Analysis and Synchronization of the Hyperchaotic Yujun Systems via Sliding Mode Control

Sundarapandian Vaidyanathan

R & D Centre, Vel Tech Dr. RR & Dr. SR Technical University
Avadi-Alamathi Road, Avadi, Chennai-600 062, India
sundarvtu@gmail.com
http://www.vel-tech.org/

Abstract. In this paper, we deploy sliding mode control (SMC) to derive new results for the global chaos synchronization of identical hyperchaotic Yujun systems (2010). The synchronization results derived in this paper are established using the Lyapunov stability theory. Numerical simulations have been provided to illustrate the sliding mode control results derived in this paper for the complete synchronization of identical hyperchaotic Yujun systems.

Keywords: Sliding mode control, chaos synchronization, hyperchaos, hyperchaotic Yujun system.

1 Introduction

Chaotic systems are nonlinear dynamical systems that are characterized by sensitive dependence on initial conditions and by having evolution through phase space that appears to be quite random. This sensitive dependence on initial conditions is commonly called as the *butterfly effect* [1].

Chaos theory has been applied to a variety of fields including physical systems [2], chemical systems [3], ecological systems [4], secure communications ([5]-[7]) etc.

Since the pioneering work by Pecora and Carroll ([8], 1990), chaos synchronization problem has been studied extensively in the literature. In the last two decades, various control schemes have been developed and successfully applied for the chaos synchronization such as PC method [8], OGY method [9], active control method ([10]-[13]), adaptive control method ([14]-[17]), time-delay feedback method [18], backstepping design method ([19]-[20]), sampled-data feedback synchronization method ([21]-[22]) etc.

In this paper, we adopt the *master-slave* formalism of the chaos synchronization approaches. If we call a particular chaotic system as the *master* system and another chaotic system as the *slave* system, then the goal of the global chaos synchronization is to use the output of the master system to control the slave system so that the states of the slave system track asymptotically the states of the master system. In other words, global chaos synchronization is achieved when the difference of the states of master and slave systems converge to zero asymptotically with time.

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In this paper, we derive new results based on the sliding mode control ([23]-[25]) for the global chaos synchronization of identical hyperchaotic Yujun systems ([26], Yujun *et al.* 2010).

The sliding mode control is often adopted in robust control theory due to its inherent advantages of easy system realization, fast response and good transient performance. The sliding mode control results are also insensitive to parameter uncertainties and external disturbances.

This paper has been organized as follows. In Section 2, we describe the problem statement and our methodology using sliding mode control. In Section 3, we describe an analysis of the hyperchaotic Yujun system (2010). In Section 4, we discuss the sliding mode controller design for the global chaos synchronization of identical hyperchaotic Yujun systems (2010). Section 5 contains the conclusions of this paper.

2 Problem Statement and Our Methodology Using Sliding Mode Control

2.1 Problem Statement

Consider the chaotic system described by

$$\dot{x} = Ax + f(x) \tag{1}$$

where $x \in \mathbb{R}^n$ is the state of the system, A is the $n \times n$ matrix of the system parameters and $f : \mathbb{R}^n \to \mathbb{R}^n$ is the nonlinear part of the system. We take the system (1) as the *master* system.

As the *slave* system, we consider the following chaotic system described by the dynamics

$$\dot{y} = Ay + f(y) + u \tag{2}$$

where $y \in \mathbb{R}^n$ is the state of the system and $u \in \mathbb{R}^m$ is the controller of the slave system.

If we define the *synchronization error* e as

$$e = y - x, \tag{3}$$

then the error dynamics is obtained as

$$\dot{e} = Ae + \eta(x, y) + u, \quad \text{where} \quad \eta(x, y) = f(y) - f(x) \tag{4}$$

The objective of the global chaos synchronization problem is to find a controller u such that

$$\lim_{t \to \infty} \|e(t)\| = 0 \quad \text{for all initial conditions} \quad e(0) \in \mathbb{R}^n \tag{5}$$

2.2 Our Methodology

First, we define the control u as

$$u(t) = -\eta(x, y) + Bv(t) \tag{6}$$

where B is a constant gain vector selected such that (A, B) is controllable.

Substituting (6) into (4), the error dynamics becomes

$$\dot{e} = Ae + Bv \tag{7}$$

which is a linear time-invariant control system with single input v.

Thus, we have shown that the original global chaos synchronization problem is equivalent to the problem of stabilizing the zero solution $e = 0$ of the linear system (7) by a suitable choice of the sliding mode control.

