One-Sided Random Context Grammars with Leftmost Derivations

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Abstract. In this paper, we study the generative power of one-sided random context grammars working in a leftmost way. More specifically, by analogy with the three well-known types of leftmost derivations in regulated grammars, we introduce three types of leftmost derivations to one-sided random context grammars and prove the following three results. (I) One-sided random context grammars with type-1 leftmost derivations characterize the family of context-free languages. (II) One-sided random context grammars with type-2 and type-3 leftmost derivations characterize the family of recursively enumerable languages. (III) Propagating one-sided random context grammars with type-2 and type-3 leftmost derivations characterize the family of context-sensitive languages. In the conclusion, the generative power of random context grammars and one-sided random context grammars with leftmost derivations is compared.

Keywords: formal languages, regulated rewriting, one-sided random context grammars, leftmost derivations, generative power.

1 Introduction

The investigation of grammars that perform leftmost derivations is central to formal language theory as a whole. Indeed, from a practical viewpoint, leftmost derivations fulfill a crucial role in parsing, which represents a key application area of formal grammars (see [1,2,7,21]). From a theoretical viewpoint, an effect of leftmost derivation restrictions to the power of grammars restricted in this way represents an intensively investigated area of this theory as clearly indicated by many studies on the subject. More specifically, [3,4,17,18,32] contain fundamental results concerning leftmost derivations in classical Chomsky grammars, [6,14,19,30,33] and Section 5.3 in [9] give an overview of the results concerning leftmost derivations in regulated grammars published until late 1980's, and [8,10,11,20,23,25] together with Section 7.3 in [24] present several follow-up results. In addition, [15,16,31] cover language-defining devices introduced with some kind of leftmost derivations, and [5] discusses the recognition complexity of derivation languages of various regulated grammars with leftmost derivations. Finally, [16,22,28] study grammar systems working under the leftmost derivation restriction, and [12,13,29] investigates leftmost derivations in terms of P systems.

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The present paper approaches this topic in terms of one-sided random context grammars. Recall that a *one-sided random context grammar* (see [26,27]) represents a variant of a random context grammar (see [9] and Chapter 3 in the second volume of [32]). In this variant, a set of *permitting symbols* and a set of *forbidding symbols* are attached to every rule, and its set of rules is divided into the set of *left random context rules* and the set of *right random context rules.* A left random context rule can rewrite a nonterminal if each of its permitting symbols occurs to the left of the rewritten symbol in the current sentential form while each of its forbidding symbols does not occur there. A right random context rule is applied analogically except that the symbols are examined to the right of the rewritten symbol.

Specifically, this paper introduces three types of leftmost derivation restrictions placed upon one-sided random context grammars. In the *type*-1 *derivation restriction*, during every derivation step, the leftmost occurrence of a nonterminal has to be rewritten. In the *type*-2 *derivation restriction*, during every derivation step, the leftmost occurrence of a nonterminal which can be rewritten has to be rewritten. In the *type*-3 *derivation restriction*, during every derivation step, a rule is chosen, and the leftmost occurrence of its left-hand side is rewritten.

The paper demonstrates the following three results. (I) One-sided random context grammars with type-1 leftmost derivations characterize the family of context-free languages. (II) One-sided random context grammars with type-2 and type-3 leftmost derivations characterize the family of recursively enumerable languages. (III) Propagating one-sided random context grammars with type-2 and type-3 leftmost derivations characterize the family of context-sensitive languages.

The paper is organized as follows. First, Section 2 gives all the necessary terminology. Then, Section 3 rigorously establishes the results mentioned above. In the conclusion, Section 4 compares the generative power of random context grammars and that of one-sided random context grammars with leftmost derivations.

2 Preliminaries and Definitions

We assume that the reader is familiar with formal language theory (see [32]). For a set Q, 2^Q denotes the power set of Q. For an alphabet (finite nonempty set) V, V^* represents the free monoid generated by V under the operation of concatenation. The unit of V^* is denoted by ε. Set $V^+ = V^* - \{\varepsilon\}$; algebraically, V^+ is thus the free semigroup generated by V under the operation of concatenation. For $x \in V^*$, $|x|$ denotes the length of x, and $\mathrm{alph}(x)$ denotes the set of symbols occurring in x.

A *context-free grammar* is a quadruple, $G = (N, T, P, S)$, where N and T are two disjoint alphabets, $S \in N$, and $P \subseteq N \times (N \cup T)^*$ is a finite relation. Set $V = N \cup T$. The components V, N, T, P, and S are called the *total alphabet*, the alphabet of *nonterminals*, the alphabet of *terminals*, the set of *rules*, and the *start symbol*, respectively. Each $(A, x) \in P$ is written as $A \to x$ throughout this paper. If $A \to x \in P$ implies that $|x| \geq 1$, then G is *propagating*. The *direct derivation relation* over V^*, symbolically denoted by $\Rightarrow$, is defined as follows: $uAv \Rightarrow uxv$ in G if and only if $u, v \in V^*$ and $A \to x \in P$. Let $\Rightarrow^n$ and $\Rightarrow^*$ denote the nth power of $\Rightarrow$, for some $n \geq 0$, and the reflexive-transitive closure of $\Rightarrow$, respectively. The *language of* G is denoted by $L(G)$ and defined as $L(G) = \{w \in T^* \mid S \Rightarrow^* w\}$.

