Chapter 4
Theory of Optical Feedback in Semiconductor Lasers

A semiconductor laser with optical feedback is an excellent model for generating chaos in its output power and the system has proven to be very useful in practical applications. This chapter concerns the theoretical background for instability and chaos induced by optical feedback in narrow-stripe edge-emitting semiconductor lasers, such as Pabry-Perot lasers, multi-quantum well (MQW) lasers, and distributed feedback (DFB) lasers. Particular dynamics of feedback-induced instability and chaos in semiconductor lasers are separately discussed in the following chapter. In this chapter, we focus on the theoretical treatment of optical feedback effects in semiconductor lasers. Lasers show the same or similar dynamics as far as rate equations are described by the same equations. We here assume single mode operations for semiconductor lasers. The dynamics for multimode cases will be discussed in Chap. 8.

4.1 Theory of Optical Feedback

4.1.1 Optical Feedback Effects and Classifications of Optical Feedback Phenomena

The effects of optical feedback in semiconductor lasers have been studied from the beginning of their development (Risch and Voumard 1977; Voumard 1977; Gavrielides et al. 1997). In early 1980, Lang and Kobayashi published a milestone paper on the effects of optical feedback in semiconductor lasers, which initiated an enormous research effort devoted to the study of the dynamics induced by optical feedback. Since then, bistability, instability, self-pulsations, and coherence collapse states have been observed in feedback-induced irregular oscillations in semiconductor lasers (Mils et al. 1980; Glas et al. 1983; Lenstra et al. 1985; Cho and Umeda 1986). In semiconductor lasers, self-optical-feedback effects are frequently used for the control of oscillation frequency, selection of mode, and suppression of side modes.

J. Ohtsubo, *Semiconductor Lasers*, Springer Series in Optical Sciences 111,
DOI: 10.1007/978-3-642-30147-6_4,

Indeed, the linewidth of laser oscillations can be stabilized by a strong optical feedback and chirping of oscillation frequency can be compensated by optical feedback (Goldberg et al. 1982; Tamburrini et al. 1983; Agrawal 1984; Lin et al. 1984). On the other hand, the semiconductor laser shows unstable oscillations for a certain range of optical feedback levels. The dynamics of semiconductor lasers induced by optical feedback in this range are very interesting not only from the viewpoint of fundamental physics but also for practical applications, since optical feedback effects appear everywhere in optical systems including optical communication systems, optical data storages, and optical measurements. These irregular oscillations are induced by the dynamics involved in laser systems known as chaos described by nonlinear delay differential equations.

Cleaved facets were frequently used as a laser resonator in semiconductor lasers in the early days. Therefore, the reflectivity of laser facets of semiconductor lasers is much lower than that of other lasers such as gas lasers. Since light in a cavity of a semiconductor laser is reflected perpendicularly to the laser facet, the internal amplitude reflectivity r_0 is given by

$$r_0 = \frac{\eta - 1}{\eta + 1} \tag{4.1}$$

where η is the refractive index of the laser material. For example, the refractive index η of the AlGaAs semiconductor laser without any optical coating is about 3.6 and the amplitude reflectivity of the facet is calculated to be $r_0 = 0.565$. The corresponding intensity reflectivity is $R_0 = r_0^2 = 0.32$. Only 32 % of the light generated by the stimulated emission is fed back into the laser cavity and the other photons dissipate from the laser cavity (Zah et al. 1987). To make a high power laser, the laser facets are coated appropriately by dielectric films. Then, the rear facet of the cavity usually has a high reflectivity of more than 90 % and the front fact has a low reflectivity of less than 10 %. This is quite different from other lasers where both facets have high reflectivities close to 100 %.

In spite of such a dissipative laser structure, laser oscillations are still possible in semiconductor lasers due to the high efficiency of the conversion from pump to light. For example, the conversion efficiency of electricity to light in semiconductor lasers is usually up to fifty percent. This makes semiconductor lasers different from other lasers. Thus, light goes away from the cavity after a few reflections within the resonator. In other words, semiconductor lasers are easily affected by external light due to optical feedback or optical injection from a different laser. Indeed, the use of optical isolators is essential in optical communication systems to prevent unstable laser operations generated by feedback light from optical components and optical fiber facets. Optical feedback induces various instabilities in semiconductor lasers, for example, noises (actually they are chaotic fluctuations as discussed later) are much enhanced by optical feedback. In optical communications, the quality of signal transmissions has priority, so that optical isolators are used at the expensive of system sizes and costs to reduce feedback noises. On the other hand, optical information equipment, for example, optical data storages, in which serious problems by optical feedback are encountered for the performance of operations, the cost of the system is most important. In those systems, the reduction and control of noises (actually chaotic oscillations) are essential issues for good systems. For such purpose, the idea of chaos control, which is discussed in Chap. 9, can be applied.

There are many parameters to characterize instabilities and chaos in semiconductor lasers. Every parameter is important for describing the characteristics, however, one important and most useful parameter to figure out the characteristics is the reflectivity of the external mirror. Tkach and Chraplyvy (1986) investigated the instabilities of semiconductor lasers with optical feedback and categorized them into the following five regimes, depending on the feedback fraction.

