Chapter 1
Introduction to Phononic Crystals and Acoustic Metamaterials

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Abstract The objective of this chapter is to introduce the broad subject of phononic crystals and acoustic metamaterials. From a historical point of view, we have tried to refer to some of the seminal contributions that have made the field. This introduction is not an exhaustive review of the literature. However, we are painting in broad strokes a picture that reflects the biased perception of this field by the authors and coauthors of the various chapters of this book.

1.1 Properties of Phononic Crystals and Acoustic Metamaterials

The field of phononic crystals (PCs) and acoustic metamaterials emerged over the past two decades. These materials are composite structures designed to tailor elastic wave dispersion (i.e., band structure) through Bragg's scattering or local resonances to achieve a range of spectral (ω-space), wave vector (k-space), and phase (ϕ-space) properties.

1.1.1 Spectral Properties

The development of phononic crystals for the control of vibrational waves followed by a few years the analogous concept of photonic crystals (1987) for electromagnetic waves [1]. Both concepts are based on the idea that a structure composed of a periodic arrangement of scatterers can affect quite strongly the propagation of classical waves

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such as acoustic/elastic or electromagnetic waves. The names photonic and phononic crystals are based on the elementary excitations associated with the particle description of vibrational waves (phonon) and electromagnetic waves (photon). The first observation of a periodic structure, a GaAs/AlGaAs superlattice, used to control the propagation of high-frequency phonons was reported by Narayanamurti et al. in 1979 [2]. Although not called a phononic crystal then, a superlattice is nowadays considered to be a one-dimensional phononic crystal. The actual birth of two-dimensional and three-dimensional phononic crystals can be traced back to the early 1990s. Sigalas and Economou demonstrated the existence of band gaps in the phonon density of state and band structure of acoustic and elastic waves in three-dimensional structures composed of identical spheres arranged periodically within a host medium [3] and in two-dimensional fluid and solid systems constituted of periodic arrays of cylindrical inclusions in a matrix [4]. The first full band structure calculation for transverse polarization of vibration in a two-dimensional periodic elastic composite was subsequently reported [5]. In 1995, Francisco Meseguer and colleagues determined experimentally the aural filtering properties of a perfectly real but fortuitous phononic crystal, a minimalist sculpture by Eusebio Sempere standing in a park in Madrid, Spain [6] (Fig. 1.1). This sculpture is a two-dimensional periodical square arrangement of steel tubes in air. They showed that attenuation of acoustic waves occurs at certain frequencies due not to absorption since steel is a very stiff material but due to multiple interferences of sound waves as the steel tubes behave as very efficient scatterers for sound waves. The periodic arrangement of the tubes leads to constructive or destructive interferences depending on the frequency of the waves. The destructive interferences attenuate the amplitude of transmitted waves, and the phononic structure is said to exhibit forbidden bands or band gaps at these frequencies. The properties of phononic crystals result from the scattering of acoustic or elastic waves (i.e., band folding effects) in a fashion analogous to Bragg scattering of X-rays by periodic crystals. The mechanism for the formation of band gaps in phononic crystals is a Bragg-like scattering of acoustic waves with wavelength comparable to the dimension of the period of the crystal i.e., the crystal lattice constant. The first experimentally observed ultrasonic full band gap for longitudinal waves was reported for an aluminum alloy plate with a square array of cylindrical holes filled with mercury [7]. The first experimental and theoretical demonstration of an absolute band gap in a two-dimensional solid-solid phononic crystal (triangular array of steel rods in an epoxy matrix) was demonstrated 3 years later [8]. The absolute band gap spanned the entire Brillouin zone of the crystal and was not limited to a specific type of vibrational polarization (i.e., longitudinal or transverse).

