Self-Growing Regularized Gaussian Mixture Models for Image Segmentation

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Abstract. Mixture Models(MMs) are a typical class of statistical models and have been applied to image processing in many situations, among which Gaussian MM (GMMs) are widely adopted. Main drawbacks of classical models involve that they need presetting the number of clusters, have not considered the influence of outliers. They will lead to unreasonable image segmentation results. This paper proposes the Self-Growing Regularized GMMs(SGRGMMs), which generalizes the classical GMMs, for image segmentation. We compute the unknown parameters using the self-branching competitive leaning and a new generalized EM algorithm, Regularized EM(REM). We carried out experiments on the segmentation of some images and our approach can automatically determine the number of clusters and efficiently erase the influence of outliers.

Keywords: Gaussian mixture models, Expectation Maximum(EM), Self-branching Competitive Learning.

1 Introduction

Image segmentation can be viewed as a process to partition an image into a series of non-overlapping regions. Currently, there have existed some approaches including histogram thresholding[1] and classification based methods. in which the pixels with same gray or color are grouped to one class. Many classification approaches are based on statistical learning models and the frequently used ones involve vector quantization, mixed models and Markov random fields. Mixture Models(MMs) are established on statistical theory and have received wide attention in image segmentation[2-9]. Gaussian mixture models(GMMs) are a kind of MMs whose components are Gaussian distributions. In image segmentation using GMMs, one assumes that data comply with Gaussian mixture distribution with unknown parameters and then determines the optimal parameters by EM algorithm. In segmentation of remotely sensed images, P. Masson and W. Pieczynski proposed a stochastic variant of EM to segment satellite images[9]. They pointed out that EM related algorithms had large dependence on the initialization. Recently, some researchers further develop GMMs and combine GMMs with Markov Random Fields for the spatial constraints of pixels[5,6]. However, there are still three drawbacks in GMMs and EM algorithm. First, one must preset the initial number of clusters, which

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is almost unreasonable in many practical cases. Second, the unsuitable initialization of centers may lead to the local converge of EM algorithm. Third, in general, the GMMs incline to be affected by outliers.

In this paper we propose the SGRGMM model and the REM algorithm by imposing a regularized item on the MAP function to obtain parameter estimation. The Self-Branching Competitive Learning[10] is used to determine the initial number and locations of clusters. We carry out experiments on remotely sensed images and standard test images and get better results compared to GMMs+EM.

2 Mixture Models and EM Algorithm

In the probability space with d dimension, given sample X and probability density function $p_i(X|\theta_i)$, $i \le r$, the mixture model has the following form

$$p(X|\Theta) = \sum_{i=1}^{M} \alpha_i p_i(X|\theta_i), \tag{1}$$

where $\Theta = (\alpha_1, \cdots, \alpha_M, \theta_1, \cdots, \theta_M)$, $\sum_{i=1}^{M} \alpha_i = 1$.

Here, $p_i(X|\theta_i)$ can be any probability density function. In particular, if $p_i(\cdot)$ is the Gaussian distribution $G_i(\cdot)$, then we get GMMs that are frequently used in image segmentation. The coefficients α_i, which may be distorted by outliers, affect the performance of the estimators.

In common, it is assumed that we know the number of clusters and then compute the maximal likelihood estimation(MLE) of parameters by EM algorithm. EM is an iterative optimal method proposed by A. P. Dempster et al in 1977 and can be used to calculate the optimal parameters of mixture models when sample are incomplete[3]. In sequent years, EM is developed and many variants of it are proposed, such as Stochastic EM(SEM)[12]. However, it is not convenient to directly apply EM to (1) when estimating parameters.

3 Regularized Mixture Models and Generalized EM Algorithm

The conventional GMMs and EM algorithm classify the pixels of an image into several groups, each of which denotes a region with specified feature values. In common, the irregular parts in images, such as isolated points, will affect the convergence of EM algorithm and the segmentation quality of images. To erase the influence, we propose a regularized EM algorithm(REM) that defines a regularized $Q(\cdot|\cdot)$ function for M-step written as

$$\begin{aligned} Q\left(\Theta|\Theta^{(k)}\right) &= E\left[l(\Theta) \mid X, \Theta^{(k)}\right] - \beta \sum_{j=1}^{N} \sum_{i=1}^{M} \left(\alpha_i^j\right)^2 \\ &= \sum_{j=1}^{N} \sum_{i=1}^{M} E\left[z_i^j \mid x_j, \Theta^{(k)}\right] \ln\left[\alpha_i^j p_i\left(x_j \mid \theta_i\right)\right] - \beta \sum_{j=1}^{N} \sum_{i=1}^{M} \left(\alpha_i^j\right)^2, \end{aligned} \tag{2}$$

where $\beta \geq 0$ is the regularized factor , $l(\Theta)$ is the likelihood function and $\sum_{i=1}^{M} \alpha_i = 1$.