Regime I. Very small feedback (the feedback fraction of the amplitude is less than 0.01 %) and small effects. The linewidth of the laser oscillation becomes broad or narrow, depending on the feedback fraction (Kikuchi and Okoshi 1982).
Regime II. Small, but not negligible effects (less than ∼0.1 % and the case for $C > 1$, where the C parameter is a measure of instability, discussed in Sect. 4.2). Generation of the external modes gives rise to mode hopping among internal and external modes (Tkach and Chraplyvy 1985).
Regime III. This is a narrow region around ∼0.1 % feedback. The mode hopping noise is suppressed and the laser may oscillate with a narrow linewidth (Tkach and Chraplyvy 1986).
Regime IV. Moderate feedback (around 1 %). The relaxation oscillation becomes undamped and the laser linewidth is broadened greatly. The laser shows chaotic behavior and sometimes evolves into unstable oscillations in a coherence collapse state. The noise level is enhanced greatly under this condition (Lenstra et al. 1985).
Regime V. Strong feedback regime (higher than 10 % feedback). The internal and external cavities behave like a single cavity and the laser oscillates in a single mode. The linewidth of the laser is narrowed greatly (Fleming and Mooradian 1981a,b).

In the above regimes, the quoted fraction is that of the actual optical feedback level into the active layer and it does not mean the reflectivity of the external mirror, since there are scattering and absorption losses of light through optical components. Furthermore, a diffraction loss of light due to a collimator lens usually put in front of the laser facet is not negligible, because the thickness of the active layer is as small as 0.1μm in ordinary edge-emitting lasers. Therefore, the fraction of optical feedback actually fed back into the active layer becomes one-tenth or less than the intensity reflectivity of the external mirror. However, semiconductor lasers are sensitive enough to destabilize their output power by a small amount of optical feedback of less than 1 % of the amplitude. Therefore, an isolation of 40 dB is usually required in optical communication systems to avoid optical feedback effects.

The investigated dynamics of the above regimes were for a DFB laser with a wavelength of 1.55 μm, so that the feedback fraction corresponding to each dynamics scenario described above is not always true for other lasers. However, the dynamics for other lasers show similar trends for the variations of feedback fraction. The lasers show the same or similar dynamics as far as the rate equations are written in the same forms. As has already been discussed, the rate equations for narrow-stripe edge-emitting semiconductor lasers, such as Fabry-Perot, MQW, and DFB lasers, are described by the same forms. Therefore, these lasers exhibit similar chaotic dynamics, though the parameters may have different values. We are very interested in regime IV that shows chaotic dynamics (Sacher et al. 1989; Mørk et al. 1990a, 1992), though it is a small level of optical feedback (the intensity fraction of the feedback is only 0.01 %). In actual applications of semiconductor lasers, this regime is important because, for example, the feedback fraction of laser amplitude in Compact Disk systems corresponds to regime IV (Gray et al. 1994). Thus, regime IV is important for the studies of both nonlinear dynamics and applications.

4.1.2 Theoretical Model

The static characteristics of semiconductor lasers with optical feedback can be theoretically investigated with the relations among the reflectivities of internal cavity and external reflector, the gain in a medium, and other static laser parameters. However, the dynamic characteristics must be described by time-dependent equations of the systems. The equations for semiconductor lasers in the presence of optical feedback are easily obtained by modifying the rate equations for the solitary laser discussed in Chap. 3. The schematic model of a semiconductor laser with optical feedback is shown in Fig. 4.1. For a while, we consider that the external reflector is a conventional plain reflection mirror. The effects of other reflectors such as grating and phase conjugate mirrors will be discussed later. Light from a laser is reflected from an external mirror and fed back into the laser cavity with time delay. We assume that the mirror is positioned within the coherence length of the laser. Also, the laser is assumed to be operated at a single mode, although this is not always true in actual situations. The laser sometimes oscillates at multimode under certain parameter conditions of optical feedback even when the laser oscillates at a single mode in the solitary condition. The external feedback effect is added to the equation for the complex field of (3.47) and the field equation is written in the following form (Lang and Kobayashi 1980):

$$\frac{\mathrm{d}E(t)}{\mathrm{d}t} = \frac{1}{2}(1 - i\alpha)G_n\{n(t) - n_{\mathrm{th}}\}E(t) + \frac{\kappa}{\tau_{\mathrm{in}}}E(t-\tau)\exp(i\omega_0\tau) \tag{4.2}$$

where κ is the feedback coefficient due to the external optical feedback, $\tau = 2L/c$ (L being the length of the external cavity) is the round trip time of light within the external cavity, ω_0 is the angular oscillation frequency of the laser. The extra term has a delay time τ and the complex field is described by a delay differential equation and this is the origin of instability and chaotic dynamics in semiconductor lasers. The equation is known as the Lang-Kobayashi equation after their derivation.