In 2000, Liu et al. [9] presented a class of sonic crystals that exhibited spectral gaps with lattice constants two orders of magnitude smaller than the relevant sonic wavelength. The formation of band gaps in these acoustic metamaterials is based on the idea of locally resonant structures. Because the wavelength of sonic waves is orders of magnitude larger than the lattice constant of the structure, periodicity is not necessary for the formation of a gap. Disordered composites made from such localized resonant structures behave as a material with effective negative elastic

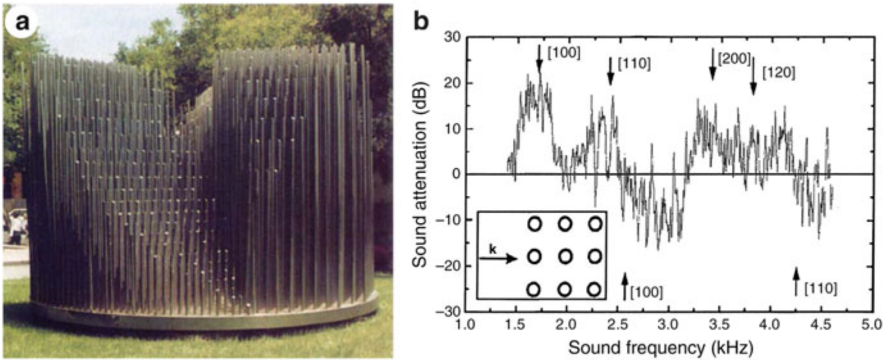

Fig. 1.1 (**a**) Eusebio Sempere's sculpture in Madrid, Spain, (**b**) Measured sound attenuation as a function of frequency. The *inset* illustrates the direction of propagation of sound waves. The *brackets* [hkl] represent, in the vocabulary of X-ray diffraction, crystallographic planes for which Bragg interferences will occur (after [6])

constants and a total wave reflector within certain tunable sonic frequency ranges. This idea was implemented with a simple cubic crystal consisting of a heavy solid core material (lead) coated with elastically soft material (silicone elastomer) embedded in a hard matrix material (epoxy). Centimeter size structures produced narrow transmission gaps at low frequencies corresponding to that of the resonances of the lead/elastomer resonator (Fig. 1.2).

While early phononic crystals and acoustic metamaterials research on spectral properties focused on frequencies in the sonic (10^2–10^3 Hz) and ultrasonic (10^4–10^6 Hz) range, phononic crystals with hypersonic (GHz) properties have been fabricated by lithographic techniques and analyzed using Brillouin Light Scattering [10]. It has also been shown theoretically and experimentally that phononic crystals may be used to reduce thermal conductivity by impacting the propagation of thermal phonons (THz) [11, 12].

Wave localization phenomena in defected phononic crystals containing linear and point defects have been also considered [13]. Kafesaki et al. [14] calculated the transmission of elastic waves through a straight waveguide created in a two-dimensional phononic crystal by removing a row of cylinders. The guidance of the waves is due to the existence of extended linear defect modes falling in the band gap of the phononic crystal. The propagation of acoustic waves through a linear waveguide, created inside a two-dimensional phononic crystal, along which a stub resonantor (point defect) was attached to its side has also been studied [15]. The primary effect of the resonator is to induce zeros of transmission in the transmission spectrum of the perfect waveguide. The transmittivity exhibits very narrow dips whose frequencies depend upon the width and the length of the stub. When a gap exists in the transmittivity of the perfect waveguide, the stub may also permit selective frequency transmission in this gap.

