

Chapter 2

Hybrid Metaheuristics for Dynamic and Stochastic Vehicle Routing

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Abstract. Recent developments in telematics, such as the wide spread use of positioning services and mobile communication technologies, allow the exact monitoring of vehicles. These advances build the basis for automatic real-time fleet management systems. To be successful such systems have to rely on optimization algorithms for solving dynamic and stochastic vehicle routing problems based on ingredients such as historical data, stochastic modeling, machine learning, fast shortest-path calculation, fast construction heuristics, and exact and (meta)heuristic optimization methods. This book documents the growing interest in and success of hybrid metaheuristics. They are often used to solve complex and large real-world optimization problems, combining advantages from various fields of computer science and mathematical optimization. Within this chapter the application of such methods for the dynamic and stochastic vehicle routing problem is studied. After a general introduction in this field, the main commonalities of dynamic and stochastic vehicle routing problems are described and a short overview of classical algorithms for these problems is given. Then, in the third part hybrid metaheuristics for dynamic problems vehicle routing problems are be described. The third part focusses on stochastic problems. The fourth part examines the combination of dynamic and stochastic problems. The chapter is concluded with an outlook towards future developments in the field as well as promising open research areas.

2.1 Introduction

In today's globalized economy fast, reliable but also flexible supply chains are among the main factors for successful enterprises. In real-world applications it is most often the case that some information about the future is available (stochastic information) and that known information is revealed over time (dynamic information). For example, an estimation for the customer demand is given at the beginning, whereas

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the actual demand will be revealed at a later date. Additionally, recent developments in telematics, such as the wide spread use of positioning services and mobile communication technologies, allow the exact monitoring of vehicles [33]. These advances build the basis for automatic real-time fleet management systems and extensive and detailed data collection. To be successful such systems have to rely on optimization algorithms for solving dynamic and stochastic vehicle routing problems based on ingredients such as historical data, stochastic models, machine learning, fast shortest-path calculation, fast construction and insertion heuristics, and exact and (meta)heuristic optimization methods.

In dynamic real-world vehicle routing applications it is fundamental that short-term decisions are accurate and made quickly, while long-term decision need to ensure a certain quality standard. Moreover, stochastic models based on previously collected data can be used in order to provide some information about upcoming events. The combination of the dynamic and stochastic problem further extends the high complexity of their static and deterministic counterparts. Therefore, such problems are especially amenable to hybrid optimization approaches, combining the advantages of different techniques. The growing interest in hybrid metaheuristics and their success is well documented in [13]. They are often used to solve complex and large real-world optimization problems, combining advantages from various fields of computer science and mathematical optimization. In the recent vehicle routing literature there is an increasing number of successful application of hybrid metaheuristics [32].

Here, we examine the application of hybrid methods to dynamic and stochastic vehicle routing problems (VRP). In a first part, we give a short overview of vehicle routing variants and a classification depending on the nature of the available data. Additionally, it includes a short summary of various algorithms for solving dynamic and stochastic VRPs. In the second part, we examine hybrid metaheuristics for dynamic problem variants, where not all information is available in advance. In the third part of this chapter, we focus on problems that consider a priori stochastic information about possible future events and progresses. The fourth part of this chapter, examines dynamic problems where some stochastic information about future events is available. We conclude with an outlook towards future developments in the field as well as promising open research areas.

2.2 Dynamic and Stochastic Vehicle Routing Problems

The Vehicle Routing Problem (VRP) is a well-known and extensively studied combinatorial optimization problem [79], [35]. In the last years, the interest in solving real world applications of the VRP has grown tremendously as information technologies now allow the gathering of relevant information about available vehicles

and scheduled requests in real time. This new problem class, where information is handled at the time it arrives, known as dynamic VRP (DVRP), has recently received increased attention from the research community [52], [27]. In the DVRP not all relevant information is known at the time of route construction and the information may change during the execution of the planned routes. Recent summaries about current developments in DVRP can be found in [75] and [66]. Additionally, in real world applications it is often the case, that some information about the future, for example about travel times or arising requests or demand, is available. This information can be incorporated into the route planning process by modeling stochastic VRPs [31], [81], [47].