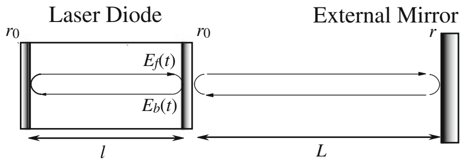

Fig. 4.1 Model of semiconductor laser with optical feedback

The feedback coefficient κ can be calculated from considering the multiple-reflection effects of light in the external cavity. In Fig. 4.1, we consider the fields propagating forward and backward within the cavity and the extra term added to the laser field from the optical feedback in front of the facet of the resonator. For the steady-state oscillation in the presence of external feedback, the relation between the forward and backward traveling fields at the laser facet, $E_f(t)\exp(-i\omega_0 t)$ and $E_b(t)\exp(-i\omega_0 t)$, is given by (Lang and Kobayashi 1980)

$$E_b(t) = r_0 \left\{ E_f(t) + \frac{1-r_0^2}{r_0} r \sum_{m=1}^{\infty} (-r_0 r)^{m-1} E_f(t - m\tau)\exp(im\omega_0\tau) \right\} \quad (4.3)$$

where r is the amplitude reflectivity of the external mirror. In the parenthesis of the above equation, the first term is the ordinary field of reflection in the internal cavity and the second is the effect of the external optical feedback. The semiconductor laser is easily destabilized and shows chaotic dynamics even for a small level of feedback less than a few percent of the amplitude reflectivity. We here consider a steady-state solution as $E_f(t - m\tau) \sim E_f(t)$ and only assume a single reflection for a small external reflection r. Then, the feedback coefficient κ is written by (Tartwijk and Lenstra 1995)

$$\kappa = (1 - r_0^2)\frac{r}{r_0} \quad (4.4)$$

We assume that the reflectivities for the front and back facets of the laser cavity are the same at r_0. It is not always true for actual lasers, but the feedback rate for different reflectivities can be calculated straightforwardly. Recent semiconductor lasers have a low intensity reflectivity of the front facet as small as 10 % or less by optical coating and, therefore, the lasers are much affected by optical feedback.

The time-dependent phase in the presence of optical feedback plays an important role, since the phase couples with the other variables. For the carrier density, we need not consider the modification of the equation. Similar to the derivations for the rate equations in (3.59)–(3.61), we obtain the rate equations in the presence of optical feedback as follows (Ohtsubo 2002):

$$\frac{\mathrm{d}A(t)}{\mathrm{d}t} = \frac{1}{2}G_n\{n(t) - n_{\mathrm{th}}\}A(t) + \frac{\kappa}{\tau_{\mathrm{in}}}A(t-\tau)\cos\theta(t) \quad (4.5)$$

$$\frac{\mathrm{d}\phi(t)}{\mathrm{d}t} = \frac{1}{2}\alpha G_n\{n(t) - n_{\mathrm{th}}\} - \frac{\kappa}{\tau_{\mathrm{in}}}\frac{A(t-\tau)}{A(t)}\sin\theta(t) \tag{4.6}$$

$$\frac{\mathrm{d}n(t)}{\mathrm{d}t} = \frac{J}{ed} - \frac{n(t)}{\tau_s} - G_n\{n(t) - n_0\}A^2(t) \tag{4.7}$$

$$\theta(t) = \omega_0\tau + \phi(t) - \phi(t-\tau) \tag{4.8}$$

We can investigate the dynamics of semiconductor lasers with optical feedback by numerically solving the above equations. In the rate equations for a solitary laser derived from (3.59)–(3.61), the phase does not affect the other variables and, therefore, a semiconductor laser is only described by the field amplitude and carrier density equations. However, we must consider the phase for a time development in the presence of optical feedback, since the phase is related to the other variables through the optical feedback term as shown in the above equations. Then, three coupled equations are essential for semiconductor lasers with optical feedback and they show unstable oscillations and chaotic dynamics in their output powers like three coupled equations in Lorenz systems. In the numerical simulations, the fourth-order Runge-Kutta algorithm is frequently used for the sake of the accuracy of the calculations (Press et al. 1986).

4.2 Linear Stability Analysis for Optical Feedback Systems

4.2.1 Linear Stability Analysis

When the fluctuation of the output power is small even in the presence of optical feedback in a semiconductor laser, we assume a steady-state solution for the average field. In this case, we obtain the steady-state solutions for $A(t) = A_s$, $\phi(t) = (\omega_s - \omega_{\mathrm{th}})t$, and $n(t) = n_s$ from (4.5)–(4.7) as follows (Tromborg et al. 1984, 1987; Agrawal and Dutta 1993):

$$A_s^2 = \frac{J/ed - n_s/\tau_s}{G_n(n_s - n_0)} \tag{4.9}$$

$$\omega_s - \omega_{\mathrm{th}} = -\frac{\kappa}{\tau_{\mathrm{in}}}\{\alpha\cos(\omega_s\tau) + \sin(\omega_s\tau)\} \tag{4.10}$$

$$n_s = n_{\mathrm{th}} - \frac{2\kappa}{\tau_{\mathrm{in}}G_n}\cos(\omega_s\tau) \tag{4.11}$$