A *one-sided random context grammar* (see [27]) is a quintuple, $G = (N, T, P_L, P_R, S)$, where N and T are two disjoint alphabets, $S \in N$, and $P_L, P_R \subseteq N \times (N \cup T)^* \times 2^N \times 2^N$ are two finite relations. Set $V = N \cup T$. The components V, N, T, P_L, P_R and S are called the *total alphabet*, the alphabet of *nonterminals*, the alphabet of *terminals*, the set of *left random context rules*, the set of *right random context rules*, and the *start symbol*, respectively. Each $(A, x, U, W) \in P_L \cup P_R$ is written as $\lfloor A \to x, U, W \rfloor$ throughout this paper. For $\lfloor A \to x, U, W \rfloor \in P_L$, U and W are called the *left permitting context* and the *left forbidding context*, respectively. For $\lfloor A \to x, U, W \rfloor \in P_R$, U and W are called the *right permitting context* and the *right forbidding context*, respectively. If $\lfloor A \to x, U, W \rfloor \in P_L \cup P_R$ implies that $|x| \geq 1$, then G is *propagating*. The *direct derivation relation* over V^*, symbolically denoted by $\Rightarrow$, is defined as follows. Let $u, v \in V^*$ and $\lfloor A \to x, U, W \rfloor \in P_L \cup P_R$. Then, $uAv \Rightarrow uxv$ in G if and only if

$$\lfloor A \to x, U, W \rfloor \in P_L, U \subseteq \text{alph}(u) \text{ and } W \cap \text{alph}(u) = \emptyset$$

or

$$\lfloor A \to x, U, W \rfloor \in P_R, U \subseteq \text{alph}(v) \text{ and } W \cap \text{alph}(v) = \emptyset$$

Let $\Rightarrow^n$ and $\Rightarrow^*$ denote the nth power of $\Rightarrow$, for some $n \geq 0$, and the reflexive-transitive closure of $\Rightarrow$, respectively. The *language of* G is denoted by $L(G)$ and defined as $L(G) = \{w \in T^* \mid S \Rightarrow^* w\}$.

2.1 Leftmost Derivations

By analogy with the discussion of leftmost derivations in [9], we next place three types of leftmost derivation restrictions on one-sided random context grammars.

In the first derivation restriction type, during every derivation step, the leftmost occurrence of a nonterminal has to be rewritten. This type of leftmost derivations corresponds to the well-known leftmost derivations in context-free grammars.

Definition 1. Let $G = (N, T, P_L, P_R, S)$ be a one-sided random context grammar. The *type*-1 *direct leftmost derivation relation* over V^*, symbolically denoted by ${}_{\text{lm}}\!\overset{1}{\Rightarrow}$, is defined as follows. Let $u \in T^*$, $A \in N$ and $x, v \in V^*$. Then, $uAv \;{}_{\text{lm}}\!\overset{1}{\Rightarrow} uxv$ in G if and only if $uAv \Rightarrow uxv$ in G.

Let ${}_{\text{lm}}\!\overset{1}{\Rightarrow}^n$ and ${}_{\text{lm}}\!\overset{1}{\Rightarrow}^*$ denote the nth power of ${}_{\text{lm}}\!\overset{1}{\Rightarrow}$, for some $n \geq 0$, and the reflexive-transitive closure of ${}_{\text{lm}}\!\overset{1}{\Rightarrow}$, respectively. The ${}_{\text{lm}}\!\overset{1}{}$-*language of* G is denoted by $L(G, {}_{\text{lm}}\!\overset{1}{\Rightarrow})$ and defined as $L(G, {}_{\text{lm}}\!\overset{1}{\Rightarrow}) = \{w \in T^* \mid S \;{}_{\text{lm}}\!\overset{1}{\Rightarrow}^* w\}$. □

Notice that if the leftmost occurrence of a nonterminal cannot be rewritten by any rule, then the derivation is blocked.

In the second derivation restriction type, during every derivation step, the leftmost occurrence of a nonterminal that can be rewritten has to be rewritten.

Definition 2. Let $G = (N, T, P_L, P_R, S)$ be a one-sided random context grammar. The *type-*2 *direct leftmost derivation relation* over V^*, symbolically denoted by ${}_{\rm lm}{\overset{2}{\Rightarrow}}$, is defined as follows. Let $u, x, v \in V^*$ and $A \in N$. Then, $uAv \ {}_{\rm lm}{\overset{2}{\Rightarrow}} \ uxv$ in G if and only if $uAv \Rightarrow uxv$ in G and there is no $B \in N$ and $y \in V^*$ such that $u = u_1Bu_2$ and $u_1Bu_2Av \Rightarrow u_1yu_2Av$ in G.

Let ${}_{\rm lm}{\overset{2}{\Rightarrow}}^n$ and ${}_{\rm lm}{\overset{2}{\Rightarrow}}^*$ denote the nth power of ${}_{\rm lm}{\overset{2}{\Rightarrow}}$, for some $n \geq 0$, and the reflexive-transitive closure of ${}_{\rm lm}{\overset{2}{\Rightarrow}}$, respectively. The ${}_{\rm lm}{\overset{2}{\Rightarrow}}$*-language of* G is denoted by $L(G, {}_{\rm lm}{\overset{2}{\Rightarrow}})$ and defined as $L(G, {}_{\rm lm}{\overset{2}{\Rightarrow}}) = \{w \in T^* \mid S \ {}_{\rm lm}{\overset{2}{\Rightarrow}}^* \ w\}$. □

In the third derivation restriction type, during every derivation step, a rule is chosen, and the leftmost occurrence of its left-hand side is rewritten.

Definition 3. Let $G = (N, T, P_L, P_R, S)$ be a one-sided random context grammar. The *type-*3 *direct leftmost derivation relation* over V^*, symbolically denoted by ${}_{\rm lm}{\overset{3}{\Rightarrow}}$, is defined as follows. Let $u, x, v \in V^*$ and $A \in N$. Then, $uAv \ {}_{\rm lm}{\overset{3}{\Rightarrow}} \ uxv$ in G if and only if $uAv \Rightarrow uxv$ in G and $\mathrm{alph}(u) \cap \{A\} = \emptyset$.

Let ${}_{\rm lm}{\overset{3}{\Rightarrow}}^n$ and ${}_{\rm lm}{\overset{3}{\Rightarrow}}^*$ denote the nth power of ${}_{\rm lm}{\overset{3}{\Rightarrow}}$, for some $n \geq 0$, and the reflexive-transitive closure of ${}_{\rm lm}{\overset{3}{\Rightarrow}}$, respectively. The ${}_{\rm lm}{\overset{3}{\Rightarrow}}$*-language of* G is denoted by $L(G, {}_{\rm lm}{\overset{3}{\Rightarrow}})$ and defined as $L(G, {}_{\rm lm}{\overset{3}{\Rightarrow}}) = \{w \in T^* \mid S \ {}_{\rm lm}{\overset{3}{\Rightarrow}}^* \ w\}$. □

Notice the following difference between the second and the third type. In the former, a leftmost occurrence of a rewritable nonterminal is chosen first, and then, a choice of a rule with this nonterminal on its let-hand side is made. In the latter, a rule is chosen first, and then, the leftmost occurrence of its left-hand side is rewritten.