2.2.1 Vehicle Routing Problem Variants and Available Information

The literature discerns various classical VRP variants that are further complemented by numerous real-world applications with additional requirements and constraints. A detailed description of numerous VRP variants can be found in [79] and [35].

In the classic VRP formulation, a set of vertices representing customers or cities, a set of arcs where each arc is associated with travel costs and a set of vehicles, stationed at a depot are considered. The aim is to construct vehicle routes where each vertex is visited exactly once, all vehicle routes start and end in the depot and the total travel costs are minimized. A more detailed definition of the classic VRP can be found in [49]. In the following, the most important VRP variants are presented.

The simplest and most studied variant is the Capacitated VRP (CVRP). It is known to be NP-hard and generalizes the well known Traveling Salesman Problem (TSP) [79]. In the CVRP, the aim is to serve all customer demands with a given fleet of vehicles located at a single depot where the capacity of the vehicles is restricted.

A related variant is the distance-constrained VRP where the route length is restricted by a maximum tour length and the VRP with Time Windows (VRPTW) is an extension of the CVRP in which the service at the customer has to start within a given time window.

Another variant is the VRP with Backhauls (VRPB) where the customer set is split into two subsets. The first subset contains of customers which require a product to be delivered (*linehaul*), whereas the second subset contains customers where a given quantity of a product has to be picked up (*backhaul*). Here, the precedence of the customers must be considered.

In the VRP with Pickup and Delivery (PDP) each customer is split into two different locations, where the goods have to be picked-up at one location and delivered to the other one. Another variant of the PDP, where people are transported instead of goods, is called the Dial-a-Ride Problem (DARP). In this problem class, additional constraints for user convenience are introduced [19].

2.2.1.1 Information Availability

In the literature, problem variants are discerned according to the availability and certainty of information [75]. A problem can be seen as *static* or *dynamic* depending on the availability of information before the start of the optimization process. Depending on the certainty of the available information problems can be considered as *deterministic* or *stochastic*. The combination of these characteristics yields four categories of routing problems.

The first category are *static and deterministic* problems. Here, all relevant information is completely known at the beginning of the route planning process and no changes take place during the execution of the routes. This leads to the classic static VRPs and solution methods ranging from exact methods to metaheuristics. These problems have been extensively studied in the literature [79], [20], [50], [63].

The next category encompasses *static and stochastic* problems, the relevant information is known a priori, but some parts of it are afflicted with a given uncertainty. Some information is given as random variables, and the aim is to generate solutions optimizing the expected value of the objective function. Commonly, stochastic programming methods are applied to such problems [69].

In *dynamic and deterministic* problems, the available information at the beginning of the planning process is incomplete and there is no information about future events. In the literature, these problems are often referred to as real-time or online optimization problems. Most commonly, some information is already known before the planning horizon starts, but other parts of the information are revealed or change during the execution phase. Solution methods for the dynamic VRP can range from reoptimization algorithms over fast insertion heuristics to queuing theory based algorithms, depending on the degree of dynamism [54], [55], [27].

In the fourth category, *dynamic and stochastic* problems, relevant information is revealed throughout the planning horizon, but additionally stochastic information about the future, most commonly gathered from historical data, is available. To deal with stochastic information, solution methods are either based on sampling approaches where possible future scenarios are included [6], [7] or considering stochastic information explicitly [26], [76].

2.2.1.2 Stochastic Information

As described above, there are problem categories which have some information about future events available. This means, there exists an estimation about the occurrence of possible future events. This often happens in real world applications where stochastic information can be obtained from historical data. There are several types of stochastic information which can be incorporated in the optimization process of VRPs [31]. In the following section, a differentiation about the most common types of stochastic information is given.

Travel times are elementary data in VRPs, thus, it is important to provide authentic travel times for the considered network. *Stochastic Travel Times* are random

variables and can be used to deal with uncertainties occurring in real world environment, like time dependent travel times, seasonal effects, car accidents, bad weather or working zones. In [49], the VRP with stochastic travel times is described in detail. Sometimes time dependent travel times defined in [27] and stochastic travel times are combined as proposed in [29].