For zero feedback coefficient $\kappa = 0$, the above equations reduce to the solutions for the solitary laser already given by (3.62)–(3.64). We rewrite (4.10) as

$$\omega_{\mathrm{th}}\tau = \omega_s\tau + C\sin(\omega_s\tau + \tan^{-1}\alpha) \tag{4.12}$$

where the C parameter already introduced in the regimes of the dynamics for the optical feedback level in Sect. 4.1.1 is defined by (Tartwijk and Lenstra 1995)

$$C = \frac{\kappa\tau}{\tau_{\text{in}}}\sqrt{1+\alpha^2} \tag{4.13}$$

From (4.12), we can calculate modes for laser oscillations in the presence of optical feedback. The relation in (4.12) can be written by

$$\Delta\omega_s\tau = -C\sin(\varphi_0 + \Delta\omega_s\tau) \tag{4.14}$$

where $\Delta\omega_s = \omega_s - \omega_{\text{th}}$ corresponds to the steady-state value of the phase difference $\phi(t) - \phi(t-\tau)$ and $\varphi_0 = \omega_{\text{th}}\tau + \tan^{-1}\alpha$. When $C < 1$, there is only one solution for (4.12), as already discussed, which is dynamically stable and can be identified as a slightly changed solitary laser state. By increasing the C parameter value, the number of the mode solutions increases but is always an odd number. The curves in (C, φ_0) space in Fig. 4.2 that separate the regions of equal number of solutions are given by

$$\varphi_0 = (2m+1)\,\pi \pm \cos^{-1}\left(\frac{1}{C}\right) \mp C\sin\left\{\cos^{-1}\left(\frac{1}{C}\right)\right\} \tag{4.15}$$

where $C \geq 1$ and m is an integer number. This causes a pattern to arise in (C, φ_0) space, as shown in Fig. 4.2 where the roman numbers represent the number of solutions. For $C > 1$, multiple steady-state solutions appear.

The solutions in (4.12) are also graphically calculated as intersections of the curves $y = \omega_{\text{th}}\tau$ and $y = \omega_s\tau + C\sin(\omega_s\tau + \tan^{-1}\alpha)$ as shown in Fig. 4.3 (Fravre 1987; Murakami et al. 1997). When $C < 1$ (for a small optical feedback and a short external cavity), (4.12) has only a single solution and the laser exhibits stable oscillation. If $C > 1$, many possible modes for the laser oscillations (external modes and anti-modes) are generated with the relation among the internal laser modes and the excited external modes, and then the laser shows unstable operations. By adjusting the position of the external mirror (which is equivalent to appropriate selection of the round-trip time τ) and setting $\varphi_0 = \omega_{\text{th}}\tau + \tan^{-1}\alpha = 0$ (where $\omega_{\text{th}}\tau = -\tan^{-1}\alpha$ and, thus, the condition $\Delta\omega_s\tau = -C\sin(\Delta\omega_s\tau)$ is satisfied), the higher bound of the coefficient C for a single mode oscillation of the laser is easily obtained from Fig. 4.3 as $\mathrm{C} \sim 3\pi/2$ (Petermann 1988). Above this value $C > 3\pi/2$, many modes are excited and the laser becomes unstable. Complicated dynamics are observable in the output power, however, the laser does not always exhibit unstable oscillations. Even for such unstable regimes, the laser may show stable oscillations. The details of the dynamics will be discussed in Sect. 5.2.

When the C parameter well exceeds the value of unity, many modes are excited in the laser output and the laser becomes truly unstable. Another representation for possible oscillation modes is frequently used in the phase space of the oscillation frequency and the carrier density. Figure 4.4 is such a representation for the parameter space in the $\Delta\omega_s\tau$ versus Δn_s plane. The relation is calculated from (4.10) and (4.11)

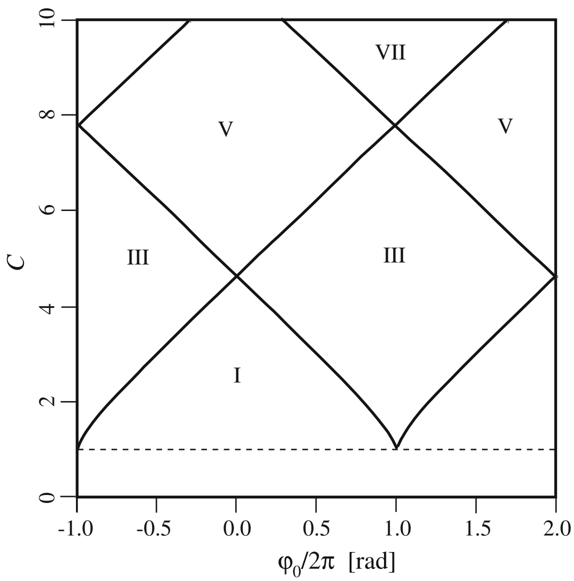

Fig. 4.2 Number of solutions for (4.14) in (C, φ_0) space. The roman numbers represent the number of solutions