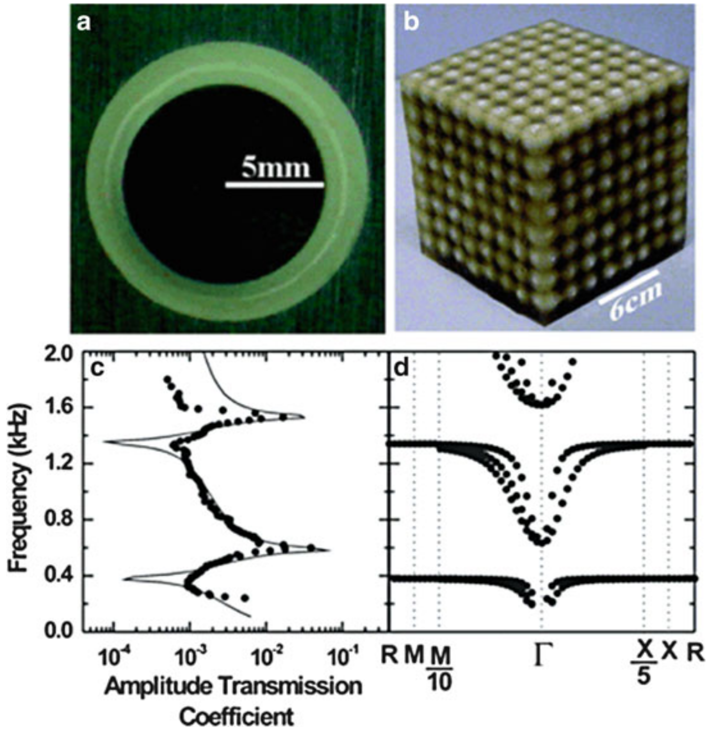

Fig. 1.2 (**a**) Cross section of a coated lead sphere that forms the basic structure unit (**b**) for an $8 \times 8 \times 8$ sonic crystal. (**c**) Calculated (*solid line*) and measured (*circles*) amplitude transmission coefficient along the [100] direction as a function of frequency, (**d**) calculated band structure of a simple cubic structure of coated spheres in very good agreement with measurements (the directions to the left and the right of the Γ point are the [110] and [100] directions of the Brillouin zone, respectively (after [9])

In addition to bulk elastic waves, various authors have studied theoretically the existence of surface acoustic waves (SAW) localized at the free surface of a semi-infinite two-dimensional phononic crystal [16–19]. For this geometry, the parallel inclusions are of cylindrical shape and the surface considered is perpendicular to their axis. Various arrays of inclusions [16, 17], crystallographic symmetries of the component materials [9], and also the piezoelectricity of one of the constituent [19] were considered. The band structures of 2D phononic crystal plates with two free surfaces [20, 21] were also calculated. This includes the symmetric Lamb mode band structure of 2D phononic crystal plates composed of triangular arrays of *W* cylinders in a Si background. Charles et al. [21] reported on the band structure of a slab made of a square array of iron cylinders embedded in a copper matrix. Hsu and Wu [22] determined the lower dispersion curves in the band structure of 2D gold-epoxy phononic crystal plates. Moreover, Manzanares-Martinez and Ramos-Mendieta have also considered the propagation of acoustic waves along a surface parallel to the cylinders in a 2D phononic crystal [23]. Sainidou and Stefanou investigated the guided elastic waves in a glass plate coated on one side with a periodic monolayer of polymer spheres immersed in water [24]. On the experimental point of view, Wu et al. [25] observed high-frequency SAW with a pair of interdigital transducers placed on both sides of a very thick silicon plate in which a square array of holes was drilled. Similar experiments were conducted by Benchabane et al. on a 2D square lattice piezoelectric phononic crystal etched in lithium niobate [26]. Zhang et al. [27] have shown the existence of gaps for acoustic waves propagating at the surface of an air-aluminum 2D phononic crystal plate through laser ultrasonic measurements.

1.1.2 Wave Vector Properties

The wave vector (k-space) properties of phononic crystals and acoustic metamaterials result from passing bands with unique refractive characteristics, such as negative refraction or zero-angle refraction. Negative refraction of acoustic waves is analogous to negative refraction of electromagnetic waves also observed in electromagnetic and optical metamaterials [28]. Negative refraction is achieved when the wave group velocity (i.e., the direction of propagation of the energy) is antiparallel to the wave vector. In electromagnetic metamaterials, the unusual refraction is associated with materials that possess negative values of the permittivity and permeability , so-called double negative materials [29]. Negative refraction of acoustic waves may be achieved with double negative acoustic metamaterials in which both the effective mass density and bulk modulus are negative [30]. The double negativity of the effective dynamical mass and bulk modulus results from the coexistence in some specific range of frequency of monopolar and dipolar resonances [31]. The monopolar resonance may be due to a breathing mode of inclusions resonating out of phase with an incident acoustic wave leading to an effective negative bulk modulus. The dipolar resonance of heavy inclusions coated with a soft material embedded in a stiff matrix can result in a displacement of the center of mass of the metamaterial that is out of phase with the acoustic wave, leading to an effective negative dynamical mass density. Negative refraction may also be achieved through band-folding effect due to Bragg's scattering using phononic crystals. Band folding can produce bands with negative slopes (i.e., negative group velocity and positive phase velocity), a prerequisite for negative refraction. A combined theoretical and experimental study of a three-dimensional phononic crystal composed of tungsten carbide beads in water has shown the existence of a strongly anisotropic band with negative refraction [32]. A slab of this crystal was used to make a flat lens [33] to focus a diverging sound beam without curved interfaces typically employed in conventional lenses. A two-dimensional phononic crystal constituted of a triangular lattice of steel rods immersed in a liquid exhibited negative refraction and was used to focus ultrasound [34, 35]. High-fidelity imaging is obtained when all-angle negative refraction conditions are satisfied, that is, the equifrequency contour of the phononic crystal is circular and matches that of the medium in which it is embedded. A flat lens of this latter crystal achieved focusing and subwavelength imaging of acoustic waves [36]. This lens beat the diffraction limit of conventional lenses by transmitting the evanescent components of a sound point source via the excitation of a vibrational mode bound to the phononic crystal slab. In contrast, a conventional lens transmits only the propagating component of the source. Negative refraction of surface acoustic waves [37] and Lamb waves [38] has also been reported.