Another extensively studied problem is the VRP with *Stochastic Demand* [10]. Here, the actual demand of customers is not known in advance, but it is known as a random variable which follows a known probability distribution. This problem arises in practical applications where unknown amount of goods have to be either delivered or collected. Commonly, this problem is solved by a two stage approach where first, all routes are constructed a priori, and later, when the demand becomes known, the returns to the depot for refilling are planned. The aim is to construct routes with minimum expected routing costs, comprising the costs of the route and the return trips to the depot.

The next problem class, VRP with *Stochastic Customers*, mainly arises in a dynamic environment where not all customer requests are known a priori, but reveal during the day of operation. In this case, stochastic information about the expected number of customer requests is considered and incorporated into the optimization algorithm, as for example shown in [7] or [42]. Another possibility of stochastic customer requests in the DARP is presented in [74] where the requests for the return trips of patients are stochastic.

The last category considers problems with multiple uncertainties where lots of information is assumed to be uncertain and modeled as stochastic variables. Thus, not only travel times or requests are stochastic, but also the location and time of requests, as well as, cancellation of requests, vehicle break downs or traffic jams. In [6] and [40] problems with rich stochastic information are described, and in [26] and [3] practical applications using stochastic information are presented.

2.2.2 Algorithms for Solving Dynamic Vehicle Routing Problems

In the dynamic VRP (DVRP), not all relevant information is known at the time the routes are planned, but it is revealed throughout the execution of the scheduled routes. An extensive overview on DVRP can be found in [75].

One common strategy for solving DVRPs is to apply a static algorithm to the already known data at the beginning for computing an initial solution, and whenever new information becomes known the current solution is updated. In order to solve the static problem, solution concepts based on exact procedures, like Linear Assignment [28] or Column Generation [18], can be applied. Another concept is rule based decision making where decision rules are defined and applied whenever dynamic events occur [70], [53]. Local Search (LS) approaches compute a feasible starting solution first, usually by repeated insertion operations, and then apply some improvement techniques. For example, *best insertion* is used to add a request to the current solution and after this *cross-exchange*, *or-opt* or *interroute exchange* moves are applied to improve the solution, as in [67], [17], [15].

The most widely applied methods for solving DVRPs are metaheuristics [61], [80]. They include Ant Colony Optimization Algorithms applied in [62], Evolutionary Algorithms used in [37] or Variable Neighborhood Search implemented in [14]. A very popular metaheuristic for the DVRP is Tabu Search as shown in [56] and [57]. In many cases, a parallel implementation is used to improve computational time of reoptimization, as presented in [4] and [30].

2.2.3 Algorithms for Solving Stochastic Vehicle Routing Problems

In stochastic optimization problems knowledge about the uncertainty of certain aspects is known a priori. This can be knowledge about future requests or travel times. In this section the focus is on different methods dealing with stochastic information.

Markov Decision Processes (MDP) are often used to model stochastic VRPs, for example in [23], a VRP with stochastic demand is formulated as a MDP. In [78], a MDP is used as well to solve a VRP with customer requests which arise with a certain probability. Other approaches use an approximate dynamic programming approach [68], [76] for solving VRP or fleet management problems.

In Stochastic Programming, mathematical programming and stochastic models are combined for solving optimization problems with uncertainties. The aim is to determine a feasible solution for all possible outcomes and the optimization of the expected value of the objective function [12]. A detailed introduction to stochastic programming in the context of transportation and logistics is given in [69].

Another method to deal with stochastic information is *Sampling*. This strategy considers already known and stochastic information and generates possible future scenarios by drawing them from a given probability distribution. A novel approach, using *Sampling* is described in [7], where a VRP with stochastic customer requests is solved. Further approaches using *Sampling* can be found in [6] and [26].

