# Weak Ties in Complex Wireless Communication Networks*

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**Abstract.** Hundreds of millions of devices—from book-sized notebooks to tiny hand-held mobile phones—are equipped with wireless communication adapters that are able to form a network among themselves. The spontaneous creation of this kind of network and the unpredictable joining and leaving of devices bring forward new challenges on network and topology organization. Network Science has proven to deliver a fruitful methodology to investigate systems such as complex communication networks, and new insights and solutions can be gained by understanding and imitating the function and structure of social networks. Following this line, this paper initially focuses on the development of models that reveal characteristics found to be inherent to social networks. In particular, we consider the finding that social networks can contain a diversity of links: we create clusters of friends, connected by strong links and, additionally, there are links to acquaintances, the so-called weak ties which, despite the name, have been hypothesized as essential for finding jobs or disseminating rumors when strong ties fail. As such links seem to be highly important to deal with the requirements of a complex network such as our own social network, we argue that bringing these structures to the design principles of complex communication networks may result in an increase of efficiency and robustness, and we describe the implementation of two algorithms for wireless communication networks using only local neighborhood information and producing features of complex social networks (weak ties in particular). The results imply that local removing promotes the emergence of weak ties, which we found by using a recently proposed link clustering algorithm for identifying link communities.

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## 1 Introduction

In the last years, it has become clear that the increasing number of wireless communication devices such as notebooks, hand-held mobile phones or even tiny sensors is generating an enormous impact in our daily lifes [6, 13]. The design of wireless networks such as an *ad hoc* or sensor network that consists of a diversity of a large number of devices is a hard task, since the paradigm of self-organization applies: these devices can join and leave the network unpredictably and form networks spontaneously.

These characteristics create new challenges on how to handle the emerging complex communication topologies that potentially can consist of thousands of devices. In order to deal with these challenges, we have to look for networks, which are used to naturally and inherently deal with the problems, and learn design principles by analogy. In fact, understanding the structure of our own social network might help finding answers of how to design a complex communication network and which patterns we have to evoke in a man-made communication network to deal with its own complexity [9].

This work focuses on the findings that our social network consists not of a single type of ties or links, rather it is built on a diversity of links. The human social network is actually a highly complex structure that is tied by different types of interdependency, such as histories, interests, trades, neighborhood, and communications. These ties or links are neither randomly nor uniformly distributed, and the characteristics of the links vary considerably. As a matter of fact, Granovetter [4] reports on the difference between friends and acquaintances, and points out that acquaintances are more useful for certain tasks such as finding a job and disseminating news or rumors. Granovetter calls the links between acquaintances as weak ties. The difference between a weak and a strong tie can be understood in different ways. For a wireless communication network, this can be interpreted by the fact that clusters should concentrate on processing information, while weak ties should dedicate mostly on information dissemination.

In this paper, we focus on the problem of evoking weak ties in ad hoc networks where devices communicate over a wireless medium without using any immediate router. This kind of wireless network belongs to the class of spatial graphs, where the links between nodes depend on the radio transmission range, which is a spatial relation between nodes [5, 8]. The main problem of emerging weak ties is that there is no formal definition available that could be used. Nevertheless, Kumpula *et al* [7] suggest a network model for emerging community-like structures, including strong and weak ties. Additionally and more restrictively, the introduction of new links is explicitly not allowed. This corresponds to the reality of self-organizing wireless networks since links can only be created if nodes are within their respectives transmission ranges. As in Kumpula *et al.* [7], our model also requires 2-hop neighborhood information for execution. On the other hand and in contrast to Kumpula *et al.* [7], our approach considers a localized topology control algorithm that does not rely on network evolution for the creation of weak ties.

Despite the significant limitations regarding link creation, we show that it is possible to build a distinction between strong and weak ties. This is accomplished for example with spatial graphs, but the algorithms work for relational graphs as well. We use (i) the clustering coefficient [12] and (ii) similarities between links [1] to control the topology, and promote the emergence of weak ties in a network. The objective is to create highly clustered regions with low average shortest path by removing superfluous links. To identify weak ties, we use a recently proposed link clustering algorithm for identifying link communities [1]. Link communities focuses on grouping links rather than nodes, and the algorithm incorporates overlap and reveals multiscale complexity in networks.

