

An Improved Symbol Mappings on 16QAM Constellation for BICM-ID

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Abstract. Bit-interleaved coded modulation with iterative decoding (BICM-ID) is a spectral efficient coding technique. The technique is therefore very attractive for next generation of wireless communication. It has been shown that when interleaver and error-control code are fixed, symbol mapping which defined by the signal constellation and the bit labeling has a critical influence on the error performance of BICM-ID. This paper presents an improved symbol mapping which combines two M-PSK with different radius and phases called (4,12). Through comparison with the conventional symbol mappings in terms of BER performance over Rayleigh channel, numerical results show that the new symbol mapping can improve the performance of BICM-ID system more than 0.1dB at low-to-medium SNR regions.

Keywords: Bit-interleaved coded modulation with Iterative Decoding (BICM-ID), QAM, symbol mapping, Rayleigh fading channel.

1 Introduction

Trellis coded modulation (TCM), proposed by Ungerboeck in [1], is based on maximizing the Hamming distance between codewords, which can provide good performances for the AWGN channel but for fading channel. A different strategy that increases the time diversity of the coded modulation which is referred as BICM, was suggested by Zehavi in [2]. The improved time diversity at expense of reducing the free squared Euclidean distance, leading to a degradation over AWGN channels [3]. With a careful design of signal mapping, iterative decoded BICM (BICM-ID) overcomes the drawback of conventional BICM over AWGN by increasing the free Euclidean distance with the knowledge of other bit values [4].

According to Chi-Hsiao Yih's results [5], symbol mapping was recognized as the crucial design parameter to achieve an excellent performance for BICM-ID. In general, a symbol mapping is defined by the signal constellation and the bit labeling [6]. Several mappings on QAM constellation were introduced in [7] [8]. Modified set-partitioning labeling (MSP) proposed in [7], optimized under the harmonic mean criterion, showing the performance relatively close to that of turbo coded modulation with less complexity. The optimal Maximum squared Euclidean weight (MSEW) mapping proposed in [8] has the minimum number of symbol pairs which have the minimum squared Euclidean distances with Hamming distance-1. There is no one mapping having the best performance over all SNR regions. Many studies mentioned previously have concentrated on finding symbol mappings lowering bit error rates

(BER) at high SNR. In this paper, an improved symbol mapping called (4,12) is proposed, which can achieve better performance than other conventional mappings at low-to-medium SNR more than 0.1dB.

This paper is organized as follows. In section 2, we briefly review the model of BICM-ID. In section 3, the conventional symbol labelings and an improved symbol mapping are shown. Then, Section 4 shows the simulation results of BICM-ID performance with different mappings. Section 5 concludes the paper.

2 System Model

The conventional BICM can be modeled as a serial concatenation of a convolutional encoder followed by a bit-by-bit interleaver and a memoryless modulator as shown in Fig.1. At the transmitter, the information sequence $\{u_t\}$ is encoded by a convolutional encoder. The coded binary sequence $\{c_t\}$ is then fed into a bit interleaver which not only break the sequential fading correlation but also increase diversity order to the minimum Hamming distance of a code.

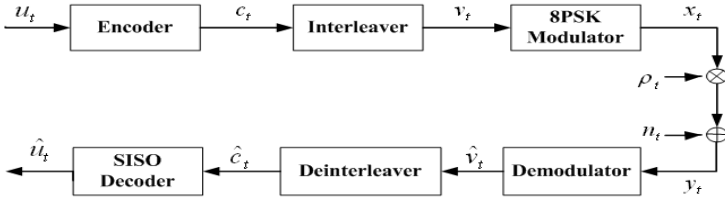


Fig. 1. BICM system model

After interleaving, the consecutive bits of the interleaved coded sequence are grouped to form $V_t = [v_t^1, v_t^2, v_t^3]$, which are mapped onto signal set χ of size $|\chi| = M = 2^m$ through the one to one mapping $\mu: \{0,1\} \rightarrow \chi$, $x = \mu(t)$ and the corresponding 8PSK signal at time t by x_t ,

$$x_t = \mu(x_t), x_t \in \chi \tag{1}$$

Where the 8PSK signal set is $\chi = \{\sqrt{E_s} e^{j2n\pi/8}, n = 0, \dots, 7\}$ and E_s is the symbol energy. For a frequency nonselective Rayleigh fading channel with coherent detection, the received discrete-time signal can be expressed as

$$y_t = \rho_t x_t + n_t \tag{2}$$

Where x_t is a transmitted symbol in M complex-valued. ρ_t is the Rayleigh random variable with $E(\rho_t) = 1$ presenting the fading amplitude of the received signal and n_t is a complex AWGN with one-sided spectral density N_0 . For the AWGN channel, ρ_t is set to 1.