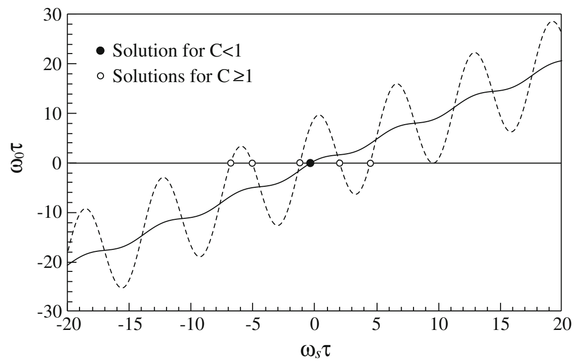

Fig. 4.3 Dependence of steady-state solutions for the phase on the parameter value C. *Solid* and *dashed lines* correspond to $C = 0.76$ and $C = 9.50$, respectively. The *black circle* denotes only one solution for $C < 1$ and *white circles* represent multiple solutions for $C > 1$

by eliminating the sine and cosine functions and is given by (Henry 1986)

$$\left(\Delta\omega_s\tau - \frac{\alpha\tau}{2}G_n\Delta n\right)^2 + \left(\frac{\tau}{2}G_n\Delta n\right)^2 = \left(\frac{\kappa\tau}{\tau_{\text{in}}}\right)^2 \tag{4.16}$$

where $\Delta\omega_s = \omega_s - \omega_{\text{th}}$ and $\Delta n_s = n_s - n_{\text{th}}$. The broken sinusoidal curve in the figure denotes the deviation from the steady state of the oscillation angular frequency $\Delta\omega_s$ and the other sinusoidal curve represents that of the carrier density Δn_s. The crossing points of these two curves are the locations of possible oscillations and they are on the ellipsoid given by (4.16) (thick solid curve in the figure).Those in the

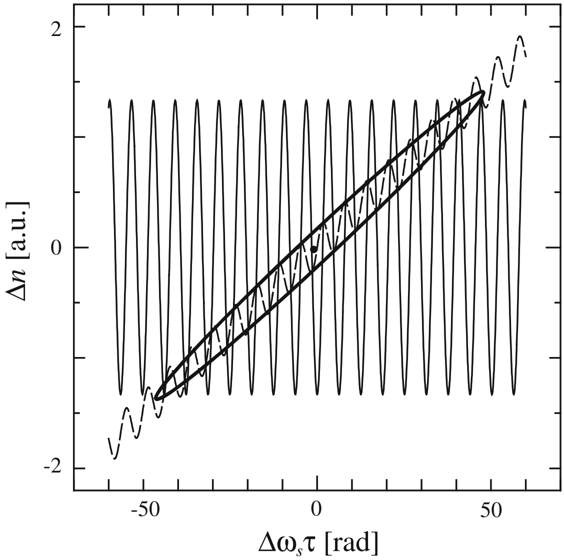

Fig. 4.4 Carrier density change Δn versus frequency change $\Delta\omega$ for the possible steady states under external feedback. The crossing points of the solid and broken sinusoidal waves are the locations of the modes. Modes are on an ellipsoid. The *solid dot* at center is the solitary oscillation mode

lower half are the solutions for stable oscillations (external modes) and those in the upper half are unstable oscillations. Solutions for unstable oscillations are sometimes called anti-mode. The laser oscillates at one of the external modes and the maximum gain mode is the most probable mode for laser oscillation. However, when the laser oscillation is unstable due to external feedback, the mode hops around among the external modes and the anti-modes, thus the laser exhibits chaotic oscillations. One typical instability is the phenomenon known as low-frequency fluctuations (LFFs), in which the laser output power shows frequent irregular dropouts having frequency from MHz to hundred MHz (Mørk et al. 1988; Fischer et al. 1996). The details for the origin of LFFs and their dynamics are again discussed in Sect. 5.3. The solid dot at the center of the ellipsoid in the figure is the solution for the laser oscillation in the solitary laser (solitary mode). The laser without optical feedback, of course, has no fluctuation in the sense of chaotic dynamics and oscillates only at this mode.

The stability and instability of laser oscillations in the presence of optical feedback are theoretically studied by the linear stability analysis for the steady-state solutions of the laser variables. In the same manner as a solitary laser, using the rate equations and taking the first order small infinities for the perturbations, the equations for the field δE, the phase $\delta\phi$, and the carrier density δn are calculated as (Tromborg et al. 1984)

$$\frac{\mathrm{d}\delta A(t)}{\mathrm{d}t} = \frac{1}{2}G_n A_s \delta n(t) - \frac{\kappa}{\tau_{\mathrm{in}}}\cos(\omega_s \tau)\{\delta A(t) - \delta A(t-\tau)\}$$
$$- \frac{\kappa}{\tau_{\mathrm{in}}} A_s \sin(\omega_s \tau)\{\delta\phi(t) - \delta\phi(t-\tau)\} \tag{4.17}$$

$$\frac{\mathrm{d}\delta \phi(t)}{\mathrm{d}t} = \frac{\alpha}{2}G_n A_s \delta n(t) + \frac{\kappa}{\tau_{\mathrm{in}}}\frac{\sin(\omega_s \tau)}{A_s}\{\delta A(t) - \delta A(t-\tau)\}$$
$$- \frac{\kappa}{\tau_{\mathrm{in}}} \cos(\omega_s \tau)\{\delta\phi(t) - \delta\phi(t-\tau)\} \tag{4.18}$$

$$\frac{\mathrm{d}\delta n(t)}{\mathrm{d}t} = -2G_n A_s (n_s - n_0)\delta A(t) - \left(G_n A_s^2 + \frac{1}{\tau_s}\right)\delta n(t) \tag{4.19}$$