2.3 Hybrid Metaheuristics for Dynamic Problems

Dynamic problems have usually been solved using reoptimization or fast insertion techniques depending on the amount of time available for reacting to new events. One of the earliest works presenting a reoptimization based hybrid metaheuristic in the dynamic vehicle routing context is the algorithm by Jih and Hsu [44] combining a Dynamic Programming (DP) algorithm and a Genetic Algorithm (GA) for the single-vehicle PDP with time windows and capacity constraints. The dynamic programming component is executed for a certain amount of time. It will either return an optimal solution or multiple partially constructed routes. Those partial solutions are used as initial population of a genetic algorithm. The hybrid approach was able to improve the results of the non-hybrid methods.

More recent algorithms combine fast insertion heuristics with background optimization techniques, allowing an almost immediate response to dynamic events but utilizing possibly available time for improving solution quality. Another promising hybridization technique for dynamic problems are various parallelization variants in general. Depending on the specific problem characteristics such as the degree of dynamism [52], and response time requirements either of the variants makes sense.

2.3.1 *Parallelization Approaches*

Parallel optimization methods are often applied to complex large scale VRPs, a recent survey on parallel solution methods in the context of vehicle routing can be found in [21].

Several variants of a parallel Tabu Search (TS) heuristic for the dynamic multi-vehicle Dial-a-Ride Problem (DARP) are proposed in Attanasio et al. [4]. The authors motivate the use of a parallel approach with high running times of classical methods. The objective of the dynamic DARP to fulfill as many requests as possible with the available number of vehicles. Whenever a request can be added to the current solution without violating the problem constraints, it is accepted. Therefore a fast mechanism for checking the possible acceptance of a request is required. The parallel TS approach is applied to generate a starting solution based on already known requests and to perform a background optimization after a new request has been inserted. The fast insertion procedure is performed randomly inserting the new request in the current solution for every thread. If a feasible solution is found, the insertion is possible. If this is not the case the parallel TS with independent thread is run with parameters set to focus on feasibility. The presented computational experiments show that parallelization significantly increases the amount of served requests in real-world instances.

Khouadjia et al. [48] present a multi-swarm based optimization algorithm for the VRP with dynamic requests. The method consists of a particle swarm optimization approach with interacting swarms, thereby maintaining population diversity. New customers are inserted into existing routes using a method resembling the ejection chain approach. In addition to parallelization a low-level hybridization using *2-opt* as local improvement heuristic is implemented. The results of the novel approach are significantly better than the ones obtained by the current state of the art for the dynamic vehicle routing problem.

The main advantage of applying parallel optimization methods in a dynamic context is speed. Especially in the case when new information becomes available a fast reaction is of great importance and the ability to guarantee fast response times will decide over the applicability of the optimization approaches.

2.3.2 *Other Hybridization Approaches*

Most other hybridization approaches are based on the principle of combining fast insertion techniques with longer running, usually metaheuristic background optimization methods. In some of these approaches further hybridizations are developed in order to achieve higher quality results in comparison to more classical algorithms.

Alvarenga et al. [1] extend a hybrid Column Generation Genetic Algorithm approach for solving the static VRP [2] towards the dynamic case. The algorithm is based on a set partitioning formulation of the VRP. In such an integer programming formulation, every vehicle route is explicitly modeled and an optimal solution is a set of those routes minimizing the objective function. Explicitly representing VRP instances of realistic sizes as set packing problems would require enormous computational resources, therefore techniques such as Column Generation and Branch and Price are often used [65]. Alvarenga et al. are generating and iteratively refining a subset of the routes using a Genetic Algorithm (GA). The resulting restricted set partitioning problem is finally solved using an integer programming solver. This hybrid approach is extended to the dynamic case by applying a fast insertion heuristic to integrate new requests in all the individuals of the GA before restarting the integer programming solver. Computational experiments show the effectiveness of this approach and the advantages over reoptimization using the static version of the algorithm.

Fabri and Recht [24] solve a capacitated DARP where all customers are occurring dynamically using a combination of an A^* -algorithm and Tabu Search (TS). Their approach extends a hybrid heuristic proposed by Caramia et al. [16]. New requests are inserted by applying a fast procedure. First, single vehicle routes are created by representing the single vehicle DARP as Shortest Path Problem (SPP) and solving it using the A^* -algorithm. In a second step, the routes are then heuristically assigned to the vehicles. Several TS variants are then presented for optimizing the routes between two-occurring requests. The presented computational results show that the additional optimization significantly increases the solution quality.