The remainder of this paper is organized as follows. Section 2 presents the system model. Section 3 describes topology control algorithms. Experiments and analysis of topological properties are in Section 4. Finally, Section 5 concludes this work.

## 2 System Model for a Wireless Network

We define an ad hoc sensor network consisting of a set of devices connected by wireless network links. We consider here that the initial network topology is a spatial graph such as a unit disk graphs [3]. The resulting wireless network can be represented as $G = (N, L)$ that is a graph with $|N|$ nodes and $|L|$ links. All nodes have the same transmission range $r$. Two nodes $u$ and $v$ can only form a link when they are in a spatial neighborhood, i.e. when their Euclidean distance $d$ is smaller than the transmission range: $d(u,v) \leq r$. We abstract away the details of the MAC and network layer. Nodes are static, i.e. they keep their initial position and they are deployed uniformly at random in a squared simulation area with an edge length $l$. Thus, all possible links are already given from the initial configuration.

Furthermore, we assume that every node is aware of its current 2-hop neighbors, listed in a device neighbor list data type. We assume that, in practice, a neighbor discovery service on each device updates the neighbor list at particular time intervals, such that the neighbor list represents—with a minor delay—the current local topology of the network. Geographical positions of the nodes are not considered.

## 3 Topology Control: Clustering and Weak Ties

Kumpula *et al.* [7] shows a model where a sparse network evolves to a dense network. Since our system model does not allow such a procedure of link addition, we researched for a method that increases the clustering and keeps low the average shortest path by removing links. It turned out that the clustering coefficient indicates clustered nodes, and it can be increased by *removing* links. Our hypothesis is that the links that keep the clustered regions connected should then be weak ties. The clustering coefficient can be used for measuring the efficiency regarding the clustering behavior. Since the clustering coefficient is locally defined we counter the challenge to implement a solution that is localized [10].

The definition of the clustering coefficient might suggest that more links in each node neighborhood result in a higher clustering coefficient. Our approach, however, is based on the observation that this statement does not hold in general. Thus, we found that even the removal of dedicated links can increase the global clustering coefficient. Our algorithm is built on this observation and provides a generic approach. We argue that since the links can only be removed, weak ties have to appear naturally in the network topology.

Unfortunately, there is no formal definition in the literature that could be used to identify weak ties. Granovetter [4] produces an informal idea of the impact of weak ties on the network structure. We propose to use a link communities algorithm recently reported in the literature [1] to identify weak ties. Our final analysis consists of three steps:

- calculate the similarities between pairs of links (i.e. Jaccard index),
- cluster the ties, using a single-linkage hierarchical clustering [1], and then
- classify link communities as strong or weak ties.

In link communities, the Jaccard index can be used to calculate the similarity $S$ between links from an undirected and unweighted network [1]. Link communities use single-linkage hierarchical clustering to find hierarchical community structures due to simplicity and efficiency, even on large-scale networks. Initially each link builds one community. The pairs of ties with higher similarity and common ties between them are grouped simultaneously. The algorithm ends when all links are clustered.

As the similarity $S$ measures the strength of the merged community, we consider that weak ties appear, in the link cluster, as single communities, i.e., with low or no similarity to other link communities. Thus, the set of weak links in a network is represented by the union of these unitary link communities.

## *3.1 A Link Removal Algorithm Based on Clustering Coefficient ($R_{cc}$)*

The first proposed algorithm verifies if a link $e_{u,v}$ is inefficient in terms of the clustering coefficient, *i.e.* if its removal increases the clustering coefficient. In the case of inefficiency, the link $e_{u,v}$ is considered as a candidate for removal. It is not removed immediately because removal in this stage would be in accordance with the criterion of that particular node only. However, since removing a link affects the local clustering coefficients of the 2-hop neighborhood of the set $u, v$, an additional removal confirmation phase must be performed, when nodes exchange the removal candidate information with their corresponding neighbors. Connectivity is guaranteed by the fact that removing $e_{u,v}$ requires at least one neighbor of $u$ to be connected to one neighbor of $v$. Thus, the resulting topology is connected, and the algorithm is therefore connectivity-preserving.