In the sliding mode control, we define the variable

$$s(e) = Ce = c_1e_1 + c_2e_2 + \cdots + c_ne_n \tag{8}$$

where $C = [\, c_1 \quad c_2 \quad \cdots \quad c_n \,]$ is a constant vector to be determined.

In the sliding mode control, we constrain the motion of the system (7) to the sliding manifold defined by

$$S = \{x \in \mathbb{R}^n \mid s(e) = 0\} = \{x \in \mathbb{R}^n \mid c_1e_1 + c_2e_2 + \cdots + c_ne_n = 0\}$$

which is required to be invariant under the flow of the error dynamics (7).

When in sliding manifold S, the system (7) satisfies the following conditions:

$$s(e) = 0 \tag{9}$$

which is the defining equation for the manifold S and

$$\dot{s}(e) = 0 \tag{10}$$

which is the necessary condition for the state trajectory $e(t)$ of the system (7) to stay on the sliding manifold S.

Using (7) and (8), the equation (10) can be rewritten as

$$\dot{s}(e) = C\,[Ae + Bv] = 0 \tag{11}$$

Solving (11), we obtain the equivalent control law given by

$$v_{\text{eq}}(t) = -(CB)^{-1}CAe(t) \tag{12}$$

where C is chosen such that $CB \neq 0$.

Substituting (12) into the error dynamics (7), we get the closed-loop dynamics as

$$\dot{e} = [I - B(CB)^{-1}C]Ae \tag{13}$$

where C is chosen such that the system matrix $[I - B(CB)^{-1}C]A$ is Hurwitz.

Then the system dynamics(13) is globally asymptotically stable.

To design the sliding mode controller for the linear time-invariant system (7), we use the constant plus proportional rate reaching law

$$\dot{s} = -q\text{sgn}(s) - ks \tag{14}$$

where $\text{sgn}(\cdot)$ denotes the sign function and the gains $q > 0, k > 0$ are determined such that the sliding condition is satisfied and sliding motion will occur.

From equations (11) and (14), we obtain the control $v(t)$ as

$$v(t) = -(CB)^{-1}[C(kI + A)e + q\text{sgn}(s)] \tag{15}$$

Theorem 1. *The master system (1) and the slave system (2) are globally and asymptotically synchronized for all initial conditions $x(0), y(0) \in \mathbb{R}^n$ by the feedback control law*

$$u(t) = -\eta(x, y) + Bv(t) \tag{16}$$

where $v(t)$ is defined by (15) and B is a column vector such that (A, B) is controllable. Also, the sliding mode gains k, q are positive.

Proof. First, we note that substituting (16) and (15) into the error dynamics (7), we obtain the closed-loop dynamics as

$$\dot{e} = Ae - B(CB)^{-1}[C(kI + A)e + q\text{sgn}(s)] \tag{17}$$

To prove that the error dynamics (17) is globally asymptotically stable, we consider the candidate Lyapunov function defined by the equation

$$V(e) = \frac{1}{2}s^2(e) \tag{18}$$

which is a positive definite function on $\mathbb{R}^n$.

Differentiating V along the trajectories of (17) or the equivalent dynamics (14), we obtain

$$\dot{V}(e) = s(e)\dot{s}(e) = -ks^2 - q\text{sgn}(s) \tag{19}$$

which is a negative definite function on $\mathbb{R}^n$.

Thus, by Lyapunov stability theory [27], it is immediate that the error dynamics (17) is globally asymptotically stable for all initial conditions $e(0) \in \mathbb{R}^n$.

This completes the proof. □

3 Analysis of the Hyperchaotic Yujun System

The 4-D Yujun dynamics is described by

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_2x_3 \\
\dot{x}_2 &= cx_1 - x_2 - x_1x_3 + x_4 \\
\dot{x}_3 &= x_1x_2 - bx_3 \\
\dot{x}_4 &= -x_1x_3 + rx_4
\end{aligned} \tag{20}$$

where x_1, x_2, x_3, x_4 are the states and a, b, c, r are constant, positive parameters of the system.

It has been shown by Yujun *et al.* [26] that the system (20) exhibits hyperchaotic behaviour when the parameter values are taken as

$$a = 35, \quad b = \frac{8}{3}, \quad c = 55, \quad 0.41 < r \leq 3 \tag{21}$$

When $r = 15$, the system (20) has the Lyapunov exponents

$$\lambda_1 = 1.4944, \quad \lambda_2 = 0.5012, \quad \lambda_3 = 0, \quad \lambda_4 = -38.9264$$

Since the system (20) has two positive Lyapunov exponents *viz.* λ_1 and λ_2, it is hyperchaotic.