2.2 Denotation of Language Families

Throughout the rest of this paper, the language families under discussion are denoted in the following way. The families of context-free languages, context-sensitive languages, and recursively enumerable languages are denoted by $\mathscr{L}^{\varepsilon}_{\rm CF}$, $\mathscr{L}_{\rm CS}$, and $\mathscr{L}^{\varepsilon}_{\rm RE}$, respectively.

The language family generated by one-sided random context grammars is denoted by $\mathscr{L}^{\varepsilon}_{\rm ORC}$. The language families generated by one-sided random context grammars with type-1 leftmost derivations, one-sided random context grammars with type-2 leftmost derivations, and one-sided random context grammars with type-3 leftmost derivations are denoted by $\mathscr{L}^{\varepsilon}_{\rm ORC}({}_{\rm lm}{\overset{1}{\Rightarrow}})$, $\mathscr{L}^{\varepsilon}_{\rm ORC}({}_{\rm lm}{\overset{2}{\Rightarrow}})$, and $\mathscr{L}^{\varepsilon}_{\rm ORC}({}_{\rm lm}{\overset{3}{\Rightarrow}})$, respectively.

The notation without ε stands for the corresponding propagating family. For example, $\mathscr{L}_{\rm ORC}$ denotes the language family generated by propagating one-sided random context grammars.

3 Results

In this section, we prove results I through III, given next.

I. One-sided random context grammars with type-1 leftmost derivations characterize $\mathscr{L}_{\mathrm{CF}}^{\varepsilon}$ (Theorem 1). An analogical result holds for propagating one-sided random context grammars (Theorem 2).
II. One-sided random context grammars with type-2 leftmost derivations characterize $\mathscr{L}_{\mathrm{RE}}^{\varepsilon}$ (Theorem 3). Propagating one-sided random context grammars with type-2 leftmost derivations characterize $\mathscr{L}_{\mathrm{CS}}$ (Theorem 4).
III. One-sided random context grammars with type-3 leftmost derivations characterize $\mathscr{L}_{\mathrm{RE}}^{\varepsilon}$ (Theorem 5). Propagating one-sided random context grammars with type-3 leftmost derivations characterize $\mathscr{L}_{\mathrm{CS}}$ (Theorem 6).

3.1 Type-1 Leftmost Derivations

First, we consider one-sided random context grammars with type-1 leftmost derivations.

Lemma 1. *For every context-free grammar G, there is a one-sided random context grammar H such that $L(H, {}_{\mathrm{lm}}\!\overset{1}{\Rightarrow}) = L(G)$. Furthermore, if G is propagating, then so is H.*

Proof. Let $G = (N, T, P, S)$ be a context-free grammar. Construct the one-sided random context grammar $H = (N, T, P', P', S)$, where

$$P' = \big\{ \lfloor A \rightarrow x, \emptyset, \emptyset \rfloor \mid A \rightarrow x \in P \big\}$$

As the rules in P' have their permitting and forbidding contexts empty, any successful type-1 leftmost derivation in H is also a successful derivation in G, so the inclusion $L(H, {}_{\mathrm{lm}}\!\overset{1}{\Rightarrow}) \subseteq L(G)$ holds. On the other hand, let $w \in L(G)$ be a string successfully generated by G. Then, it is well known that there exists a successful leftmost derivation of w in G. Observe that such a leftmost derivation is also possible in H. Thus, the other inclusion $L(G) \subseteq L(H, {}_{\mathrm{lm}}\!\overset{1}{\Rightarrow})$ holds as well. Finally, notice that whenever G is propagating, then so is H. Hence, the theorem holds. □

Lemma 2. *For every one-sided random context grammar G, there is a context-free grammar H such that $L(H) = L(G, {}_{\mathrm{lm}}\!\overset{1}{\Rightarrow})$. Furthermore, if G is propagating, then so is H.*

Proof. Let $G = (N, T, P_L, P_R, S)$ be a one-sided random context grammar. In what follows, symbols $\langle$ and $\rangle$ are used to clearly unite more symbols into a single compound symbol. Construct the context-free grammar

$$H = \big(N', T, P, \langle S, \emptyset \rangle\big)$$

in the following way. Initially, set $N' = \{\langle A, Q \rangle \mid A \in N,\ Q \subseteq N\}$ and $P = \emptyset$ (without any loss of generality, we assume that $N' \cap V = \emptyset$). Perform (1) and (2), given next:

(1) for each $\lfloor A \rightarrow y_0Y_1y_1Y_2y_2 \cdots Y_hy_h, U, W\rfloor \in P_R$, where $y_i \in T^*$, $Y_j \in N$, for all i and j, $0 \leq i \leq h$, $1 \leq j \leq h$, for some $h \geq 0$, and for each $\langle A, Q\rangle \in N'$ such that $U \subseteq Q$ and $W \cap Q = \emptyset$, add the following rule to P:

$$\begin{aligned}\langle A, Q\rangle \rightarrow y_0 & \langle Y_1, Q \cup \{Y_2, Y_3, \dots, Y_h\}\rangle y_1 \\ & \langle Y_2, Q \cup \{Y_3, \dots, Y_h\}\rangle y_2 \\ & \vdots \\ & \langle Y_h, Q\rangle y_h\end{aligned}$$

(2) for each $\lfloor A \rightarrow y_0Y_1y_1Y_2y_2 \cdots Y_hy_h, \emptyset, W\rfloor \in P_L$, where $y_i \in T^*$, $Y_j \in N$, for all i and j, $0 \leq i \leq h$, $1 \leq j \leq h$, for some $h \geq 0$, and for each $\langle A, Q\rangle \in N'$, add the following rule to P:

$$\begin{aligned}\langle A, Q\rangle \rightarrow y_0 & \langle Y_1, Q \cup \{Y_2, Y_3, \dots, Y_h\}\rangle y_1 \\ & \langle Y_2, Q \cup \{Y_3, \dots, Y_h\}\rangle y_2 \\ & \vdots \\ & \langle Y_h, Q\rangle y_h\end{aligned}$$

Before proving that $L(H) = L(G, {}_{\mathrm{lm}}{\overset{1}{\Rightarrow}})$, let us give an insight into the construction. As G always rewrites the leftmost occurrence of a nonterminal, we use compound nonterminals of the form $\langle A, Q\rangle$ in H, where A is a nonterminal, and Q is a set of nonterminals that appear to the right of this occurrence of A. When simulating rules from P_R, the check for the presence and absence of symbols is accomplished by using Q. Also, when rewriting A in $\langle A, Q\rangle$ to some y, the compound nonterminals from N' are generated instead of nonterminals from N.