Creput et al. [22] propose a novel approach combining a Self Organizing Map (SOM) with an Evolutionary Algorithm (EA) for solving the VRP with dynamic requests. Creput et al. describe the SOM as a center-based clustering algorithm preserving the density and the topology of the data distribution. The approach is based on applying SOM to the Travelling Salesman Problem (TSP). Cities of the TSP are mapped to the SOM network, local moves increasingly approach the vertices of the SOM network to the cities. By mapping the SOM vertices to the closest city a solution to the TSP is generated. This approach is extended to the VRP by embedding it into an solution pool based iterative improvement algorithm. New customers are added to the existing routes by simple insertion satisfying the relative route duration constraints. Extensive computational results show the advantages of the presented approach.

Berbeglia et al. [8] consider a dynamic dial-a-ride problem (DARP) in which some requests are static and the others arrive in real time. In this work, a hybrid algorithm is introduced which combines an exact constraint programming (CP)

algorithm and a tabu search (TS) heuristic. A crucial point in dynamic DARP is to determine whether a new incoming request can be accepted and satisfied or not. Generally, the TS algorithm manages the insertion of new requests well when the problem is not too tightly constrained and CP is fairly effective in proving infeasibility in tight settings. Thus, the idea is to combine the advantages of these two methods. In their approach, the CP algorithm returns either a feasible solution for a given instance or proves that none exists. To tackle the dynamic aspects, the CP algorithm for the static DARP in [9] is extended by additional constraints to state that the solution must consider the partial routes followed up to now. The TS algorithm constructs a feasible starting solution, continually optimizes the current solution and also tries to insert new requests. Thus, when a new request arrives the CP and TS algorithm run in parallel for insertion. The TS algorithm uses three scheduling schemes determining the arrival times, begin of service times and departure times for each request vertex, which has a considerable impact on the algorithm performance. Concluding, it is shown that the hybrid algorithm clearly outperforms each of the two algorithms when executed separately.

2.4 Hybrid Metaheuristics with Stochastic Information

There are several examples of hybrid metaheuristics incorporating stochastic information. Similar to methods for other problem classes, many of the proposed methods combination different search algorithms. Another possibility is to combine approximated and exact stochastic models for computing the expected value of the objective function.

2.4.1 Hybridization of Search Techniques

Hvattum and Løkketangen [38] consider the stochastic Inventory Routing Problem (IRP). This problem is a combination of inventory management, vehicle routing, and stochastic demands. The problem is solved by applying the progressive hedging algorithm to a scenario tree representation of the problem. The problem is originally modeled using a Markov Decision Process (MDP). Based on the observation that most probably it is sufficient to consider a finite horizon, the authors propose to approximate the MDP by using a scenario tree based integer programming formulation (STP). The authors adapt the Progressive Hedging Algorithm (PHA) [73] to the STP. The PHA decomposes the scenario tree and solves the scenarios separately as subproblems and iteratively joins them using penalty terms in the adapting the respective objective functions. The subproblems are solved using a Greedy Randomized Adaptive Search Procedures (GRASP) based approach [41]. Although calibration of the PHA is reportedly difficult, results of the combined PHA and GRASP approach are more robust than the ones by any of the presented methods examined separately.

In Laporte et al. [51], a capacitated Arc-Routing Problem with stochastic demand (CARPSD) is considered. This problem arises for example in practical applications like garbage collection. Here, the customer locations are known in advance but the demand is random and not known until the location is reached. Thus, a capacity constraint violation can happen at some point in the planned solution. In this case, the vehicle has to interrupt its route and empty the vehicle by going to the dump site early and returns to the point of failure to restart its route. The objective is to minimize the costs of the planned route and the expected costs of the recourse action. The CARPSD is conventionally formulated as a stochastic program, where a first-stage solution is computed, realization of random variables are revealed and then a recourse action is applied. In contrast to that, an alternative approach is proposed where a first-stage solution is constructed which will take the expected cost of the recourse action into account. This is realized by an Adaptive Large Neighborhood Search (ALNS) heuristic. The garbage quantities are assumed to be independent random variables with known probability distribution and expected costs of recourse are defined for the discrete and continuous case. A construction heuristic, *stochastic path scanning*, is proposed. Several removal and insertion heuristics, destroying and repairing the current solution are also used. At each iteration one of these heuristics is selected based on the *roulette-wheel selection principle*. The acceptance criterion for a new solution and the stopping criterion are obtained by using an annealing-based search framework. The approach was tested on self generated instances, based on instances for CARP from Golden et al. [34]. The comparison between the deterministic and stochastic case shows clearly that improved results could be computed with the presented approach.