The phase portrait of the hyperchaotic Yujun system is depicted in Figure 1.

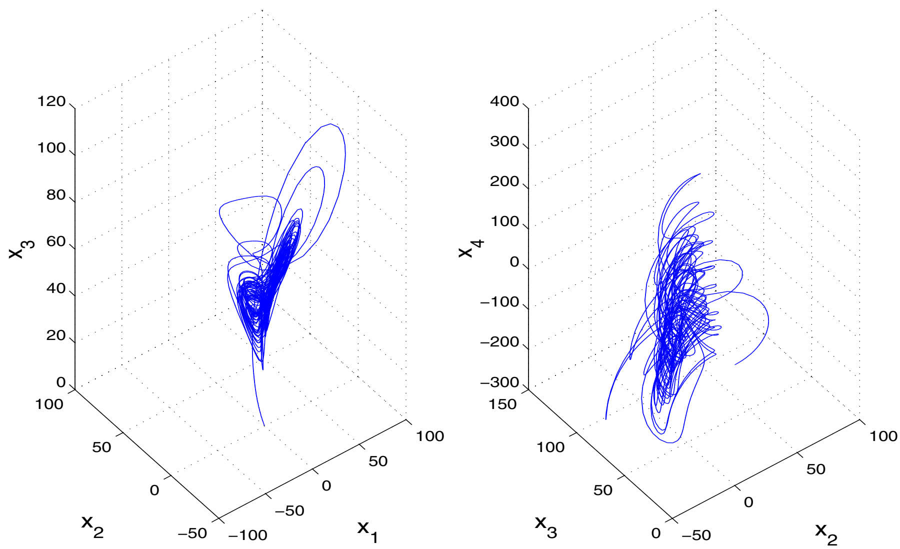

Fig. 1. State Portrait of the Hyperchaotic Lorenz System

4 Global Chaos Synchronization of the Identical Hyperchaotic Yujun Systems

4.1 Main Results

In this section, we apply the sliding mode control results derived in Section 2 for the global chaos synchronization of identical hyperchaotic Yujun systems ([26], 2010).

Thus, the master system is described by the hyperchaotic Yujun dynamics

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\ \dot{x}_2 &= c x_1 - x_2 - x_1 x_3 + x_4 \\ \dot{x}_3 &= x_1 x_2 - b x_3 \\ \dot{x}_4 &= -x_1 x_3 + r x_4\end{aligned} \tag{22}$$

where x_1, x_2, x_3, x_4 are the states of the system and a, b, c, r are the constant, positive parameters of the system.

The slave system is also described by the hyperchaotic Lorenz dynamics

$$\begin{aligned}\dot{y}_1 &= a(y_2 - y_1) + y_2 y_3 + u_1 \\ \dot{y}_2 &= c y_1 - y_2 - y_1 y_3 + y_4 + u_2 \\ \dot{y}_3 &= y_1 y_2 - b y_3 + u_3 \\ \dot{y}_4 &= -y_1 y_3 + r y_4 + u_4\end{aligned} \tag{23}$$

where y_1, y_2, y_3, y_4 are the states of the system and u_1, u_2, u_3, u_4 are the controllers to be designed.

The chaos synchronization error e is defined by

$$e_i = y_i - x_i, \quad (i = 1, 2, 3, 4) \tag{24}$$

The error dynamics is easily obtained as

$$\begin{aligned}\dot{e}_1 &= a(e_2 - e_1) + y_2 y_3 - x_2 x_3 + u_1 \\ \dot{e}_2 &= c e_1 - e_2 + e_4 - y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 &= -b e_3 + y_1 y_2 - x_1 x_2 + u_3 \\ \dot{e}_4 &= r e_4 - y_1 y_3 + x_1 x_3 + u_4\end{aligned} \tag{25}$$

We can write the error dynamics (25) in the matrix notation as

$$\dot{e} = Ae + \eta(x, y) + u \tag{26}$$

where the associated matrices are

$$A = \begin{bmatrix} -a & a & 0 & 0 \\ c & -1 & 0 & 1 \\ 0 & 0 & -b & 0 \\ 0 & 0 & 0 & r \end{bmatrix}, \quad \eta(x, y) = \begin{bmatrix} y_2 y_3 - x_2 x_3 \\ -y_1 y_3 + x_1 x_3 \\ y_1 y_2 - x_1 x_2 \\ -y_1 y_3 + x_1 x_3 \end{bmatrix} \text{ and } u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \tag{27}$$

The sliding mode controller design is carried out as detailed in Section 2.

