The Efficacy of Diagrams in Syllogistic Reasoning: A Case of Linear Diagrams

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Abstract. We study the efficacy of external diagrams in syllogistic reasoning, focusing on the effectiveness of a linear variant of Euler diagrams. We tested subjects' performances in syllogistic reasoning tasks where linear diagrams were externally supplied. The results indicated that the linear diagrams work as effectively as Euler diagrams. It is argued that the relational information such as inclusion and exclusion is crucial for understanding the efficacy of diagrams in syllogistic reasoning.

1 Introduction

In psychology of deduction, it has long been known that solving categorical syllogisms is a difficult task for those who are untrained in logic. Certain external representations such as Euler and Venn diagrams are traditionally regarded as effective tools to support deductive reasoning. However, it is still open to discussion whether and how such diagrams could aid untrained people to conduct deductive reasoning in a successful way. For instance, Calvillo et al. [1] reported some negative effects of traditional Euler diagrams in syllogistic reasoning.

Sato et al. [4] examined the efficacy of Euler diagrams in solving syllogisms, in comparison to sentential reasoning and reasoning with Venn diagrams. In the experiments of [4], subjects were divided into three groups, the Euler group, the Venn group, and the Linguistic group. The Euler and Venn groups were presented with two sentential premises, together with the corresponding two diagrams, and asked to choose a valid conclusion. The Linguistic group was presented only with sentential premises and asked to choose an answer without any aid of diagrams. The results indicated that the performance of the Euler and Venn groups was significantly better than that of the Linguistic group, and that the performance of the Euler group was significantly better than that of the Venn group.

The differences in performance between the three groups can be explained on the basis of the dual roles played by diagrams in the overall process of reasoning, namely, interpretational and inferential roles [4,5]. More specifically, a categorical sentence in syllogisms is intended to be interpreted as denoting a *relation* between sets. Thus, a universal sentence of the form All A are B is to be interpreted as expressing $\mathbf{A} \subseteq \mathbf{B}$, i.e. the inclusion relation, and No A are B as $\mathbf{A} \cap \mathbf{B} = \emptyset$, i.e. the exclusion relation. Such relational semantic information

P. Cox, B. Plimmer, and P. Rodgers (Eds.): Diagrams 2012, LNAI 7352, pp. 352–355, 2012.

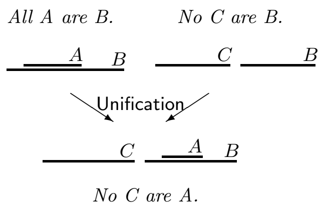

Fig.1. Solving a syllogism with linear diagrams

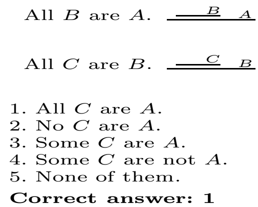

Fig.2. An example of a syllogistic reasoning task in the Linear group

is often not directly accessible to untrained reasoners [4]. Euler and Venn diagrams could then help the reasoners realize the semantic relationships implicit in quantificational sentences in terms of the spatial relationships between objects (circles or points), and thereby avoid reasoning errors due to misinterpretation. For the inferential side, note first that a deductive reasoning task in general requires the reasoner to assemble the information contained in the premises. In the case of the Euler group, then, such a task could naturally be replaced with the task of manipulating diagrams, specifically, of unifying premise diagrams and extracting information [5]. Moreover, the manipulations of diagrams are expected to be spontaneously triggered without much effort, if the spatial relations holding on external diagrams are governed by natural constraints—constrains that depend solely upon spatial properties of diagrams so that they are accessible even to untrained users. By contrast, Venn diagrams lack this kind of inferential efficacy, due largely to the fact that the manipulation of them to solve deductive reasoning tasks requires prior understanding of some conventions (cf. [4,5]).

The key assumption here is that the form of diagrams mirrors the semantic information required in a given reasoning task, specifically, the relational information in the case of syllogisms. If this is correct, then it is expected that any diagram which can make explicit the relational information encoded in a categorical sentence would be effective in supporting syllogistic reasoning. The aim of the present study is to investigate whether this holds good of a linear variant of Euler diagrams, where set-relationships are represented by one-dimensional lines, rather than by circles in a plane. Although it is known that linear diagrams have limited expressive power (cf. [2]), they are expressive enough to represent categorical syllogisms. We hypothesize that linear diagrams would be effective ways of representing relational structures and of reasoning about them. An example of linear diagrams and the process of solving a syllogism with them are indicated in Fig. 1 (cf. the case with Euler diagrams in [4]). Here by unifying the two linear diagrams in premises the reasoner could almost automatically obtain the desired information about the relationship between A and C. If such linear diagrams would work as effectively as Euler diagrams, it could count as evidence that the effectiveness of external diagrams in syllogistic reasoning is not due to particular shapes such as circles of Euler diagrams.

2 Experiment and Result

The semantics of the linear diagrams used is essentially the same as that of Euler diagrams in [4,3]. The experiment was conducted in the same manner as that of [4]; the only difference is that in syllogistic reasoning tasks, Euler diagrams associated with premises are replaced by the corresponding linear diagrams. Note that we adopted a system of categorical syllogisms without the existential import, hence in the syllogism of Fig. 2 we do not count 3 as a correct answer.

Method. Thirty-three undergraduates (mean age 22.72 ± 8.72 SD) participated in the experiment, which we call the Linear group. Of them, we excluded five students who did not follow our instruction. Subjects were first provided with an instruction on the meaning of linear diagrams, and asked to take a pretest to check whether they understood the instruction correctly. Then the subjects were asked to solve syllogistic reasoning tasks supported by linear diagrams. An example is shown in Fig. 2. In this task, the subjects were presented with two sentential premises and asked to choose a correct answer. We presented 31 syllogisms in total. The test was a 20-minute test.

Result. In the following analysis, we exclude the seven subjects who failed the pretest. The average accuracy rate of the total 31 tasks in the Linear group was 80.7%. The data were compared with those of the Linguistic group, the Venn group, and the Euler groups reported in [4] by one-way Analysis of Variance. There was a significant main effect, F $(3, 140) = 37.734, p < .001$. Multiple comparison tests by Ryan's procedure yield the following results. (i) The accuracy rate of the Linear group was higher than that of the Linguistic group: 46.7% for the Linguistic group (F $(1, 64) = 7.112, p < .001$.). (ii) The accuracy rate of the Linear group was higher than that of the Venn group: 66.5% for the Venn group (F $(1, 49) = 2.741, p < .05$.). (iii) There was no significant difference between the accuracy rate of the Linear group and that of the Euler group: 85.2% for the Euler group. It should be noted that if we include those subjects who failed the pretest, we still obtain similar results in each comparison: for (i) and (ii), there were significant differences, $p < .001$; for (iii), there was no significant difference.

These results support our prediction that linear diagrams work as effectively as Euler diagrams in syllogistic reasoning. This in turn provides evidence that the efficacy of external diagrams in syllogistic reasoning depends upon the fact that the diagrams make explicit the semantic relations such as inclusion and exclusion relations in such a way that they are suitable for syntactic manipulation.

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