A Modified Satellite Selection Algorithm Based on Satellite Contribution for GDOP in GNSS

Guangyao Li[1,2], Chengdong Xu[1,2], Pengfei Zhang[2], and Chunsheng Hu[2]

[1] Key Laboratory of Dynamics and Control of Flight Vehicle, Ministry of Education of the People's Republic of China
[2] School of Aerospace Engineering, Beijing Institute of Technology, No.5 South Zhongguancun Street, Haidian District, Beijing, 100081, China
Liguangyao283@163.com

Abstract. In Global Navigation Satellite System (GNSS), the visible satellites combination which has a better geometry distribution can provide better positioning accuracy. Geometry Dilution of Precision (GDOP) can be used to measure the positioning accuracy. The direct satellite selection algorithm (DSSA) is a time-consuming algorithm because it would calculate all GDOP values associated with different combinations to choose the smallest one. In this paper, based on each satellite's contribution for GDOP, a modified satellite selection algorithm (MSSA) is proposed which is dependent on a fixed contribution value. Through the theory analysis, the computational complexity of the MSSA is far less than that of the DSSA. Meanwhile, the result of simulation experiment indicates that the calculation precision of the MSSA is almost close to that of the DSSA.

Keywords: GNSS, satellite contribution value, modified satellite selection algorithm, GDOP.

1 Introduction

Global Navigation Satellite System (GNSS) is a time measuring and positioning navigation system based on satellite radio. It can provide positioning data with different accuracy online or offline for different users on aeronautics, astronautics, land, and sea. With the development of GNSSs, the number of global navigation satellites continues to increase[1]. Thus, time and coordinate transformations among different GNSSs should be taken into account for calculation, and they have been researched in some papers[2][3]. The combination of different GNSSs will provide more visible satellites in the same epoch moment. This will improve the navigation accuracy and reliability, but the computational complexities will increase exponentially[4]. Furthermore, the requirement of calculation speed is increasing for receivers of users in engineering, especially for high dynamic users' receivers, and this will add the receivers' burden. For satellite navigation and positioning calculation, when the number of visible satellites used to calculate reaches a certain value, the improvement of positioning accuracy is inconspicuous[5]. In addition, for the number of the receiver channels is restricted, it needs to choose a combination

D. Jin and S. Lin (Eds.): Advances in Mechanical and Electronic Engineering, LNEE 176, pp. 415–421.
springerlink.com

which has a good geometry distribution for calculation. Thus, how to choose the visible satellites to form the combination fast is significant. Geometry Dilution of Precision (GDOP) is used to be a measurement for satellite selection. Based on each satellite's contribution for GDOP, a modified satellite selection algorithm (MSSA) is proposed in this paper.

2 Related Knowledge of GDOP

2.1 The Geometrical Meaning of GDOP

The accuracy of positioning is decided by two main factors: one is observations' accuracy which is caused by the navigation satellite itself, the signal transmission error and the receiver's error; the other is geometry distribution of visible satellites. The amplification degree of measurement error which caused by geometry distribution of visible satellites is called GDOP[6][7]. A smaller GDOP value indicates that the geometry is better, which yields a better positioning accuracy. The more dispersed the distribution of satellites is, the smaller the GDOP value is. On the contrary, the more concentrated the distribution of satellites is, the larger the GDOP value is. It can be shown in Fig. 1.

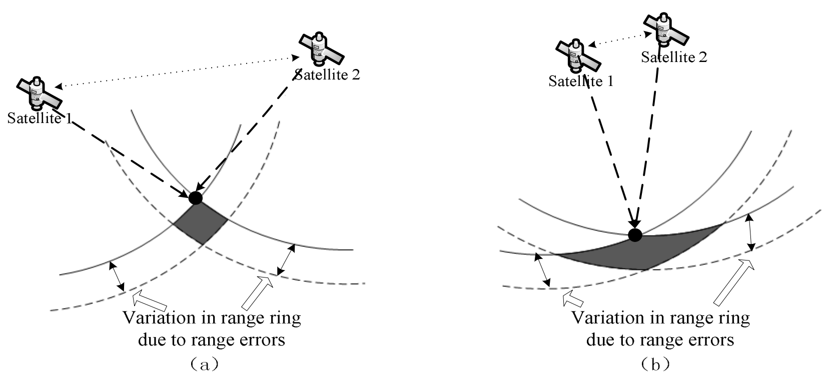

Fig. 1. Relative geometry of satellites and possible locations of receivers

The range between a satellite and a receiver is measured including an error. The solid range rings in Fig. 1 are formed by the true range between a satellite and a receiver, and the regions between the solid range ring and the dashed range ring indicate the error bands. Furthermore, the shaded regions indicate the set of locations of receivers with error. With the same measurement error variation, the accuracy of the computed location is very different for the two cases. The Fig.1 (a) is the situation that the GDOP value is smaller. We can see the range of satellite 1 and satellite 2 is larger, and the error band is smaller. The Fig.1 (b) is the situation that the GDOP value is bigger, and the error band is also bigger. The situation of more satellites is similar to that of the two satellites. This is the geometrical meaning of GDOP.