Another significant stochastic routing problem is the Probabilistic Traveling Salesman Problem (PTSP) [43], and since nature inspired intelligence became increasingly popular, Marinakis and Marinaki present a hybrid algorithm based on nature inspired approaches in [58]. Here, a hybrid scheme incorporating Particle Swarm Optimization (PSO) [46] and two further metaheuristics, Greedy Randomized Adaptive Search Procedure (GRASP) [72] and Expanding Neighborhood Search (ENS) [59] is introduced. The combination of PSO and GRASP is used to produce as good as possible initial populations, and the ENS strategy, speeds up the optimization process. The performed computational experiments demonstrate that the proposed approach leads to an effective handling of the PTSP, resulting in fast computational run-times and good results for very large problem instances.

Rei et al. [71] solve the single VRP with stochastic demands by combining Monte-Carlo sampling and local branching [25]. The authors consider an a priori optimization setting based on a two-stage stochastic programming model of the problem. In the first stage of the stochastic program a route visiting all customers once is constructed. In the second stage the route is executed with the actual demands and possibly necessary predetermined recourse actions. Starting from an optimal solution to the original first stage problem, the approach partitions the search space using the local branching principle in an iterative multi-descent search. The subproblems are solved to optimality or until a certain time limit is reached using the L-shaped method. The expected value of the recourse action is approximated

using Monte-Carlo sampling. The computational results show the competitiveness of the presented approach, yielding comparable solution quality in significantly less run-time than an exact solution approach.

Mendoza et al. [60] solve the multi-compartment vehicle routing problem with stochastic demands (MC-VRPSD) using a Memetic Algorithm (MA) combining a genetic algorithm with local search including novel evaluation and repair procedures taking into account the stochastic nature of the problem. In a first step the problem is modeled as two-stage stochastic program, where the recourse actions consist of trips back to depot in order to reload the vehicle. The authors then propose an MA for solving the problem using an approximation of the expected cost as objective function. The initial solutions are created using a stochastic best insertion heuristic. A combination of relocate and *2-opt* moves are used as local improvement operators. Finally, the repair and evaluation of individuals is done by applying a stochastic extension of the split algorithm [5]. The presented algorithm is evaluated on random test instances as well as benchmark instances from the literature, showing an improved performance compared to the state of the art.

2.4.2 Objective Function Hybridization

Bianchi et al. [11] focus on the most commonly studied problem in this class, the VRP with stochastic demand (VRPSD). The idea is, to analyze the hybridization of different approximations of the objective function (minimizing the expected costs of the tour) with well known metaheuristics for this problem. The aim is to test the impact of interleaving the exact VRPSD objective function with the a priori tour length as an approximation. Therefore, five well known metaheuristics, Simulated Annealing, Iterated Local Search (ILS), Tabu Search (TS), Ant Colony Optimization (ACO) and Evolutionary Algorithms (EA) are presented. All considered metaheuristics use the common OrOpt Local search (LS) as proposed in [81], in order to obtain meaningful comparisons. The basic operator in the OrOpt LS considers a starting tour and moves sets of consecutive customers from one position in the tour to another one. For the computation of the moving costs two types of approximation schemes are described. In the *VRPSD approximation scheme* the costs are composed of the savings from extracting the customers from the tour and the costs of inserting them back, whereas the *TSP approximation scheme* only computes the length difference of the tour. The starting solution for all metaheuristics are generated by the *Farthest Insertion Construction Heuristic* [45]. The first hybridization shows the impact of using approximate move costs in local search, by running each proposed metaheuristic with the *VRPSD approximation scheme* and the *TSP approximation scheme*. The tests show that metaheuristics which use the local search as a black box (EA, ACO, ILS) perform better with the *TSP approximation*, while the other perform better with the *VRPSD approximation*. The second hybridization further explores the TSP objective function. Therefore, they expand the best algorithms determined in the first hybridization (ILS, EA) with the *3-opt LS for TSP* [77] and show significant improvements of the performance. The proposed approach is