The algorithm requires 2-hop synchronization to remove a link, since 2-hop topological information is required in order to plan the action. Then again, this local link removal affects the 2-hop neighbors. For reasons of simplicity the algorithm has been implemented in a synchronous network, and a desynchronization procedure is not detailed here. We notice, however, that any synchronous algorithm (*i.e.* an algorithm for synchronous networks) can be transformed in its asynchronous counterpart by using synchronizers [2].

### 3.2 A Link Removal Algorithm Based on Link Similarity ($R_{simil}$)

The second proposed algorithm verifies if a link $e_{u,v}$ is inefficient in terms of similarity, *i.e.* if the link has a very high average similarity between its neighboring links. In this case, we propose that $e_{u,v}$ can be considered a strong or redundant link because its removal should not considerably affect the average shortest path. Otherwise, if the link $e_{u,v}$ has a low average similarity between its neighboring links, $e_{u,v}$ can be a weak tie, since removing this link may significantly increase the average shortest path.

An approach for the removing decision is based on a variable probability $p$ that is proportional to the average similarity between the link $e_{u,v}$ and its neighboring links. This means that links with high average similarity are strong removal candidate with probability $p$. On the other hand, links with low average similarity, or weak ties, may be preserved in the network due to its low probability of removal.

As in the previous algorithm, the connectivity is guaranteed and the resulting topology is connected. In both algorithms, the stop condition is given by the choice of a percentage of links removed from the initial spatial network.

## 4 Simulation Study

The first experiments were run on a set of 200 nodes uniformly deployed at random in a square with edge length $l = 450$ units and transmission range $r = 60$ units. The initial topology was created using the unit disk graph model described in Section 3. An example is shown in Fig. 1(a). Table 1 shows statistics from 50 spatial networks. Fig. 1(b) shows an example of a resulting network using the link removal algorithm based on clustering coefficient. Each color represents different link communities. Fig. 1(c) shows an example of a resulting network using the link removal algorithm based on link similarities. Each dotted line represents an unitary link communities.

Fig. 2(a) reveals that the clustering coefficient increases approximately 30% using the algorithm based on clustering coefficient after removing 25% of links. Our experiments have indicated that this algorithm reaches its stop condition in 25% removed links. However, with the same percentage of removal, the clustering coefficient decreases approximately 30% using the algorithm based on link similarities. The working principle of the first model is the optimization of the clustering coefficient by selectively removing links. We observe that link removals are more likely to occur in sparse regions, while highly clustered regions are mostly unaffected (see Fig. 1(b)).

The average path length is showed in Fig. 2(b). The link removal algorithm based on link similarities increases the APL in only 20% after removing half of links in the

**Table 1.** Statistics from 50 spatial networks with $n = 200$ nodes, $l = 450$ u, $r = 60$

| Metric | Average | Standard Deviation |
|---|---|---|
| $\|L\|$ initial links | 979.7037 | 43.0071 |
| Clustering Coefficient | 0.6309 | 0.0152 |
| Average Shortest Path | 5.6571 | 0.2432 |

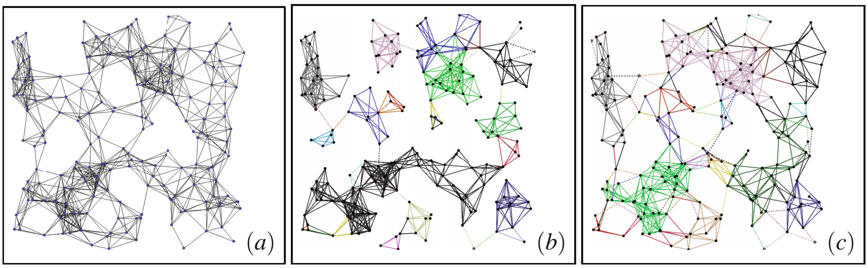


**Fig. 1.** Examples: (a) Spatial network with $n = 200$ nodes, $l = 450$ u, $r = 60$ u, $|L| = 1,226$ links; (b) Network with 20% nodes removed by the link removal algorithm based on clustering coefficient $R_{cc}$; (c) Network with 30% nodes removed by the link removal algorithm based on link similarities $R_{simil}$

