Vertex Cover, Dominating Set and My Encounters with Parameterized Complexity and Mike Fellows

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Abstract. In this report, I start with a historic view of how, the two problems Vertex Cover and Dominating Set that were influential to the birth of the area of parameterized complexity, also led me to this area and introduced me to Mike Fellows. I also discuss early research and meetings in Parameterized Complexity, Mike's influence in community building and some personal anecdotes with Mike. I conclude with some recent results on these two problems and also discuss open problems in the area.

1 Introduction (How It All Started for Me!)

Having done my PhD on Sorting algorithms, I took a natural liking to the family of directed graphs called tournaments, as a transitive tournament exactly models a totally ordered set. I started looking at papers [2] that showed connections between sorting algorithms and finding directed hamiltonian paths in tournaments. With some colleagues, I wrote a couple of papers describing efficient algorithms and lower bounds to find vertices with specific degrees in tournaments. Along the way, I stumbled into the paper [10] that discussed the complexity of dominating set in tournaments. From the paper, I learnt that every tournament on n vertices has a dominating set of size at most $\lceil \log n \rceil$, and I found it interesting that there are tournaments [6] on n vertices, constructed using simple number theoretic properties where the *minimum* dominating set size is $\Omega(\log n)$.

Using these properties (particularly the existence of tournaments with large dominating sets), I managed to give a reduction from dominating set in general directed graphs to dominating set in tournaments.. The reduction, pretty much, preserved the parameter (k went to $k+1$), but it used $O(2^k n)$ additional vertices and time. Essentially I had proved that the dominating set problem is $W[2]$-hard in tournaments [14], though, at that time, I didn't know the terminology of W-hardness.

This led me to the paper by Papadimitriou and Yannakakis [13] where this problem was discussed in the context of needing limited non-determinism. There, I got the notions of parameterized complexity (in the last paragraph) and $O^*(4^k)$

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algorithm for Vertex Cover (Theorem 5 in that paper). I could quickly improve this to $O^*(2^k)$ through a different algorithm. With these two results ($W[2]$-hardness result for dominating set in tournaments and $O^*(2^k)$ algorithm for Vertex Cover), I sent a mail to Mike Fellows explaining what I had proved.

The response I got was initially dampening, as Mike cited his paper [4] with Rod Downey, where both these results (Theorem 2.1 and Theorem 4.1) had already been proved. However, the response also opened me to the exciting world of parameterized complexity and got me in touch with Mike.

Later, Mike followed up by sending me a lot of his surveys, and some excerpts of the book he was writing with Rod Downey. He also invited me to visit him in Victoria if I was interested in working in the area. I jumped at the offer as I was scheduled to visit Waterloo, Canada that summer (of 1995) anyway.

In his response, Mike also asked whether 2^k for the $f(k)$ for Vertex Cover can be improved further. I discussed this with my colleague R. Balasubramanian (who is the current director of our institute) and together we could improve the 2^k function to c^k for some constant $c < 2$. After my visit to Victoria, and discussions with Mike, we could exhibit a much improved c (close to 1.32) in c^k [1] which was the beginning of the long race of improvements for Vertex Cover. Subsequently, I was quite happy to start the races for MaxSAT, and Undirected Feedback Vertex Set. Jianer Chen's group at Texas, and Rolf Niedermier's group at Jena were the early participants in these races.

2 Early Meetings in Parameterized Complexity

I visited Mike again at Victoria two years later, and there had been big changes in his life since my previous visit (but one could hardly notice anything about them in discussions with him). I also visited him at New Zealand (on the way to COCOON at Australia to present our paper [8]) and there I got to meet Fran. Also during that visit[1] we worked on irredundant sets [5] and planar directed feedback vertex set (with not much progress on the later problem).

During those times, Mike was giving invited talks at various conferences and workshops and his infectious enthusiasm for giving talks caught my attention. I gave talks in this area at various places including the University of Waterloo, Canada, Max Planck Insitute for Informatik, Saarbrucken, National Seminar [15] in India and IMSc. In fact our work on parameterizing above the guarantee [9] grew out of a question asked by someone during one of my talks. He felt cheated when I was showing that the standard parameterized question for MaxSAT was fixed-parameter tractable without giving a new algorithm.

While most of the parameterized complexity community maybe aware of the first meeting on parameterized complexity at Chennai in December 2000 (Mike mentions about this in the preface of the first IWPEC proceedings), not

[1] Interestingly, I landed up in a court with Mike and Fran in Wellington, as a witness to help them; the subtenant, to whom they had rented their apartment during their visit to Canada, refused to vacate and Mike and Fran had to settle that in the court; coincidentally the judge in the case was an Indian!

many would know about the 'mini-symposium' on parameterized complexity that Mike organized at the SIAM Discrete Mathematics conference at Toronto (www.siam.org/meetings/dm98/ms6.htm) in which some of us participated and gave talks. This is a flagship conference for Discrete Mathematics, and it is usually attended by big names in the field.