2.2 The Calculation Method of GDOP

In GNSS, in order to calculate the receiver's 3-dimensional coordinate (x_u, y_u, z_u) and the clock offset t_u, it needs at least four satellites. The formula (1) can be used to calculate single pseudorange ρ_j [6]. The (x_j, y_j, z_j) denotes the *j*th satellite's position in three dimensions.

$$\rho_j = \sqrt{(x_j - x_u)^2 + (y_j - y_u)^2 + (z_j - z_u)^2} + ct_u = f(x_u, y_u, z_u, t_u) \tag{1}$$

It is known that the unknown user's position and the unknown receiver's clock offset can be considered to consist of an approximate component and an incremental component. This can be shown in formula (2).

$$f(x_u, y_u, z_u, t_u) = f(\hat{x}_u + \triangle x_u, \hat{y}_u + \triangle y_u, \hat{z}_u + \triangle z_u, \hat{t}_u + \triangle t_u) \tag{2}$$

When the number of satellites is equal to or more than four, the pseudorange equations can be linearized with Taylor's series expansion at the approximate point and associated predicted receiver clock offset (x_u, y_u, z_u, t_u). It can be expressed as formula (3).

$$\begin{bmatrix} \triangle\rho_1 \\ \triangle\rho_2 \\ \triangle\rho_3 \\ \vdots \\ \triangle\rho_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 1 \\ a_{21} & a_{22} & a_{23} & 1 \\ a_{31} & a_{32} & a_{33} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & 1 \end{bmatrix} \begin{bmatrix} \triangle x_u \\ \triangle y_u \\ \triangle z_u \\ -c\triangle t_u \end{bmatrix} + \begin{bmatrix} v_{\rho 1} \\ v_{\rho 2} \\ v_{\rho 3} \\ \vdots \\ v_{\rho n} \end{bmatrix} \tag{3}$$

In the formula (3), $\triangle\rho_j$ is the pseudorange increment and $(\triangle x_u, \triangle y_u, \triangle z_u)$ is the user's 3-dimensional coordinate increment. $v_{\rho i}$ denotes a random noise with an expected value of *0*. *H* is the $n\times 4$ matrix and (a_{j1}, a_{j2}, a_{j3}) are the unit vectors pointing from the linearization point to the location of the *i*th satellite. GDOP can be calculated by matrix *H*, and the calculation formula is as follows:

$$GDOP = \sqrt{trace(H^T \bullet H)^{-1}} \tag{4}$$

It involves matrix inversion and matrix multiplication, whose computational complexity will be greatly increased with the dimensions of matrix *H* growing.

3 Satellite Selection Algorithm

As mentioned earlier, it only needs to choose the combination which has a good geometry distribution for calculation. The combination with minimum GDOP value is considered as the best one. The direct satellite selection algorithm (DSSA) is to calculate all GDOP values of different combinations, but the computational complexity is large. In order to reduce the computational complexity, the researchers have proposed many approximate calculation methods, such as the neural network algorithm, the support vector machine, the genetic algorithm and so on[8]. They have reduced the computational complexity partly. In this paper, visible satellites are selected based on each satellite's contribution for GDOP of the combination. For avoiding too many matrix inversions and matrix multiplications, it can improve the efficiency of calculation.

3.1 Single Satellite's Contribution for GDOP

Supposing that H_m is an observation matrix of m satellites, and the observation matrix that removed the *i*th satellite from the m satellites is H_{m-1}^i. As pointed out in [5], the relationship between the two matrixes is as follow:

$$H_m^T H_m = {H_{m-1}^i}^T H_{m-1}^i + h_i^T h_i \tag{5}$$

Recording $(H_m^T H_m)^{-1}$ as G_m, the following formula can be got according to Sherman-Morrison formula[5]:

$$G_{m-1}^i = ({H_{m-1}^i}^T H_{m-1}^i)^{-1} = (H_m^T H_m - h_i^T h_i)^{-1} = G_m + G_m {h^T}_i (1 - h_i G_m h_i^T)^{-1} h_i G_m \tag{6}$$

where $(1 - h_i G_m h_i^T)$ is a scalar, and recorded as λ_{mi}. The further formula about the GDOP values of the $m-1$ satellites and the m satellites is as follow:

$$ {GDOP_{m-1}^i}^2 = trace G_{m-1}^i = GDOP_m^2 + trace(G_m h_i^T h_i G_m / \lambda_{mi}) \tag{7}$$