Rules from P_L are simulated analogously; however, notice that if the permitting set of such a rule is nonempty, it is never applicable in G. Therefore, such rules are not introduced to P'. Furthermore, since there are no nonterminals to the left of the leftmost occurrence of a nonterminal, no check for their absence is done.

Clearly, $L(G, {}_{\mathrm{lm}}{\overset{1}{\Rightarrow}}) \subseteq L(H)$. The opposite inclusion, $L(H) \subseteq L(G, {}_{\mathrm{lm}}{\overset{1}{\Rightarrow}})$, can be proved by analogy with the proof of Lemma 1 by simulating the leftmost derivation of every $w \in L(H)$ by G. Observe that since the check for the presence and absence of symbols in H is done in the second components of the compound nonterminals, each rule introduced to P in (1) and (2) can be simulated by a rule from P_R and P_L from which it is created.

Finally, notice that whenever G is propagating, then so is H. Hence, the theorem holds. □

Theorem 1. $\mathscr{L}^{\varepsilon}_{\mathrm{ORC}}({}_{\mathrm{lm}}{\overset{1}{\Rightarrow}}) = \mathscr{L}^{\varepsilon}_{\mathrm{CF}}$

Proof. By Lemma 1, $\mathscr{L}^{\varepsilon}_{\mathrm{CF}} \subseteq \mathscr{L}^{\varepsilon}_{\mathrm{ORC}}({}_{\mathrm{lm}}{\overset{1}{\Rightarrow}})$. By Lemma 2, $\mathscr{L}^{\varepsilon}_{\mathrm{ORC}}({}_{\mathrm{lm}}{\overset{1}{\Rightarrow}}) \subseteq \mathscr{L}^{\varepsilon}_{\mathrm{CF}}$. Consequently, $\mathscr{L}^{\varepsilon}_{\mathrm{ORC}}({}_{\mathrm{lm}}{\overset{1}{\Rightarrow}}) = \mathscr{L}^{\varepsilon}_{\mathrm{CF}}$, so the theorem holds. □

Theorem 2. $\mathscr{L}_{\mathrm{ORC}}({}_{\mathrm{lm}}{\overset{1}{\Rightarrow}}) = \mathscr{L}_{\mathrm{CF}}$

Proof. Since it is well-known that any context-free grammar that does not generate the empty string can be converted to an equivalent propagating context-free grammar, this theorem follows from Lemmas 1 and 2. □

3.2 Type-2 Leftmost Derivations

Next, we turn our attention to one-sided random context grammars with type-2 leftmost derivations.

Lemma 3. *For every one-sided random context grammar G, there is a one-sided random context grammar H such that $L(H, {}_{\mathrm{lm}}{}^{2}\!\Rightarrow) = L(G)$. Furthermore, if G is propagating, then so is H.*

Proof. Let $G = (N, T, P_L, P_R, S)$ be a one-sided random context grammar. We construct the one-sided random context grammar H in such a way that always allows it to rewrite an arbitrary occurrence of a nonterminal. Construct

$$H = \big(N', T, P'_L, P'_R, S\big)$$

as follows. Initially, set $\bar{N} = \{\bar{A} \mid A \in N\}$, $\hat{N} = \{\hat{A} \mid A \in N\}$, $N' = N \cup \bar{N} \cup \hat{N}$, and $P'_L = P'_R = \emptyset$ (without any loss of generality, we assume that N, $\bar{N}$, and $\hat{N}$ are pairwise disjoint). Define the function ψ from 2^N to $2^{\bar{N}}$ as $\psi(\emptyset) = \emptyset$ and

$$\psi\big(\{A_1, A_2, \dots, A_n\}\big) = \{\bar{A}_1, \bar{A}_2, \dots, \bar{A}_n\}$$

Perform (1) through (3), given next:

(1) for each $A \in N$,
  (1.1) add $\lfloor A \to \bar{A}, \emptyset, N \cup \hat{N} \rfloor$ to P'_L,
  (1.2) add $\lfloor \bar{A} \to \hat{A}, \emptyset, N \cup \bar{N} \rfloor$ to P'_R,
  (1.3) add $\lfloor \hat{A} \to A, \emptyset, \bar{N} \cup \hat{N} \rfloor$ to P'_R;
(2) for each $\lfloor A \to y, U, W \rfloor \in P_R$, add $\lfloor A \to y, U, W \rfloor$ to P'_R;
(3) for each $\lfloor A \to y, U, W \rfloor \in P_L$, add $\lfloor A \to y, \psi(U), \psi(W) \cup N \cup \hat{N} \rfloor$ to P'_L.

Before proving that $L(H) = L(G)$, let us informally explain (1) through (3). Rules from (2) and (3) simulate the corresponding rules from P_R and P_L, respectively. Rules from (1) allow H to rewrite any occurrence of a nonterminal.

Consider a sentential form $x_1 A x_2$, where $x_1, x_2 \in (N \cup T)^*$ and $A \in N$. To rewrite A in H using type-2 leftmost derivations, all occurrences of nonterminals in x_1 are first rewritten to their barred versions by rules from (1.1). Then, A can be rewritten by a rule from (2) or (3). By rules from (1.1), every occurrence of a nonterminal in the current sentential form is then rewritten to its barred version. Rules from (1.2) then start rewriting barred nonterminals to hatted nonterminals. This is done from the right to the left. Finally, hatted nonterminals are rewritten to their original versions by rules from (1.3). This is also done from the right to the left.