Assuming that the perturbations take the forms of $\delta x(t) = \delta x \exp(\gamma t)$ ($x = A$, ϕ, and n), the characteristic equations for the condition having non-trivial solutions for the variables δA, $\delta\phi$, and δn are calculated from the following:

$$\begin{pmatrix} \gamma + \frac{\kappa}{\tau_{\mathrm{in}}}K\cos(\omega_s\tau) & \frac{\kappa}{\tau_{\mathrm{in}}}K A_s \sin(\omega_s\tau) & -\frac{1}{2}G_n A_s \\ -\frac{\kappa}{\tau_{\mathrm{in}}}\frac{K}{A_s}\sin(\omega_s\tau) & \gamma + \frac{\kappa}{\tau_{\mathrm{in}}}K\cos(\omega_s\tau) & -\frac{1}{2}\alpha G_n \\ 2A_s G_n(n_s - n_0) & 0 & \gamma + G_n A_s^2 + \frac{1}{\tau_s} \end{pmatrix} = 0 \tag{4.20}$$

where $K = 1 - \exp(-\gamma\tau)$. The oscillation modes for the perturbations are calculated by solving the characteristic equation

$$D(\gamma) = \gamma^3 + 2\{-\Gamma_R + \frac{\kappa}{\tau_{\mathrm{in}}}K\cos(\omega_s\tau)\}\gamma^2$$
$$+ \left\{\omega_R^2 - \frac{4\kappa K\Gamma_R}{\tau_{\mathrm{in}}}\cos(\omega_s\tau) + \left(\frac{\kappa}{\tau_{\mathrm{in}}}K\right)^2\right\}\gamma$$
$$- \frac{2\kappa K^2\Gamma_R}{\tau_{\mathrm{in}}} + \frac{\kappa K\omega_R^2}{\tau_{\mathrm{in}}}\{\cos(\omega_s\tau) - \alpha\sin(\omega_s\tau)\} = 0 \tag{4.21}$$

In the above equation, Γ_R and ω_R are the previously defined parameters of the damping factor and angular frequency of the relaxation oscillation at the solitary mode.

We cannot calculate explicit forms of the solutions for (4.21), since the equation includes the exponential form for the variable γ and, then, the solutions are numerically calculated. The real part of the solution is related to the stability of the mode and the imaginary part of it represents the oscillation frequency of the mode as has already been discussed. When the real part (damping factor) takes a negative value, the mode is stable and the excited oscillation damps out for the time development with a frequency calculated from the imaginary part. On the other hand, the mode is unstable for a positive value of the real part and the laser shows either regular or irregular oscillations with a typical frequency corresponding to the imaginary part. If the level of optical feedback is low or the condition $\kappa\tau/\tau_{\text{in}} \ll 1$ is satisfied, we can assume $|\gamma\tau| \ll 1$ and obtain the analytical form of the solution for γ. Then, the real and imaginary parts, Γ'_R and ω'_R, of the solution are given by (Agrawal and Dutta 1993)

$$\Gamma'_R = \Gamma_R \tag{4.22}$$

$$\omega'_R = \omega_R \sqrt{\frac{1 + (\kappa_c - \alpha\kappa_s)\tau/\tau_{\text{in}}}{(1 + \kappa_c\tau/\tau_{\text{in}})^2 + (\kappa_s\tau/\tau_{\text{in}})^2}} \tag{4.23}$$

where $\kappa_c = \kappa\cos(\omega_s\tau)$ and $\kappa_s = \kappa\sin(\omega_s\tau)$. Of course, (4.22) and (4.23) are equal to (3.70) and (3.71) at no optical feedback, respectively.

The relaxation frequency in the presence of optical feedback shifts from that of the solitary oscillation. Increase or decrease of the frequency shift depends on the signs of κ_c and κ_s, however, it is usually enhanced at moderate optical feedback and takes a larger value than that of the solitary oscillation. For a laser oscillation, the sign of the expression inside the square root in (4.23) must be positive and we obtain the stability condition (Acket et al. 1984; Lenstra et al. 1984)

$$1 + C\cos(\omega_s\tau + \tan^{-1}\alpha) > 0 \tag{4.24}$$

Equation (4.24) denotes that the laser becomes unstable for $C > 1$ as expected, while it is stable for $C < 1$ even if optical feedback is present in semiconductor lasers. We calculated oscillation modes for perturbations of the steady-state values for the variables. The solutions obtained from such characteristic equations are called linear modes, the name comes from the linear stability analysis.