A broader range of unusual refractive properties was also reported in a study of a phononic crystal consisting of a square array of cylindrical polyvinylchloride (PVC) inclusions in air [39]. This crystal exhibits positive, negative, or zero refraction depending on the angle of the incident sound beam. Zero angle refraction can lead to wave guiding/localization without defects. The refraction in this crystal is highly anisotropic due to the nearly square shape of the fourth vibrational band. For all three cases of refraction, the transmitted beam undergoes splitting upon exiting the crystal because the equifrequency contour on the incident medium (air) in which a slab of the phononic crystal is immersed is larger than the Brillouin zone of the crystal. In this case Block modes in the extended Brillouin zone are excited inside the crystal and produce multiple beams upon exit.

1.1.3 Phase properties

Only recently has progress been made in the extension of properties of phononic crystals beyond ω-k space and into the space of acoustic wave phase (ϕ-space). The concept of phase control between propagating waves in a phononic crystal can be realized through analysis of its band structure and equi-frequency contours [40]. The dominant mechanism behind the control of phase between propagating acoustic waves in two-dimensional phononic crystals arises from the non-colinearity of the wave vector and the group velocity.

1.2 Beyond Macroscopic, Linear Elastic, Passive Structures and Media

Until recently, phononic crystals and acoustic metamaterials have been constituted of passive media satisfying continuum linear elasticity. A richer set of properties is emerging by utilizing dissipative media or media obeying nonlinear elasticity. Lossy media can be used to modify the dispersive properties of phononic crystals. Acoustic structures composed of nonlinear media can support nondispersive waves. Composite structures constituted of active media, media responding to internal or external stimuli, enable the tunability of their band properties.

1.2.1 Dissipative Media

Psarobas studied the behavior of a composite structure composed of close packed viscoelastic rubber spheres in air [41]. He reported the existence of an appreciable omnidirectional gap in the transmission spectrum in spite of the losses. The existence of band gaps in phononic crystals constituted of viscoelastic silicone rubber and air was also reported [42]. It was also shown that viscoelasticity did not only attenuate acoustic waves traversing a rubber-based phononic crystal but also modified the frequency of passing bands in the transmission spectrum [43]. A theory of damped Bloch waves [44] was employed to show that damping alters the shape of dispersion curves and reduces the size of band gaps as well as opens wave vector gaps via branch cutoff [45]. Loss has an effect on the complete complex band structure of phononic systems including the group velocity [46].

1.2.2 Nonlinear Media

In this subsection, we introduce only the nonlinear behavior of granular-type acoustic structures. The nonlinearity of vibrational waves in materials at the atomic scales due to the anharmonic nature of interatomic forces will be addressed in Sect. 1.2.4. The nonlinearity of contact forces between grains in granular materials has inspired the design of strongly nonlinear phononic structures. Daraio has demonstrated that a one-dimensional phononic crystal assembled as a chain of polytetrafluoroethylene (PTE-Teflon) spheres supports strongly nonlinear solitary waves with very low speed [47]. Using a similar system composed of a chain of stainless-steel spheres, Daraio has also shown the tunability of wave propagation properties [48]. Precompression of the chain of spheres lead to a significant increase in solitary wave speed. The study of noncohesive granular phononic crystals lead to the prediction of translational modes but also, due to the rotational degrees of freedom, of rotational modes and coupled rotational and translational modes [49]. The dispersion laws of these modes may also be tuned by an external loading on the granular structure.