To establish $L(H, {}_{\mathrm{lm}}{}^{2}\!\Rightarrow) = L(G)$, we prove two claims. First, Claim 1 shows how derivations of G are simulated by H. Then, Claim 2 demonstrates the converse—that is, it shows how derivations of H are simulated by G.

Claim 1. If $S \Rightarrow^n x$ in G, where $x \in V^*$, for some $n \geq 0$, then $S \;{}_{\mathrm{lm}}{}^{2}\!\Rightarrow^* x$ in H.

Proof. This claim is established by induction on $n \geq 0$.

Basis. For $n = 0$, this claim obviously holds.

Induction Hypothesis. Suppose that there exists $n \geq 0$ such that the claim holds for all derivations of length ℓ, where $0 \leq \ell \leq n$.

Induction Step. Consider any derivation of the form $S \Rightarrow^{n+1} w$ in G, where $w \in V^*$. Since $n + 1 \geq 1$, this derivation can be expressed as $S \Rightarrow^n x \Rightarrow w$, for some $x \in V^+$. By the induction hypothesis, $S \ {}_{\mathrm{lm}}^{\ 2}\!\!\Rightarrow^* x$ in H. Next, we consider all possible forms of $x \Rightarrow w$ in G, covered by the following two cases—(i) and (ii).

(i) *Application of* $\lfloor A \rightarrow y, U, W \rfloor \in P_R$. Let $x = x_1 A x_2$ and $r = \lfloor A \rightarrow y, U, W \rfloor \in P_R$, where $x_1, x_2 \in V^*$ such that $U \subseteq \mathrm{alph}(x_2)$ and $W \cap \mathrm{alph}(x_2) = \emptyset$, so $x_1 A x_2 \Rightarrow x_1 y x_2$ in G. If $x_1 \in T^*$, then $x_1 A x_2 \ {}_{\mathrm{lm}}^{\ 2}\!\!\Rightarrow x_1 y x_2$ in H by the corresponding rule introduced in (2), and the induction step is completed for (i). Therefore, assume that $\mathrm{alph}(x_1) \cap N \neq \emptyset$. Let $x_1 = z_0 Z_1 z_1 Z_2 z_2 \cdots Z_h z_h$, where $z_i \in T^*$ and $Z_j \in N$, for all i and j, $0 \leq i \leq h$, $1 \leq j \leq h$, for some $h \geq 1$. By rules introduced in (1.1),

$$z_0 Z_1 z_1 Z_2 z_2 \cdots Z_h z_h A x_2 \ {}_{\mathrm{lm}}^{\ 2}\!\!\Rightarrow^* z_0 \bar{Z}_1 z_1 \bar{Z}_2 z_2 \cdots \bar{Z}_h z_h A x_2 \text{ in } H$$

By the corresponding rule to r introduced in (2),

$$z_0 \bar{Z}_1 z_1 \bar{Z}_2 z_2 \cdots \bar{Z}_h z_h A x_2 \ {}_{\mathrm{lm}}^{\ 2}\!\!\Rightarrow z_0 \bar{Z}_1 z_1 \bar{Z}_2 z_2 \cdots \bar{Z}_h z_h y x_2 \text{ in } H$$

By rules introduced in (1.1) through (1.3),

$$z_0 \bar{Z}_1 z_1 \bar{Z}_2 z_2 \cdots \bar{Z}_h z_h y x_2 \ {}_{\mathrm{lm}}^{\ 2}\!\!\Rightarrow^* z_0 Z_1 z_1 Z_2 z_2 \cdots Z_h z_h y x_2 \text{ in } H$$

which completes the induction step for (i).

(ii) *Application of* $\lfloor A \rightarrow y, U, W \rfloor \in P_L$. Let $x = x_1 A x_2$ and $r = \lfloor A \rightarrow y, U, W \rfloor \in P_L$, where $x_1, x_2 \in V^*$ such that $U \subseteq \mathrm{alph}(x_1)$ and $W \cap \mathrm{alph}(x_1) = \emptyset$, so $x_1 A x_2 \Rightarrow x_1 y x_2$ in G. To complete the induction step for (ii), proceed by analogy with (i), but use a rule from (3) instead of a rule from (2).

Observe that cases (i) and (ii) cover all possible forms of $x \Rightarrow w$ in G. Thus, the claim holds. □

Set $V = N \cup T$ and $V' = N' \cup T$. Define the homomorphism τ from V'^* to V^* as $\tau(A) = \tau(\bar{A}) = \tau(\hat{A}) = A$, for all $A \in N$, and $\tau(a) = a$, for all $a \in T$.

Claim 2. If $S \ {}_{\mathrm{lm}}^{\ 2}\!\!\Rightarrow^n x$ *in* H*, where* $x \in V'^*$*, for some* $n \geq 0$*, then* $S \Rightarrow^* \tau(x)$ *in* G*, and either* $x \in (\bar{N} \cup T)^* V^*$*,* $x \in (\bar{N} \cup T)^* (\hat{N} \cup T)^*$*, or* $x \in (\hat{N} \cup T)^* V^*$*.*

Proof. This claim is established by induction on $n \geq 0$.

Basis. For $n = 0$, this claim obviously holds.

Induction Hypothesis. Suppose that there exists $n \geq 0$ such that the claim holds for all derivations of length ℓ, where $0 \leq \ell \leq n$.

Induction Step. Consider any derivation of the form $S\ {}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}^{n+1}\ w$ in H, where $w \in V'^*$. Since $n+1 \geq 1$, this derivation can be expressed as $S\ {}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}^{n}\ x\ {}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}\ w$, for some $x \in V'^+$. By the induction hypothesis, $S \Rightarrow^* \tau(x)$ in G, and either $x \in (\bar{N} \cup T)^* V^*$, $x \in (\bar{N} \cup T)^*(\hat{N} \cup T)^*$, or $x \in (\hat{N} \cup T)^* V^*$. Next, we consider all possible forms of $x\ {}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}\ w$ in H, covered by the following five cases—(i) through (v).