4.2.2 Linear Mode, and Stability and Instability in Semiconductor Lasers

For certain ranges of optical feedback level, the output of a semiconductor laser evolves from stable states to chaotic states via unstable periodic oscillations. One or a few frequencies for the solutions derived from the characteristic equation in (4.21) are equal to or close to the typical frequency corresponding to the response of the system. Periodic oscillations in chaotic states are generally not harmonic oscillations, but they include an obscure fundamental frequency and its higher harmonics. In quasi-periodic oscillations, frequency peaks become obscured due to irregular oscillations and no clear spectral peak is observable in complete chaotic states, like white noises. In semiconductor lasers with optical feedback, modes generated by the internal and external cavities are mixed and the laser oscillates at one or several modes. The other

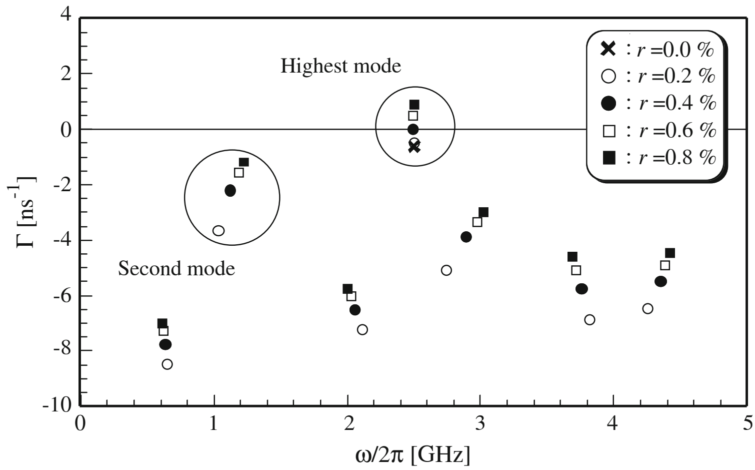

Fig. 4.5 Linear mode distributions at the external cavity length of $L = 10$ cm and the bias injection current of $J = 1.3\ J_{\mathrm{th}}$. The highest mode corresponds to the relaxation oscillation and the second mode to the external cavity mode. With increasing external feedback, the real part of each mode increases and the laser becomes less stable

important frequency of laser oscillations besides these modes is the frequency of the relaxation oscillation. Since chaos is a nonlinear phenomenon, many modes are not only related to the internal and external modes and the relaxation oscillation modes but also their sums and differences, and higher harmonics are excited (Cohen et al. 1988; Helms and Petermann 1990; Levine et al. 1995). For a chaotic bifurcation, the laser first becomes unstable with a frequency close to the relaxation oscillation, which is called period-1 oscillation. Next, the external mode is also excited. After that, many modes are excited and the laser oscillates at quasi-periodic oscillation. Then, the laser evolves into chaotic oscillations with complicated and broadened frequency components.

Figure 4.5 is an example of numerically calculated linear modes from (4.21) (Murakami and Ohtsubo 1998). In the figure, the change of modes is shown for the increase of the amplitude reflectivity from the external mirror. The vertical axis is the damping factor (the real part of the solution of the characteristic equation) and the horizontal axis is the frequency of the oscillation (the imaginary part of the solution). For a negative value of the damping factor, the mode damps out for a time evolution even if it is once excited. The value of the real part is negative for the highest mode in the absence of optical feedback (around the frequency of 2.5 GHz in this case) and the laser never gets into unstable oscillations. The frequency corresponds to the relaxation oscillation at the solitary mode. With the increase of the external feedback, the real part of the highest mode at first exceeds zero and the laser becomes unstable with a frequency of the relaxation oscillation (period-1 oscillation). Under the condition in this figure, the C parameter at which the laser at first exhibits unstable oscillation has a value of $C = 2.8$ (calculated from the external reflection of 0.4 %). The value is slightly less than $C = 3\pi/2$, which was estimated in the previous section, but the assumption in the previous section is proved to be reasonable. With further increase of the external reflectivity, the damping rates for all the modes increase and the laser becomes less stable. The second highest mode in the presence of optical feedback corresponds to the external cavity mode and the frequency is about 1.5 GHz (approximately equal to the frequency calculated from the external cavity length of 10 cm). With this mode, the laser shows higher periodic oscillations and it evolves into chaotic oscillations through a bifurcation for the increase of optical feedback (Ye and Ohtsubo 1998). As we recognize from the figure, the external frequency does not have a fixed value, but shifts with the increase of the reflectivity, except for the relaxation oscillation mode, which always almost has a fixed value.

4.2.3 Gain Reduction Due to Optical Feedback

Though direct analyses for the rate equations are essential for investigating the dynamics of semiconductor lasers with optical feedback, the steady-state analysis is still useful and important to obtain parameter conditions for stable and unstable laser operations. Here, we calculate the gain in the presence of optical feedback under a steady-state condition. We assume the same reflectivities calculated in (4.4) (the internal reflectivity r_0 and the external reflectivity r), the effective reflectivity at the front facet taking into account the external mirror at steady state is given by (Koelink et al. 1992; Osmundsen and Gade 1983; Kakiuchida and Ohtsubo 1994; Katagiri and Hara 1994)

$$r_{\mathrm{eff}} = \frac{r_0 + r\exp(i\omega_0\tau)}{1 + r_0 r\exp(i\omega_0\tau)} \tag{4.25}$$