The first 'workshop' on Parameterized Complexity at IMSc Chennai in 2000 was organized at a short notice. This was organized largely by Mike and Fran during their first visit to India. Apart from a set of excellent talks (including by Jochen Alber, Marco Cesati and Liezhen Cai to name a few), the highlight of the meeting was a 'problem solving' workshop and an 'auto-rickshaw' trip to the Chennai beach, all organized by Mike and Fran.

Then again in 2002, we had a meeting on parameterized complexity as a pre-conference workshop to FSTTCS 2002 at IIT Kanpur. The big news at that time was the polynomial time algorithm for primality that came from IIT Kanpur. The first Dagstuhl workshop on parameterized complexity was eventful with the dissection of the new lower bound result by Cai and Juedes [3] that had appeared at that time. The series of IWPEC (which later became IPEC) workshops was born in that Dagstuhl meeting.

3 Conclusions

Parameterized complexity is a paradigm whose time has arrived. The large number of papers in almost all algorithms conference is a witness to this fact. While some of us may have been anguished by the lack of speed at which the area has penetrated among the theory community, even the current spread would not have been possible if not for Mike's vigorous campaign, in all possible platforms, the practical uses of the paradigm, his efforts in community building at various places and his eagerness to encourage and work with anyone interested in the field.

Let me conclude by saying a few words about Mike and then with some recent developments on both the problems – Vertex Cover and Dominating Set.

3.1 Mike Fellows

An article dedicated to Mike's birthday can not do justice if it doesn't mention Mike's generosity. Mike's generosity in readily sharing his ideas, perceptions, open problems, and his time and even money (even though he was living 'on the edge' most of the time) are well known. During my visits to Victoria and in some of his sessions in Chennai, I could see his deep passion for popularizing Mathematics. And I have seen first hand some of his Mathematics 'fairs' at Victoria. In Chennai, we could get to see the other side of Mike and Fran, as they were quite popular among the vendors, helpers, waiters and the auto rickshaw drivers, as their compassion to them knew no bounds. Mike's breadth and passion for science in general was quite visible when he was rubbing shoulders with the physicists and mathematicians at our institute during his various visits. He was

quite an enthusiastic player in the team that did a review of our entire institute, a few years ago. Personally I have been inspired by many of his qualities, and let me use this occasion to say, 'Thank you and Happy birthday Mike!'.

3.2 Recent Work on (above Matching Guarantee) Vertex Cover

A brief look at the table of FPT races [12] shows that Vertex Cover continues to be one of the very few problems having an FPT running time $< 2^k$ (not including problems that have subexponential algorithms on special classes of graphs) which grew out of a question by Mike quite early on. For example, the running time of the closely related feedback vertex set problem in undirected graphs, is still above $O^*(3^k)$ and it required a new technique (iterated compression) to even reach this stage.

Recent attempts [11] to use linear programming to get improved parameterized algorithms for vertex cover above the matching size (AGVC) could pave for a new direction and race, to get improved parameterized algorithms for Vertex Cover as well as for other problems. I am also glad that the 'above guarantee parameterization' is not just a natural paradigm (that took a life of its own), but AGVC in particular, has become a central parameterized problem to which several other natural problems could be reduced [11].

3.3 Recent Work on Dominating Set (in Graphs with Excluded Subgraphs)

The W-hardness proof of dominating set in tournaments continues to be one of the few parameterized reductions that is NOT a polynomial time reduction, but simple enough to be done in a first lecture on parameterized reductions.

On the algorithmic front on dominating sets, while a number of fixed parameter algorithms are known for planar, bounded genus and bounded treewidth graphs, I'd like to point to a (perhaps not so well known) result that the problem is fixed-parameter tractable in graphs having no 'short' cycles [16]. This has been generalized to graphs not having $K_{i,j}$ as a subgraph for fixed values of i and j, and this result eventually led to a polynomial kernel for bounded degenerate graphs [7]. These continue to be one of the few classes of graphs that are characterized by 'forbidden subgraphs' (as opposed to forbidden minors) for which fixed parameter algorithms are known for the dominating set problem.

3.4 Open Problems

I end with a couple of concrete problems that are still open from our first workshop on Parameterized complexity in Chennai. What is the parameterized complexity of the following problems (here k is the parameter)?

1. Given an undirected graph G and an integer k, does G have a complete bipartite graph as an induced subgraph with k vertices in each part?
2. Given an undirected planar graph G and an integer k, does G have an independent set on $n/4 + k$ vertices?

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