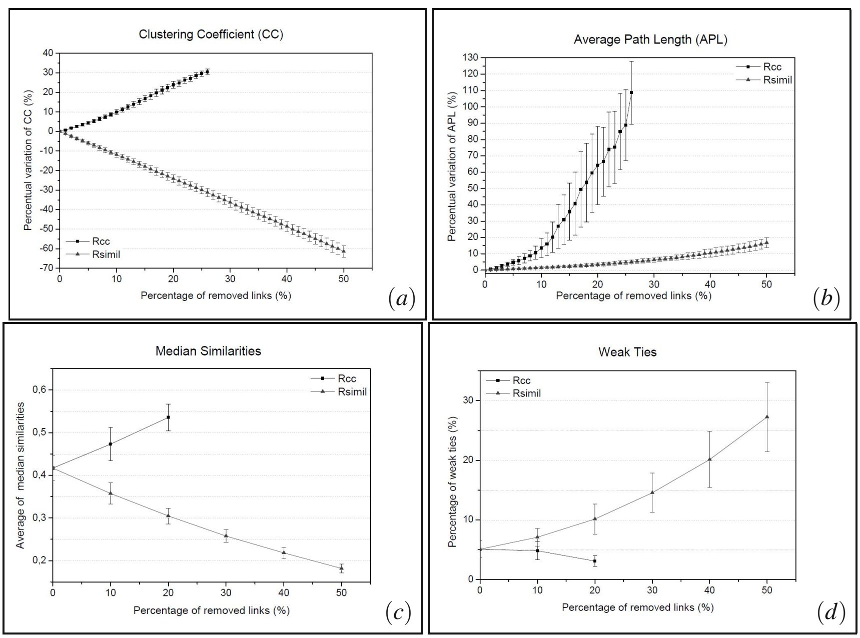


**Fig. 2.** Results from 50 final networks using link removal algorithms based on clustering coefficient ($R_{cc}$) and link similarities ($R_{simil}$)

network. This means that $R_{simil}$ keeps weak ties in the network. However, the link removal algorithm based on clustering coefficient increases the APL up to 125% after removing only 25% of links in the network. In this case, $R_{cc}$ removes mainly weak ties, that significantly increase the average shortest path.

The removal of strong *vs.* weak ties is clearer in Fig. 2(c). $R_{cc}$ significantly increases the median similarity of networks, since removing weak ties. However, $R_{simil}$ decreases the median similarity of networks after removing strong ties.

These algorithms aim at increasing the clustering coefficient and at keeping low the average shortest path, but as a side effect, isolated links that connect clustered regions may appear. These isolated links seem to have the same functions and structures as weak ties have in social networks. If the resulting network topology is powerful in terms of information dissemination, action taking (information processing etc.) as a complex social network, but using less resources than the initial network (because links have been removed), then the presented algorithm can be used to release efficiency reserves of complex communication network design.

After calculating similarities between pairs of links and clustering links, single communities – *i.e.*, unitary communities with low similarity to other sets of links – were classified as weak ties. See Fig. 2(d) for results. The algorithm $R_{simil}$ transforms about 30% of network connections in weak ties, after removing half of links in the network. However, the link removal algorithm based on clustering coefficient decreases the number of weak ties. As a matter of fact, the resulting topologies for the experiments based on similarities reveal visible dotted links where weak ties start to dominate (see Fig. 1(c)).

## 5 Conclusions

The link removal algorithm based on clustering coefficient and introduced in this paper shows that clustering does not lead to the emergence of weak ties. On the other hand, the control based on link similarities efficiently creates weak ties, but significantly decreases the clustering coefficient.

Weak ties appear to be important for the transfer of certain information that is filtered by a clustered set of nodes. And these links can be successfully classified by the link communities algorithm. Importantly, our approaches do not allow addition of new links, so our approaches rely on a procedure by removing dedicated links.

Whereby our work focuses on manipulation of an existing network to form weak ties, human social networks seem to apply different principles: they are driven by the joining and leaving of nodes and thus, use network evolution as the driving force for emerging patterns. For example, the likelihood for two persons with a common friend to become friends is higher than the possibility for two persons with no common friend to become friends [11].

In spite of these interesting considerations, it is important to keep in mind that the results were obtained for spatial networks such as unit disk graphs. We expect similar results for relational graphs, studies and analysis on network composed of dynamic nodes, and combination between link removal algorithms, but these are subjects of further investigations. In an extended version, weights can be assigned to links to conduct the removing process.

Finally, it remains an open question if weak ties can be produced with a microscopic or localized model that does not make use of more than one-hop neighborhood information.

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