(i) *Application of a rule introduced in* (1.1)*.* Let $\lfloor A \rightarrow \bar{A}, \emptyset, N \cup \hat{N} \rfloor \in P'_L$ be a rule introduced in (1.1). Observe that this rule is applicable only if $x = x_1 A x_2$, where $x_1 \in (\bar{N} \cup T)^*$ and $x_2 \in V^*$. Then,

$$x_1 A x_2\ {}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}\ x_1 \bar{A} x_2 \text{ in } H$$

Since $\tau(x_1 \bar{A} x_2) = \tau(x_1 A x_2)$ and $x_1 \bar{A} x_2 \in (\bar{N} \cup T)^* V^*$, the induction step is completed for (i).

(ii) *Application of a rule introduced in* (1.2)*.* Let $\lfloor \bar{A} \rightarrow \hat{A}, \emptyset, N \cup \bar{N} \rfloor \in P'_R$ be a rule introduced in (1.2). Observe that this rule is applicable only if $x = x_1 \bar{A} x_2$, where $x_1 \in (\bar{N} \cup T)^*$ and $x_2 \in (\hat{N} \cup T)^*$. Then,

$$x_1 \bar{A} x_2\ {}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}\ x_1 \hat{A} x_2 \text{ in } H$$

Since $\tau(x_1 \hat{A} x_2) = \tau(x_1 \bar{A} x_2)$ and $x_1 \hat{A} x_2 \in (\bar{N} \cup T)^*(\hat{N} \cup T)^*$, the induction step is completed for (ii).

(iii) *Application of a rule introduced in* (1.3)*.* Let $\lfloor \hat{A} \rightarrow A, \emptyset, \bar{N} \cup \hat{N} \rfloor \in P'_R$ be a rule introduced in (1.3). Observe that this rule is applicable only if $x = x_1 \hat{A} x_2$, where $x_1 \in (\hat{N} \cup T)^*$ and $x_2 \in V^*$. Then,

$$x_1 \hat{A} x_2\ {}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}\ x_1 A x_2 \text{ in } H$$

Since $\tau(x_1 A x_2) = \tau(x_1 \hat{A} x_2)$ and $x_1 A x_2 \in (\hat{N} \cup T)^* V^*$, the induction step is completed for (iii).

(iv) *Application of a rule introduced in* (2)*.* Let $\lfloor A \rightarrow y, U, W \rfloor \in P'_R$ be a rule introduced in (2) from $\lfloor A \rightarrow y, U, W \rfloor \in P_R$, and let $x = x_1 A x_2$ such that $U \subseteq \mathrm{alph}(x_2)$ and $W \cap \mathrm{alph}(x_2) = \emptyset$. Then,

$$x_1 A x_2\ {}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}\ x_1 y x_2 \text{ in } H$$

and

$$\tau(x_1) A \tau(x_2) \Rightarrow \tau(x_1) y \tau(x_2) \text{ in } G$$

Clearly, $x_1 y x_2$ is of the required form, so the induction step is completed for (iv).

(v) *Application of a rule introduced in* (3)*.* Let $\lfloor A \rightarrow y, \psi(U), \psi(W) \cup N \cup \hat{N} \rfloor \in P'_L$ be a rule introduced in (3) from $\lfloor A \rightarrow y, U, W \rfloor \in P_L$, and let $x = x_1 A x_2$ such that $\psi(U) \subseteq \mathrm{alph}(x_1)$ and $\big(\psi(W) \cup N \cup \hat{N}\big) \cap \mathrm{alph}(x_1) = \emptyset$. Then,

$$x_1 A x_2\ {}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}\ x_1 y x_2 \text{ in } H$$

and

$$\tau(x_1) A \tau(x_2) \Rightarrow \tau(x_1) y \tau(x_2) \text{ in } G$$

Clearly, $x_1 y x_2$ is of the required form, so the induction step is completed for (v).

Observe that cases (i) through (v) cover all possible forms of $x\ {}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}\ w$ in H. Thus, the claim holds. □

We now prove that $L(H, {}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}) = L(G)$. Consider Claim 1 with $x \in T^*$. Then, $S \Rightarrow^* x$ in G implies that $S\ {}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}{}^*\ x$ in H, so $L(G) \subseteq L(H, {}_{\mathrm{lm}}{\overset{2}{\Rightarrow}})$. Consider Claim 2 with $x \in T^*$. Then, $S\ {}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}{}^*\ x$ in H implies that $S \Rightarrow^* x$ in G, so $L(H, {}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}) \subseteq L(G)$. Consequently, $L(H, {}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}) = L(G)$.

Finally, notice that whenever G is propagating, then so is H. Hence, the theorem holds. □

Lemma 4. $\mathscr{L}^{\varepsilon}_{\mathrm{ORC}}({}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}) \subseteq \mathscr{L}^{\varepsilon}_{\mathrm{RE}}$

Proof. This inclusion can be obtained by standard simulations, so we leave the proof to the reader. □

Theorem 3. $\mathscr{L}^{\varepsilon}_{\mathrm{ORC}}({}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}) = \mathscr{L}^{\varepsilon}_{\mathrm{RE}}$

Proof. Since $\mathscr{L}^{\varepsilon}_{\mathrm{ORC}} = \mathscr{L}^{\varepsilon}_{\mathrm{RE}}$ (see Theorem 2 in [27]), Lemma 3 implies that $\mathscr{L}^{\varepsilon}_{\mathrm{RE}} \subseteq \mathscr{L}^{\varepsilon}_{\mathrm{ORC}}({}_{\mathrm{lm}}{\overset{2}{\Rightarrow}})$. By Lemma 4, $\mathscr{L}^{\varepsilon}_{\mathrm{ORC}}({}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}) \subseteq \mathscr{L}^{\varepsilon}_{\mathrm{RE}}$. Consequently, we have that $\mathscr{L}^{\varepsilon}_{\mathrm{ORC}}({}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}) = \mathscr{L}^{\varepsilon}_{\mathrm{RE}}$, so the theorem holds. □

Lemma 5. $\mathscr{L}_{\mathrm{ORC}}({}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}) \subseteq \mathscr{L}_{\mathrm{CS}}$