First, we set u as

$$u = -\eta(x, y) + Bv \tag{28}$$

where B is chosen such that (A, B) is controllable. We take B as

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{29}$$

In the hyperchaotic case, the parameter values are

$$a = 35, \quad b = 8/3, \quad c = 55 \quad \text{and} \quad r = 1.5$$

The sliding mode variable is selected as

$$s = Ce = [\, -1 \quad -2 \qquad 0 \quad 1 \,]\, e \tag{30}$$

which makes the sliding mode state equation asymptotically stable.

We choose the sliding mode gains as $k = 6$ and $q = 0.2$.

We remark that a large value of k can cause chattering and q must be chosen appropriately to speed up the time taken to reach the sliding manifold as well as to reduce the system chattering.

From equation (15), we can obtain $v(t)$ as

$$v(t) = -40.5e_1 - 22.5e_2 + 2.75e_4 + 0.1\,\text{sgn}(s) \tag{31}$$

Thus, the required sliding mode controller is obtained as

$$u(t) = -\eta(x, y) + Bv(t) \tag{32}$$

where $\eta(x, y)$, B and $v(t)$ are defined in equations (27), (29) and (31).

By Theorem 1, we obtain the following result.

Theorem 2. *The identical hyperchaotic Yujun systems (22) and (23) are globally and asymptotically synchronized for all initial conditions with the sliding mode controller u defined by (32).* □

4.2 Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-8}$ is used to solve the hyperchaotic Yujun systems (22) and (23) with the sliding mode controller u given by (32) using MATLAB.

For the hyperchaotic Lorenz systems, the parameter values are taken as

$$a = 35, \quad b = 8/3, \quad c = 55, \quad r = 1.5$$

The sliding mode gains are chosen as $k = 6$ and $q = 0.2$.

The initial values of the master system (22) are taken as

$$x_1(0) = 2, \;\; x_2(0) = 17, \;\; x_3(0) = 22, \;\; x_4(0) = 16$$

and the initial values of the slave system (23) are taken as

$$y_1(0) = 14, \;\; y_2(0) = 26, \;\; y_3(0) = 38, \;\; y_4(0) = 5$$

Figure 2 depicts the synchronization of the hyperchaotic Yujun systems (22) and (23).

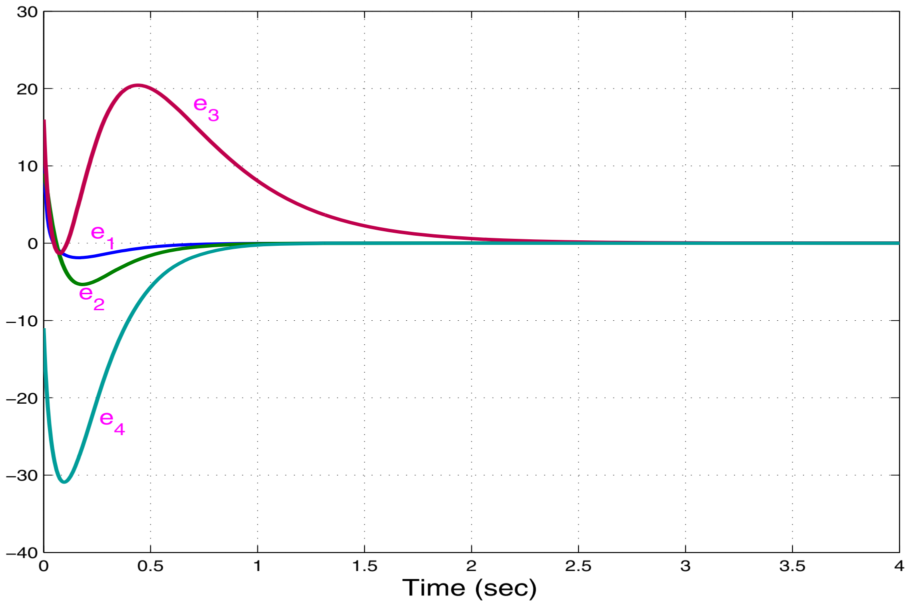

Fig. 2. Synchronization of the Identical Hyperchaotic Yujun Systems

5 Conclusions

In this paper, we have used sliding mode control (SMC) to achieve global chaos synchronization for the identical hyperchaotic Yujun systems (2010). Our synchronization results for the identical hyperchaotic Yujun systems have been established using the Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the sliding mode control method is very effective and convenient to achieve global chaos synchronization for identical hyperchaotic Yujun systems. Numerical simulations have been shown to demonstrate the effectiveness of the synchronization results derived in this paper using sliding mode control.

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