The receiver of BICM-ID in Fig.2 uses demodulation and convolutional decoding as a suboptimal iterative method. The demapper processes the received complex symbols y_t , and the corresponding priori loglikelihood ratios (LLR) of the coded bits.

$$L_a(c_k(i)) = \log \left[\frac{P(c_k(i) = 0)}{P(c_k(i) = 1)} \right] \quad (3)$$

And outputs the extrinsic LLRs

$$L_e(c_k(i)) = \log \left[\frac{P(c_k(i) = 0 | r_k, L_a(c_k))}{P(c_k(i) = 1 | r_k, L_a(c_k))} \right] - L_a(c_k(i)) \quad (4)$$

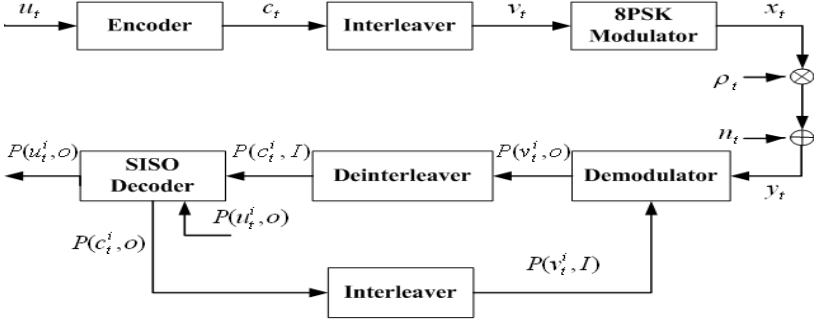


Fig. 2. BICM-ID system model

3 Signal Labeling

Different signal constellation labels are critical for optimizing different decoding methods. The comparison of five labels for 16QAM is shown in Fig.3. The influence of signal labeling to BICM-ID can be quantified by the two Euclidean distances [5] [9]. Unlike AWGN channels, there is no dominating term in the performance bound for the Rayleigh channel.

The asymptotic performance of BICM over Rayleigh fading [3] can be approximated by

$$\log_{10} P_b \approx \frac{-d_2(C)}{10} [(Rd_h^2(\mu))_{dB} + (\frac{E_b}{N_0})_{dB}] + const \quad (5)$$

Where P_b is the probability of bit error, $d_2(C)$ is the minimum Hamming distance of the code, R is the information rate and d_h^2 is the harmonic mean of the minimum squared Euclidean distance. For any M -ary constellation with a labeling map μ , d_h^2 can be calculated by

$$d_h^2(\mu) = \left(\frac{1}{m2^m} \sum_{i=1}^m \sum_{b=0}^1 \sum_{x \in \mathcal{X}_b^i} \frac{1}{\|x - z\|} \right) \quad (6)$$

Where $m=\log_2(M)$, $z = \hat{z}(x) \in \mathcal{X}_b^i$ denote the nearest neighbor of x . Specifically, $d_2(C)$ controls the slope of the probability of bit error P_b curve while d_h^2 provide the horizontal offset. Given ideal feedback for each $x \in \mathcal{X}_b^i, \mathcal{X}_b^i$ contains only one term $\tilde{z} = \tilde{z}(x)$ whose label has the same binary bit values as those of x except at the i 'th bit position.

From (6), the general rule is to design a labeling map μ that $\|x - \tilde{z}\|$ is larger than $\|x - \hat{z}\|$ for all x (if possible) in order to achieve the iterative decoding gain. It is preferable to have a labeling map that maximizes \tilde{d}_h^2 while having sufficiently large original d_h^2 to make the first iteration work well.

For BICM-ID with convolutional code, numerical results from calculating the harmonic mean of the minimum Euclidean distance are shown in Table 1. This gives a quick comparison between various labeling schemes. It is shown that Gray yields the best first round performance, however, the performance gain with feedback is very small. MSP and MSEW yield great iterative decoding performance after feedback, but they both have the poor d_h^2 , which leading to a degradation at low SNR. The weaker performance improvement of d_h^2 obtained by SP labeling is also shown for comparison.