evaluated on self generated instances, where four factors are considered: the customer position, the capacity over demand ratio, variance of the stochastic demand and the number of customer, where the size of the instances is between 50 and 200 customers, with the customers uniformly distributed or grouped in clusters. As a conclusion it is shown, that the new hybrid approach clearly outperforms the state of the art.

2.5 Hybrid Metaheuristics for Dynamic and Stochastic Problems

In most dynamic real-world application, data is gathered and stored allowing to develop stochastic models for predicting future events. The hybrid approaches developed here can be discerned in two groups. On the one hand those working with single solutions and incorporating the stochastic knowledge directly in the optimization procedure. On the other hand approaches relying on solution pools, where multiple solutions are generated based on sampling, which are then further reconciled into a single solution. Such approaches can also be used to provide multiple solutions as suggestions to human dispatchers responsible for taking final decisions.

2.5.1 Single Solution Approaches

Hvattum et al. [39] consider a dynamic and stochastic VRP, based on a case from a large distribution company. Stochastic information about customer requests, like the location and its demand and the frequency of appearance, is gathered from historical data. It is shown how this problem can be formulated as a multistage stochastic programming problem with recourse. Dynamic events are captured by dividing the time horizon into a specified number of intervals and construct a plan for each interval using the currently known requests, in contrast to [7], who proposed an event-driven model. They developed a dynamic stochastic hedging heuristic which uses sample scenarios. In each time interval the solution from the previous interval plus the requests which became known during the past interval are considered and a plan for the current time interval is constructed. Sample scenarios are solved as static VRPs and the customers which are visited most frequently are determined iteratively. Then, the solution is built by assigning the request to the vehicles according to a ranking which states which customer is serviced first most often. The algorithm was tested against a myopic dynamic heuristic and was able to reduce travel distances significantly. In [40], they present a branch-and-regret heuristic which is based on the approach described above to additionally tackle the stochastic demand of customer, thus, the customer location is already known, but not its demand. Improvements with this new approach are shown as well as the capability to cope with different stochastic information.

Attanasio et al. [3] present a real-time fleet management system for a same-day courier service, describing forecast and optimization methodologies. A *forecast* and an *allocation* module are introduced, where the forecast module generates reliable near future predictions of travel times and demand and hands this information to the allocation module which is responsible for the assignment of customer requests to the couriers and the relocation of idle couriers. In order to make reliable predictions they divided the service area into geographical zones and time periods, which show typical traffic and demand patterns. Both, demand and travel time forecasting are based on a classical decomposition approach followed by an artificial neural networks which incorporates real time information. The demand forecasting specifies the number of requests from one region to another one at a given time period and the travel time forecasting provides expected travel times from one address to another address during a time period. The methodology for the travel time forecasting also takes traffic and real time information into account and is trained through artificial neural networks. When a new request passes the feasibility check the algorithm post optimization procedure tries to improve the current solution in the background. Therefore a parallel implementation of the Tabu Search (TS) algorithm is proposed. As a result of applying this system the efficiency of the couriers raised and less dispatchers for the fleet management are needed.

Schilde et al. [74] investigate whether using stochastic information about future requests can improve the solution quality of a dynamic stochastic Dial-a-Ride Problem (DSDARP). Here, a special type of customer request is considered as with a certain probability, a request for patient transportation to the hospital creates a corresponding transportation request to return the patient back home on the same day. In order to investigate the benefit of using stochastic information about return trips two approaches for the dynamic DARP are implemented and extended to deal with stochastic information. The Variable Neighborhood Search (VNS) approach introduced by [64] is adapted for the dynamic case (DVNS) and then extended to a dynamic S-VNS due to the S-VNS concept proposed in [36], and the Multiple Plan Approach (MPA) and Multiple Scenario Approach (MSA) described by [7] are applied. Computational results show that incorporating stochastic information about return trips yields better solution quality than the myopic methods, and that it is most beneficial to consider possible return transports in the very near future (up to 20 minutes).