It can also be expressed as follows:

$$ {GDOP_{m-1}^i}^2 - GDOP_m^2 = trace(G_m h_i^T h_i G_m / \lambda_{mi}) \tag{8}$$

From the formula, it can be seen that the changed value of $GDOP^2$ resulted from the *i*th satellite is $trace(G_m h_i^T h_i G_m / \lambda_{mi})$, which is recorded as ΔG. The bigger the ΔG_i is, the larger the *i*th satellite's contribution for $GDOP_m$ is. And it indicates that the *i*th satellite should not be removed because the $GDOP_m$ will change largely. So the *r* satellites with higher ΔG can be chosen from *m* satellites, and form a combination which has a better geometry distribution to replace the all visible satellites. This method can reduce the times of matrix inversion and matrix multiplication, so the calculation efficiency will be improved.

3.2 Satellite Selection Algorithm Design

Obviously, at a certain moment, the GDOP value associated with *m* satellites changes very little when removing off a few satellites for calculation. The algorithm in paper [9] is based on a fixed satellite number six over time, but it has limitation when the number of satellites with higher ΔG is more than six. So the following modified satellite selection algorithm (MSSA) is designed.

Algorithm: If the *i*th satellite's contribution values ΔG_i is smaller than a threshold γ, the GDOP value will changes very little when removing off it. According to the experiments, the threshold can be set as an empirical value $\gamma = 0.15$, which can ensure that the difference of GDOP values are small than 0.2. Remove the satellites whose contribution values are smaller than γ from visible satellites. In this way, it can find out the satellites which are important for calculation, and the number of the selected satellites is changing over time. The algorithm process is as follows:

- Calculate the current satellites' positions through ephemeris data.
- Find out the current visible satellites by earth obscuration angle.
- Calculate all visible satellites' contribution values ΔG_i.
- Rank the visible satellites in the order of ΔG_i.
- If the ΔG of each visible satellite is larger than or equal to γ, the selection result is the all visible satellites. Otherwise proceed to the next step.
- Remove the satellites whose ΔG are smaller than the γ, and the rest of the satellites are selected.

The number of selected satellites is changing over time. The threshold γ can be reset according to requirement of accuracy.

3.3 Computational Complexity Analyses

The calculation complexities mainly depend on the times of matrix inversion and matrix multiplication. If using the DSSA to select the *n* satellites combination whose GDOP value is minimum from the *m* visible satellites, it must calculate all GDOP values of *n* satellites combinations. The times of matrix inversion and matrix multiplication are both C_m^n, this is a time consuming work. Using the MSSA, for calculating *m* satellites' ΔG, it only needs one matrix inversion and $5 \times m + 1$ matrix multiplications. When the satellites are selected more frequently, the calculation complexities are greatly different. Supposing that four satellites are selected, the calculation times are compared between DSSA and MSSA. The result is shown in table 1:

Table 1. Comparison of the computational complexity for selection

The number of Visible satellites	DSSA		MSSA	
	matrix inversion times	Matrix multiplication times	matrix inversion times	matrix multiplication times
8	70	70	1	41
9	126	126	1	46
10	210	210	1	51
11	330	330	1	56
12	495	495	1	61
13	715	715	1	66
14	1001	1001	1	71

The table 1 indicates that the calculation times of the DSSA is far more than that of the MSSA when the difference between selection number and the visible satellite number is larger. And with the number of visible satellites increasing, the accelerated speed of calculation times in the DSSA is faster.