Table 1. Harmonic mean of the minimum squared Euclidean distance d_h^2 before and after feedback and the d_h^2 asymptotic gain over six labelings

Labeling	d_h^2 (before)	d_h^2 (after)	Gain(dB)
Gray	0.492	0.514	0.19
SP	0.441	1.119	3.56
MSP	0.420	2.279	6.65
MSEW	0.400	2.364	1.77
Antigray	0.320	0.992	1.49
(4,12)	0.739	2.118	1.46

In this paper, we propose an improved type of symbol labeling scheme, which is composed of two M-PSK with different radius and phases called (4,12). The ratio between the radius of the outer and inner circles is approximate to 2.6, which is larger than the ratio in [10]. The new signal constellation depicts in Fig.3. Via computer simulations, it is verified that the new mapping could provide the better performance than Gray, SP, MSP and MSEW at low-to-medium SNR regions, due to largest d_h^2 .

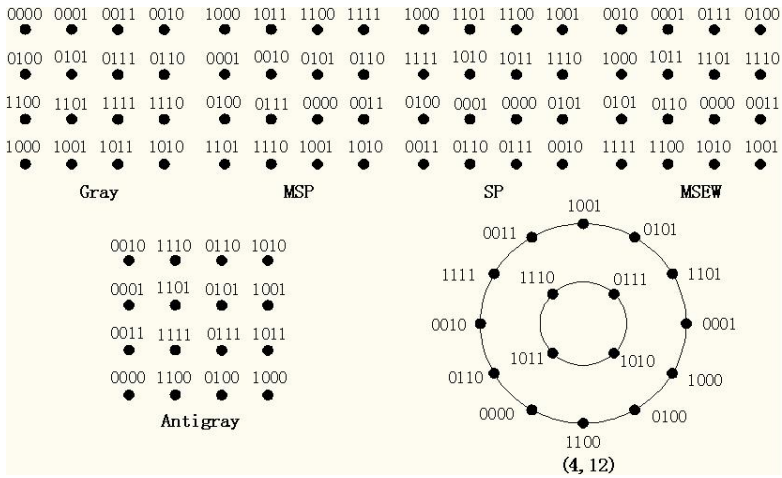


Fig. 3. Six labeling methods for 16QAM

4 Simulation Results

In our simulations, a rate-1/2, systematic convolutional (RSC) code with polynomials (131, 171) is assumed. The pseudorandom bit interleaver with a length of 5114 bits is used. Linear MAP algorithm is used for the decoding of each RSC code. Fig.4 and Fig.5 compare the BER performance with 10 iterations employing various mappings mentioned previously over Rayleigh channel.

The effect of signal labeling is shown in Fig.4. It can be observed that without iterative decoding, Gray yields the best performance. MSP and MSEW desperately achieve the worst performance at low SNR, although they have lower error floor. Furthermore, when the BER is close to 10^{-3} , Antigray mapping achieves at least more than 0.5dB than the conventional symbol mappings. And then, the (4,12) can yield 0.1dB code gains over Rayleigh fading channel compared with Antigray mapping in Fig.5.

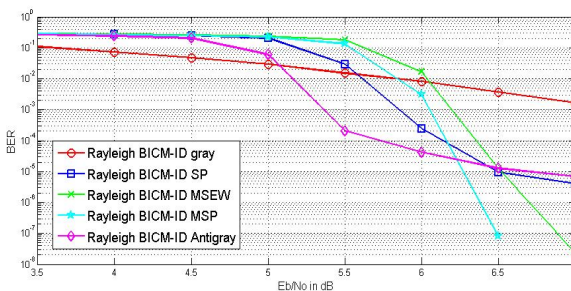


Fig. 4. Performance comparison of conventional mappings

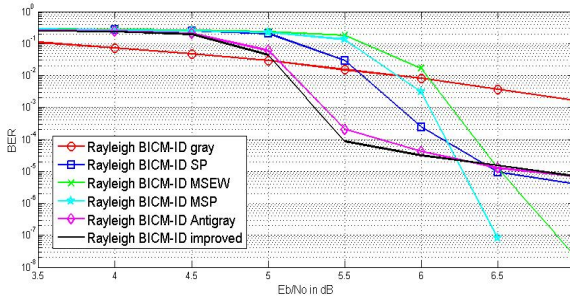


Fig. 5. Performance comparison of conventional and (4,12) mappings

5 Summaries

In this paper, a new symbol mapping (4,12) is proposed. We have analyzed it in terms of BER performance via theoretical analysis and computer simulation. To compare its performance with some previously mentioned signal labeling maps, such as Gray, MSEW, Antigray, SP, and MSP, some computer simulations were completed. Simulation results show that the newly proposed signal mapping can obtain more than 0.1dB code gains compared with the conventional mappings at low-to-medium SNR regions over Rayleigh channel.

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