2.5.2 Algorithms Based on Solution Pools

Bent et al. [7] consider a dynamic VRP with time windows (VRPTW) with stochastic customers. The aim is, to investigate how to exploit stochastic information about customers to accept and serve as many requests as possible, thus to miss fewer requests. This is achieved with a Multiple Plan Approach (MPA) dealing with dynamic customer request. Then, a related approach, called Multiple Scenario Approach (MSA) is introduced. It significantly outperforms the MPA by exploiting stochastic information about customers. The MPA is an event driven approach where

a pool of plans is maintained. Because only one specific plan can be executed, just one plan is chosen from the pool and routing plans that correspond to the current situation are generated. The selection is done by a consensus function that selects the plan most similar to the other plans in the pool. To take advantage of the stochastic information, the MSA generates new routing plans for scenarios that include a priori known requests and possible future events, which are obtained by sampling their probability distributions. Experimental results show that using stochastic information yields significantly better results and that selecting plans by consensus function brings benefits for problems with many stochastic customers.

In a subsequent paper, Bent et al. [6] examine the MSA, described in [7], for a dynamic VRP with stochastic information about customer requests. In contrast to [7], the behavior of the MSA on a less constrained but more stochastic problem is studied, in order to capture long-distance mail services. The difference is, that now customer, customer locations and service times are random variables and that the focus is on the objective function, i.e., the aim is to minimize the total travel distance. Additionally, the MSA is improved by using stochastic information to delay the departure of vehicles in order to place new stochastic requests. To optimize the plans a large neighborhood search is used basically. In the case Large Neighborhood Search (LNS) does not find any improvements also a *nearest neighbor* heuristic is applied. Experimental results on a variety of models show that with this approach travel distance could be reduced scientifically.

2.6 Conclusion

Dynamic and stochastic VRPs are currently the most challenging class of problems in the vehicle routing area, especially if they are tackled in the real-world context where instances are often much larger than those usually considered in the literature. In most cases it is not possible to solve such problems in an exact way due to the limitation of computational resources and time. Furthermore, only partial, incomplete, and uncertain information is available, thereby making the quest for optimality impossible. The surveyed literature shows that in recent years hybrid metaheuristics have been increasingly successfully applied to such complex problems. Most often the diverse aspects of the problems can be solved by applying a combination of methods sometimes even requiring an interdisciplinary approach.

In our survey, we have discerned between hybrid metaheuristics for solving vehicle routing problems with three different types of available information: dynamic, stochastic, dynamic and stochastic. These three variants have then been further divided according to predominant hybridization principles. In the case of dynamic problems, where the major challenge is the realization of quick response times, parallelization approaches and fast insertion combined with background optimization were the most common patterns of hybridization. When only a priori stochastic information is considered, we found that most commonly different complementing search algorithms are combined. Another form of hybridization was the combination of exact and approximate computation of the objective function. The

combination of dynamic and stochastic information is most often addressed using sampling based approaches, the proposed algorithms are most often working with a single solution, but important results have also been obtained with methods based on solution pools where the diversity comes from solving multiple scenarios. Usually a single solution is then recommended, but such approaches can become necessary when multiple solutions have to be recommended to a human decision maker. Depending on application specific requirements and the nature of the available information an appropriate approach will have to be chosen.

The area of dynamic and stochastic optimization in general and vehicle routing in particular is recently getting increasing attention. This is mainly due to technological advances in fields such as telematics and computing but also in algorithmic advances in the vehicle routing area. In our opinion, multi-disciplinary approaches combining strong stochastic modeling and combinatorial optimization skills will gain importance able to solve complex academic and real-world problems.

Acknowledgements. This work, partially funded by the Austrian Federal Ministry for Transport, Innovation and Technology (BMVIT) within the strategic program FIT-IT Mod-Sim under grant 822739 (project HealthLog).

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