary condition of the 1st kind or DIRICHLET boundary condition (also referred to as essential, fundamental, geometric or kinematic boundary condition):
  A boundary condition of the 1st kind exists, if the boundary condition is being expressed in parameters in which the differential equation is being formulated.
- Boundary condition of the 2nd kind or NEUMANN boundary condition (also referred to as natural or static boundary condition):
  A boundary condition of the 2nd kind exists, if the boundary condition contains the derivation in the direction of the normal of the boundary Γ.
- Boundary condition of the 3rd kind or CAUCHY boundary condition (also referred to as mixed or ROBIN boundary condition):
  Defines a weighted sum of DIRICHLET and NEUMANN condition on the boundary.

These three types of boundary conditions are summarized in Table 1.2, along with their formulas.

Table 1.2 Different boundary conditions of a differential equation

Differential equation	Dirichlet	Neumann	Cauchy
$\mathcal{L}\{u(x)\} = b$	u	$\frac{\mathrm{d}u}{\mathrm{d}x}$	$\alpha u + \beta \frac{\mathrm{d}u}{\mathrm{d}x}$

It needs to be considered at this point that one talks about *homogeneous boundary conditions* if the corresponding variables are zero on the boundary.
Within this book, the finite element method will be highlighted from the view of mathematics, physics or the engineering application. From a mathematical view, the finite element method is an appropriate tool to solve partial differential equations. From a physical view a multitude of physical problems can be worked on via the finite element method. The areas go from electrostatics via the diffusion problem all the way to elasticity theory. Engineers make use of the finite element method for the configuration and the dimensioning of products. Regarding the physical problems, at this point solely elastomechanical problems will be discussed. Within statics

- the tension bar,
- the torsion bar, and
- the bending beam with and without shear contribution,

will be covered. Vibrations of bars and beams will be covered as dynamic problems.

Reference

1. Reddy JN (2006) An Introduction to the Finite Element Method. McGraw Hill, Singapore