4 Simulation Experiments

The MSSA can be used to all GNSSs, so the experiment takes GPS navigation system for example. Supposing that the receiver is in Beijing (116°E, 40°N), the earth obscuration angle is 5°, the observation time is 24 hours and the sampling time-interval is 30 seconds. The experimental result of MSSA is as follows:

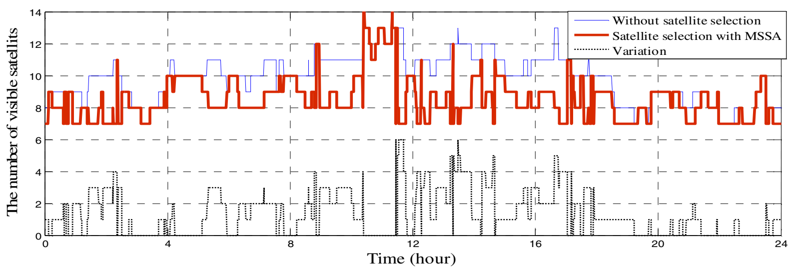

Fig. 2. The number of visible satellites for calculation

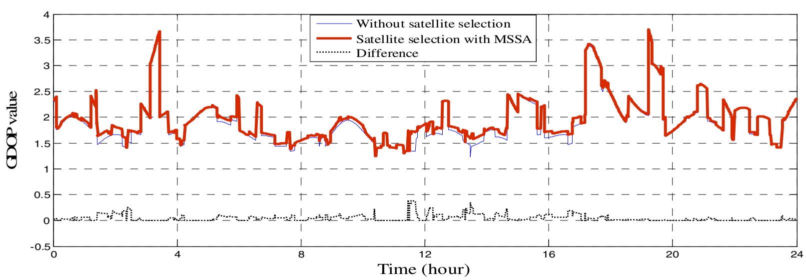

Fig. 3. The GDOP values in different time

In Fig. 2, the fine line denotes the number of all visible satellites in different time, and the bold line denotes the satellite number using the MSSA with a threshold 0.15, and the dot line is the difference between them. The result shows that the reduction of the numbers is obvious at some time. The average difference is 2.1, and the selection result is satisfactory. At the same time, the variation of accuracy is very little showed in Fig. 3. The fine line denotes the GDOP values in different time calculated by all visible satellites. The bold line denotes the GDOP values using the MSSA. The difference between them is very little, and the average is 0.09, and the maximum value is 0.4.The variation of accuracy is very little.

5 Conclusions

This paper describes the necessity of the selection on visible satellites when the number of receiver's channel is restricted and the requirement of calculation speed is increasing for high dynamic users' receivers. Based on each satellite's contribution for GDOP, the MSSA is proposed. The visible satellites are selected by a fixed contribution value γ .The theoretical analysis result shows that the calculation complexity of the MSSA is less than that of the DSSA. So it increases the speed of satellite selection. Meanwhile, by comparing the difference of GDOP values, the result of simulation experiment shows that it can reach a high precision. In addition, if the empirical value γ is set more reasonable, the selected result will be more perfect. This will be considered in the future work.

Acknowledgments. This work was supported by the National High-Tech. R&D Program, China (No.2011AA120505) and the National Natural Science Foundation, China (No.61173077).

References

1. Hofmann-Wellenhof, B., Lichtenegger, H., Wasle, E.: GNSS-Global Navigation Satellites Systems GPS,GLONASS,Galileo & More. Springer Wien, NewYork (2008)
2. Zhang, P., Xu, C., Hu, C., Chen, Y.: Coordinate transformations in satellite navigation systems. In: 2011 International Conference on Electronic Engineering, Communication and Management, EECM 2011, Beijing, pp. 249–257 (2011)
3. Zhang, P., Xu, C., Hu, C., Chen, Y.: Time Scales and Time Transformations among Satellite Navigation Systems. In: The 3rd China Satellite Navigation Conference, CSNC 2012, Guangzhou (2012)
4. Bo, X., Shao, B.: Satellite selection algorithm for combined GPS-Galileo navigation receiver. In: Forth International Conference on Autonomous Robots and Agents, Wellington, New Zealand, pp. 149–154 (2009)
5. Cong, L., Abidat, A.I., Tan, Z.: Analysis and Simulation of GDOP of Satellite Navigation. Acta Electronica Sinica 34(12), 2204–2208 (2006)
6. Kaplan, E.D., Hegarty, C.J.: Uderstanding GPS: Principles and Applications, 2nd edn. Artech House Publisher (2006)
7. Kihara, M.: Study of a GPS satellite selection policy to improve positioning accuracy. In: Proc. of IEEE Position Location Navigation Symp., Las Vegas,NV,USA, pp. 267–273 (1994)
8. Wu, C.-H., Ho, Y.-W.: Genetic Programming for the Approximation of GPS GDOP. In: 2010 International Conference on Machine Learning and Cybernetics, pp. 2944–2949 (2010)
9. Cong, L., Tan, Z.: Satellite selection algorithm to improve precision and real-time performance of GPS positioning. Systems Engineering and Electronics 30(10), 1914–1917 (2008)