Proof. Since the length of sentential forms in derivations of propagating one-sided random context grammars is nondecreasing, propagating one-sided random context grammars can be simulated by linear bounded automata. A rigorous proof of this lemma is left to the reader. □

Theorem 4. $\mathscr{L}_{\mathrm{ORC}}({}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}) = \mathscr{L}_{\mathrm{CS}}$

Proof. Since $\mathscr{L}_{\mathrm{ORC}} = \mathscr{L}_{\mathrm{CS}}$ (see Theorem 1 in [27]), Lemma 3 implies that $\mathscr{L}_{\mathrm{CS}} \subseteq \mathscr{L}_{\mathrm{ORC}}({}_{\mathrm{lm}}{\overset{2}{\Rightarrow}})$. By Lemma 5, $\mathscr{L}_{\mathrm{ORC}}({}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}) \subseteq \mathscr{L}_{\mathrm{CS}}$. Consequently, we have that $\mathscr{L}_{\mathrm{ORC}}({}_{\mathrm{lm}}{\overset{2}{\Rightarrow}}) = \mathscr{L}_{\mathrm{CS}}$, so the theorem holds. □

3.3 Type-3 Leftmost Derivations

Finally, we consider one-sided random context grammars with type-3 leftmost derivations.

Lemma 6. *For every one-sided random context grammar G, there is a one-sided random context grammar H such that $L(H, {}_{\mathrm{lm}}{\overset{3}{\Rightarrow}}) = L(G)$. Furthermore, if G is propagating, then so is H.*

Proof. Let $G = (N, T, P_L, P_R, S)$ be a one-sided random context grammar. We prove this lemma by analogy with the proof of Lemma 3. That is, we construct the one-sided random context grammar H in such a way that always allows it to rewrite an arbitrary occurrence of a nonterminal. Construct

$$H = \big(N', T, P'_L, P'_R, S\big)$$

as follows. Initially, set $\bar{N} = \{\bar{A} \mid A \in N\}$, $N' = N \cup \bar{N}$, and $P'_L = P'_R = \emptyset$ (without any loss of generality, we assume that $N \cap \bar{N} = \emptyset$). Define the function ψ from 2^N to $2^{\bar{N}}$ as $\psi(\emptyset) = \emptyset$ and

$$\psi\big(\{A_1, A_2, \ldots, A_n\}\big) = \{\bar{A}_1, \bar{A}_2, \ldots, \bar{A}_n\}$$

Perform (1) through (3), given next:

(1) for each $A \in N$,
  (1.1) add $\lfloor A \to \bar{A}, \emptyset, N \rfloor$ to P'_L;
  (1.2) add $\lfloor \bar{A} \to A, \emptyset, \bar{N} \rfloor$ to P'_R;
(2) for each $\lfloor A \to y, U, W \rfloor \in P_R$, add $\lfloor A \to y, U, W \rfloor$ to P'_R;
(3) for each $\lfloor A \to y, U, W \rfloor \in P_L$, let $U = \{X_1, X_2, \ldots, X_k\}$, and for each

$$U' \in \big\{\{Y_1, Y_2, \ldots, Y_k\} \mid Y_i \in \{X_i, \bar{X}_i\}, 1 \leq i \leq k\big\}$$

add $\lfloor A \to y, U', W \cup \Psi(W) \rfloor$ to P'_L ($U' = \emptyset$ if and only if $U = \emptyset$).

Before proving that $L(G) = L(H, {}_{\mathrm{lm}}\!\overset{3}{\Rightarrow})$, let us give an insight into the construction. Rules introduced in (1) allow H to rewrite an arbitrary occurrence of a nonterminal. Rules from (2) and (3) simulate the corresponding rules from P_R and P_L, respectively.

Consider a sentential form $x_1 A x_2$, where $x_1, x_2 \in (N \cup T)^*$ and $A \in N$, and a rule, $r = \lfloor A \to y, U, W \rfloor \in P'_L \cup P'_R$, introduced in (2) or (3). If $A \in \mathrm{alph}(x_1)$, all occurrences of nonterminals in x_1 are rewritten to their barred versions by rules from (1). Then, r is applied, and all barred nonterminals are rewritten back to their non-barred versions. Since not all occurrences of nonterminals in x_1 need to be rewritten to their barred versions before r is applied, all combinations of barred and non-barred nonterminals in the left permitting contexts of the resulting rules in (3) are considered.

The identity $L(H, {}_{\mathrm{lm}}\!\overset{3}{\Rightarrow}) = L(G)$ can be established by analogy with the proof given in Lemma 3, and we leave its proof to the reader. Finally, notice that whenever G is propagating, then so is H. Hence, the theorem holds. □

Lemma 7. $\mathscr{L}^{\varepsilon}_{\mathrm{ORC}}({}_{\mathrm{lm}}\!\overset{3}{\Rightarrow}) \subseteq \mathscr{L}^{\varepsilon}_{\mathrm{RE}}$

Proof. This inclusion can be obtained by standard simulations, so we leave the proof to the reader. □

Theorem 5. $\mathscr{L}^{\varepsilon}_{\mathrm{ORC}}({}_{\mathrm{lm}}\!\overset{3}{\Rightarrow}) = \mathscr{L}^{\varepsilon}_{\mathrm{RE}}$

Proof. Since $\mathscr{L}^{\varepsilon}_{\mathrm{ORC}} = \mathscr{L}^{\varepsilon}_{\mathrm{RE}}$ (see Theorem 2 in [27]), Lemma 6 implies that $\mathscr{L}^{\varepsilon}_{\mathrm{RE}} \subseteq \mathscr{L}^{\varepsilon}_{\mathrm{ORC}}({}_{\mathrm{lm}}\!\overset{3}{\Rightarrow})$. By Lemma 7, $\mathscr{L}^{\varepsilon}_{\mathrm{ORC}}({}_{\mathrm{lm}}\!\overset{3}{\Rightarrow}) \subseteq \mathscr{L}^{\varepsilon}_{\mathrm{RE}}$. Consequently, we have that $\mathscr{L}^{\varepsilon}_{\mathrm{ORC}}({}_{\mathrm{lm}}\!\overset{3}{\Rightarrow}) = \mathscr{L}^{\varepsilon}_{\mathrm{RE}}$, so the theorem holds. □

Lemma 8. $\mathscr{L}_{\mathrm{ORC}}({}_{\mathrm{lm}}\!\overset{3}{\Rightarrow}) \subseteq \mathscr{L}_{\mathrm{CS}}$

Proof. This lemma can be established by analogy with the proof of Lemma 5.