When $p_i\left(x_j \mid \theta_i\right)$ satisfies Gaussian distribution (3), $\theta_i = \left(\mu_i, \Sigma_i^{'}\right)$ with d=2, then (13) becomes

$$Q\left(\Theta \middle| \Theta^{(k)}\right) = \sum_{j=1}^{N}\sum_{i=1}^{M}\left\{\left(\omega_i^j\right)^{(k)} \ln \alpha_i^j - \beta \sum_{j=1}^{N}\sum_{i=1}^{M}\left(\alpha_i^j\right)^2 \right.$$

$$\left. -\left(\omega_i^j\right)^{(k)}\left(\ln 2\pi + \frac{1}{2}\ln\left|\Sigma_i\right| + \frac{1}{2}\left(x_j - \mu_i\right)^{'} \Sigma_i^{-1}\left(x_j - \mu_i\right)\right)\right\}, \tag{3}$$

where

$$E\left[z_i^j \mid x_j, \Theta^{(k)}\right] = \left(\omega_i^j\right)^{(k)} = \frac{\alpha_i^j p_i\left(x_j \mid \Theta_i^{(k)}\right)}{\sum_{l=1}^{M} \alpha_l^j p_l\left(x_j \mid \Theta_l^{(k)}\right)}. \tag{4}$$

To compute the unknown parameter α_i^j it needs to solve the following Lagrange equation. From (3) we get

$$\frac{\partial}{\partial \alpha_i^j}\left[Q\left(\Theta \mid \Theta^{(k)}\right) - \left(\sum_{j=1}^{N} \lambda_j\left(\sum_{i=1}^{M} \alpha_i^j - 1\right)\right)\right] = 0,$$

where λ_j is the Lagrange multiplier. Solve above equation and we have

$$2\beta\left(\alpha_i^j\right)^2 + \lambda_j \alpha_i^j - \left(\omega_i^j\right)^{(k)} = 0,$$

Since $\sum_{i=1}^{M} \omega_i^j = 1$ and $\sum_{i=1}^{M} \alpha_i^j = 1$, then we get the iteration at kth step

$$\lambda_j^{(k)} = 1 - 2\beta \sum_i \left(\left(\alpha_i^j\right)^{(k)}\right)^2. \tag{5}$$

Obviously, $1 \geq \lambda_j^{(k)} \geq 1 - 2\beta$. Then we obtain the iterative equation at kth step as

$$\left(\alpha_i^j\right)^{(k+1)} = \frac{1}{4\beta}\left(\sqrt{\left(\lambda_j^{(k)}\right)^2 + 8\beta\left(\omega_i^j\right)^{(k)}} - \lambda_j^{(k)}\right). \tag{6}$$

To find the priors of each component we take the mean of all α_i^j about j and then get

$$\alpha_i = \frac{1}{N}\sum_{j \leq N} \alpha_i^j. \tag{7}$$

The α_i need normalizing about i so as to satisfy $\sum_i \alpha_i^j = 1$ for j. The prior probabilities of α_i will be used to calculate ω_i^j in every iteration. Let

$$\alpha_i = \frac{\alpha_i}{\sum_{i=1}^{M} \alpha_i}. \tag{8}$$

In the iteration process, the normalized α_i is used to update ω_i^j in (4). For μ_i, we have

$$\mu_i^{(k+1)} = \frac{\sum_{j=1}^{N} \left(\omega_i^j\right)^{(k)} x_j}{\sum_{j=1}^{N} \left(\omega_i^j\right)^{(k)}} . \tag{9}$$

From (3), we get the covariance matrix for multivariate data as

$$\Sigma_i^{(k+1)} = \frac{\sum_{j=1}^{N} \left(\omega_i^j\right)^{(k)} \left(x_j - \mu_i\right)\left(x_j - \mu_i\right)^T}{\sum_{j=1}^{N} \left(\omega_i^j\right)^{(k)}} . \tag{10}$$

4 Experiment Results on Remotely Sensed Images

Remotely sensed images are widely adopted in city planning, Agriculture and resource survey. Here, we apply our algorithm to segmentation of color remotely sensed images. Figure 1 (a) show a remotely sensed image and its segmented image (b) by our approach with $\sigma = 10$, K=10. The results present a favorable segmentation result.

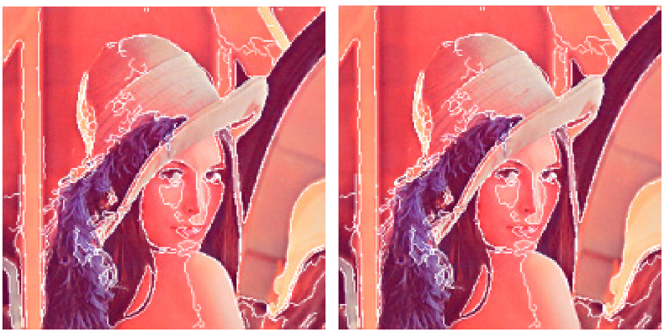
(a) origin (b) segmented image

Fig. 1. The segmentation results by our approach

5 Experiment Results on Standard Test Images

We used two standard figures to compare the performance of classical EM and REM in segmentation quality. With the same set of preset parameters, we got the final variances and priors. From the data distribution in feature space as shown in Figure 2, we knew that the data distribution of Donna was more irregular than that of Lena, which made the final priors for Donna had more adjustment. So the difference in segmentation results between (c) and (d) in Figure 2 was less than that between (a) and (b). Since classical EM algorithm was sensitive to initialization and noise, thus the change of priors improved the segmentation quality.

(a) EM (b) REM



(c) EM (d) REM

Fig. 2. The segmentation results by our approach and GMM+EM

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