1.2.3 Tunable Structures

To date the applications of phononic crystals and acoustic metamaterials have been limited because their constitutive materials exhibit essentially passive responses. The ability to control and tune the phononic/acoustic properties of these materials may overcome these limitations. Tunability may be achieved by changing the geometry of the inclusions [50] or by varying the elastic characteristics of the constitutive materials through application of contact and noncontact external stimuli [51]. For instance, some authors have proposed the use of electro-rheological materials in conjunction with application of an external electric field [52]. Some authors have considered the effect of temperature on the elastic moduli [53, 54]. Other authors [55] have controlled the band structure of a phononic crystal by applying an external stress that alters the crystal's structure. Tunability can also be achieved by using active constitutive materials. Following this approach, some authors [56, 57] have studied how the piezoelectric effect can influence the elastic properties of a PC and subsequently change its dispersion curves and gaps. Several studies have also reported noticeable changes in the band structures of magneto-electro-elastic phononic crystals when the coupling between magnetic, electric, and elastic phenomena are taken into account [58, 59] or when external magnetic fields are applied [60].

1.2.4 Scalability

The downscaling of phononic structures to nanometric dimensions requires an atomic treatment of the constitutive materials. At the nanoscale, the propagation of phonons may not be completely ballistic (wave-like) and nonlinear phenomena such as phonon–phonon scattering (Normal and Umklapp processes) occur. These nonlinear phenomena are at the core of the finiteness of the thermal conductivity of materials. Gillet et al. investigated the thermal-insulating behavior of atomic-scale three-dimensional nanoscale phononic crystals [11]. The phononic crystal consists of a matrix of diamond-cubic Silicon with a periodic array of nano-particles of Germanium (obtained by substitution of Si atoms by Ge atoms inside the phonoic crystal unit cell). These authors calculated the band structure of the nanoscale phononic crystal with classical lattice dynamics. They showed a flattening of the dispersion curves leading to a significant decrease in the phonon group velocities. This decrease leads to a reduction in thermal conductivity. In addition to these linear effects associated with Bragg scattering of the phonons by the periodic array of inclusions, another reduction in thermal conductivity is obtained from multiple inelastic scattering of the phonons using Boltzmann transport equation. The nanomaterial thermal conductivity can be reduced by several orders of magnitude compared with bulk Si. Atomistic computational methods such as molecular dynamics and the Green-Kubo method were employed to shed light on the transport behavior of thermal phonons in models of graphene-based nanophononic crystals comprising periodic arrays of holes [61]. The phonon lifetime and thermal conductivity as a function of the crystal filling fraction and temperature were calculated. These calculations suggested a competition between elastic Bragg's scattering and inelastic phonon–phonon scattering and an effect of elastic scattering via modification of the band structure on the phonon lifetime (i.e., inelastic scattering).

1.3 Phoxonic Structures

Recent effort has been aimed at designing periodic structures that can control simultaneously the propagation of phonons and photons. Such periodic materials possess band structure characteristics such as the simultaneous existence of photonic and phononic band gaps. For this reason, these materials are named "phoxonic" materials. Maldovan and Thomas have shown theoretically that simultaneous two-dimensional phononic and photonic band gaps exist for in-plane propagation in periodic structures composed of square and triangular arrays of cylindrical holes in silicon [62]. They have also shown localization of photonic and phononic waves in defected phoxonic structures. Simultaneous photonic and phononic band gaps have also been demonstrated computationally in two-dimensional phoxonic crystal structures constituted of arrays of air holes in lithium niobate [63]. Planar structures such as phoxonic crystal composed of arrays of void cylindrical holes in silicon slabs with a finite thickness have been shown to possess simultaneous photonic and phononic band gaps [64]. Other examples of phoxonic crystals include three-dimensional lattices of metallic nanospheres embedded into a dielectric matrix [65]. Phoxonic crystals with spectral gaps for both optical and acoustic waves are particularly suited for applications that involve acousto-optic interactions to control photons with phonons. The confinement of photons and phonons in a one-dimensional model of a phoxonic cavity incorporating nonlinear acousto-optic effects was shown to lead to enhanced modulation of light by acoustic waves through multiphonon exchange mechanisms [66].

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