Theorem 6. $\mathscr{L}_{\mathrm{ORC}}({}_{\mathrm{lm}}\!\overset{3}{\Rightarrow}) = \mathscr{L}_{\mathrm{CS}}$

Proof. Since $\mathscr{L}_{\mathrm{ORC}} = \mathscr{L}_{\mathrm{CS}}$ (see Theorem 1 in [27]), Lemma 6 implies that $\mathscr{L}_{\mathrm{CS}} \subseteq \mathscr{L}_{\mathrm{ORC}}({}_{\mathrm{lm}}\!\overset{3}{\Rightarrow})$. By Lemma 8, $\mathscr{L}_{\mathrm{ORC}}({}_{\mathrm{lm}}\!\overset{3}{\Rightarrow}) \subseteq \mathscr{L}_{\mathrm{CS}}$. Consequently, we have that $\mathscr{L}_{\mathrm{ORC}}({}_{\mathrm{lm}}\!\overset{3}{\Rightarrow}) = \mathscr{L}_{\mathrm{CS}}$, so the theorem holds. □

4 Concluding Remarks

In this final section, we compare the results achieved in the previous section with some well-known results of formal language theory. More specifically, we relate the language families generated by one-sided random context grammars with leftmost derivations to the language families generated by random context grammars with leftmost derivations (in what follows, by random context grammars, we always mean random context grammars with both permitting and forbidding contexts, see [9] for the details).

The language families generated by random context grammars, random context grammars with type-1 leftmost derivations, random context grammars with type-2 leftmost derivations, and random context grammars with type-3 leftmost derivations are denoted by $\mathscr{L}^{\varepsilon}_{\mathrm{RC}}$, $\mathscr{L}^{\varepsilon}_{\mathrm{RC}}({}_{\mathrm{lm}}\!\overset{1}{\Rightarrow})$, $\mathscr{L}^{\varepsilon}_{\mathrm{RC}}({}_{\mathrm{lm}}\!\overset{2}{\Rightarrow})$, and $\mathscr{L}^{\varepsilon}_{\mathrm{RC}}({}_{\mathrm{lm}}\!\overset{3}{\Rightarrow})$, respectively (see [9] for the definitions of all these families). The notation without ε stands for the corresponding propagating family. For example, $\mathscr{L}_{\mathrm{RC}}$ denotes the language family generated by propagating random context grammars.

The fundamental relationships between these families are summarized next.

Corollary 1. $\mathscr{L}^{\varepsilon}_{\mathrm{CF}} \subset \mathscr{L}_{\mathrm{RC}} \subset \mathscr{L}_{\mathrm{ORC}} = \mathscr{L}_{\mathrm{CS}} \subset \mathscr{L}^{\varepsilon}_{\mathrm{ORC}} = \mathscr{L}^{\varepsilon}_{\mathrm{RC}} = \mathscr{L}^{\varepsilon}_{\mathrm{RE}}$

Proof. This corollary follows from Theorems 1 and 2 in [27] and from Theorems 1.2.4 and 1.2.5 in [9]. □

Considering type-1 leftmost derivations, we significantly decrease the power of both one-sided random context grammars and random context grammars.

Corollary 2. $\mathscr{L}^{\varepsilon}_{\mathrm{ORC}}({}_{\mathrm{lm}}\!\overset{1}{\Rightarrow}) = \mathscr{L}^{\varepsilon}_{\mathrm{RC}}({}_{\mathrm{lm}}\!\overset{1}{\Rightarrow}) = \mathscr{L}^{\varepsilon}_{\mathrm{CF}}$

Proof. This corollary follows from Theorem 1 in the previous section and from Theorem 1.4.1 in [9]. □

Type-2 leftmost derivations increase the generative power of propagating random context grammars, but the generative power of random context grammars remains unchanged.

Corollary 3

(i) $\mathscr{L}_{\mathrm{ORC}}({}_{\mathrm{lm}}\!\overset{2}{\Rightarrow}) = \mathscr{L}_{\mathrm{RC}}({}_{\mathrm{lm}}\!\overset{2}{\Rightarrow}) = \mathscr{L}_{\mathrm{CS}}$

(ii) $\mathscr{L}^{\varepsilon}_{\mathrm{ORC}}({}_{\mathrm{lm}}\!\overset{2}{\Rightarrow}) = \mathscr{L}^{\varepsilon}_{\mathrm{RC}}({}_{\mathrm{lm}}\!\overset{2}{\Rightarrow}) = \mathscr{L}^{\varepsilon}_{\mathrm{RE}}$

Proof. This corollary follows from Theorems 3 and 4 in the previous section and from Theorem 1.4.4 in [9]. □

Finally, type-3 leftmost derivations are not enough for propagating random context grammars to generate the family of context-sensitive languages.

Corollary 4

(i) $\mathscr{L}_{\mathrm{RC}}({}_{\mathrm{lm}}\!\overset{3}{\Rightarrow}) \subset \mathscr{L}_{\mathrm{ORC}}({}_{\mathrm{lm}}\!\overset{3}{\Rightarrow}) = \mathscr{L}_{\mathrm{CS}}$

(ii) $\mathscr{L}^{\varepsilon}_{\mathrm{ORC}}({}_{\mathrm{lm}}\!\overset{3}{\Rightarrow}) = \mathscr{L}^{\varepsilon}_{\mathrm{RC}}({}_{\mathrm{lm}}\!\overset{3}{\Rightarrow}) = \mathscr{L}^{\varepsilon}_{\mathrm{RE}}$

Proof. This corollary follows from Theorems 5 and 6 in the previous section, from Theorem 1.4.5 in [9], and from Remarks 5.11 in [10]. □

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