We investigate the gain of laser oscillation in the presence of optical feedback under the condition of a small external reflectivity $r \ll 1$. From the above equation, the effective reflectivity is written by

$$r_{\mathrm{eff}} = |r_{\mathrm{eff}}|\exp(i\phi_r) \approx r_0 + (1 - r_0^2)r\exp(i\omega_0\tau) \tag{4.26}$$

where ϕ_r is the phase of the effective reflectivity. Also, the effective reflectivity $\kappa = (1 - r_0^2)r/r_0$ defined in (4.4) is small enough. Then, the absolute value and phase of the effective reflectivity are approximated as

$$|r_{\mathrm{eff}}| = r_0\{1 + \kappa\cos(\omega_0\tau)\} \tag{4.27}$$

$$\phi_r = \kappa\sin(\omega_0\tau) \tag{4.28}$$

The condition of laser oscillation under optical feedback is also given by the same equation as (3.3) and reads as

$$r_0 r_{\mathrm{eff}}\exp\{2ikl + (g - a)l\} = 1 \tag{4.29}$$

Therefore, the condition of the gain is

$$g_c = a + \frac{1}{l} \ln\left(\frac{1}{r_0|r_{\text{eff}}|}\right) \tag{4.30}$$

The difference between the gains with and without optical feedback for a small value of κ is given by

$$g_c - g_{\text{th}} = -\frac{\kappa}{l}\cos(\omega_0\tau) \tag{4.31}$$

The gain in the presence of optical feedback depends on the round-trip time τ and it changes periodically for the variation of the external cavity length. The mode for the maximum gain is attained at $\omega_0\tau = 2m\pi$ (m being an integer). As the gain varies depending on the optical feedback level, we can control or suppress the adjacent modes from the main oscillation mode by using the gain difference in accordance with (4.31) when the external mirror is positioned close to the laser facet. The difference of gains between successive modes in edge-emitting semiconductor lasers is as small as 0.1cm^{-1} and the condition $\kappa/l < 0.1$ is required for stable laser oscillations (Petermann 1988). For example, with an internal reflectivity of the laser facet of $r_0 = 0.56$ and the internal cavity length of $l = 300\,\mu\text{m}$, we obtain the condition of the stable laser oscillation for the external amplitude reflectivity as about $r < 2 \times 10^{-3}$. This value corresponds to that in regimes III to IV already discussed in Sect. 4.1.1 and is equal to the boundary of the regimes between the stable and unstable oscillations.

4.2.4 Linewidth in the Presence of Optical Feedback

The linewidth of laser oscillations in the presence of optical feedback is also calculated in the same manner as in Sect. 3.5.6. We consider small perturbations for the steady-state values of the variables in the presence of optical feedback and derive the linewidth from the power spectrum for the time derivative equations for the perturbations. The calculation is rather lengthy but straightforward, so that only the result is given here (Tromborg et al. 1984). Using the linewidth $\Delta\nu$ without optical feedback, the linewidth $\Delta\nu_{\text{ex}}$ in the presence of optical feedback is calculated as

$$\Delta\nu_{\text{ex}} = \frac{\Delta\nu}{F^2} \tag{4.32}$$

The coefficient $F = \mathrm{d}\omega_{\text{th}}/\mathrm{d}\omega_s$ for the reduction (or the broadening) of the spectral line width is calculated from (4.12) and given by

$$F = \frac{\mathrm{d}\omega_{\text{th}}}{\mathrm{d}\omega_s} = 1 + C\cos(\omega_s\tau + \tan^{-1}\alpha) \tag{4.33}$$

The minimum spectral linewidth is attained when the phase adjustment condition $\omega_s \tau = -\tan^{-1} \alpha$ is satisfied. Then, the spectral linewidth at the minimum condition is given by

$$\Delta \nu_{\text{ex}} = \frac{\Delta \nu}{(1 + C)^2} \tag{4.34}$$

On the other hand, the linewidth for the maximum gain condition at $\omega_s \tau = 2m\pi$ is calculated to be

$$\Delta \nu_{\text{ex}} = \frac{\Delta \nu}{\left(1 + \kappa \frac{\tau}{\tau_{\text{in}}}\right)^2} \tag{4.35}$$

The linewidth with optical feedback at the maximum gain condition is always less than the value of the solitary oscillation. These results hold for stable laser operations even when the laser is subjected to optical feedback. However, for optical feedback above a certain level, the laser does not oscillate at one of the modes but many modes are simultaneously excited or even drifting or wandering among the modes (external modes and anti-modes) occur. Such oscillations give rise to much noise (actually chaotic fluctuations) and even result in the collapse of coherence. These are the typical features in regimes III and IV in the preceding discussion. At this state, the linewidth of the laser is much broadened to as large as over GHz or more. However, the coherence of the laser recovers and the linewidth becomes narrow for a sufficiently strong optical feedback at regime V.

4.3 Feedback from a Grating Mirror

Other than conventional optical feedback reflectors, a grating mirror is frequently used to select the oscillation line in a semiconductor laser or stabilize the oscillation frequency. Grating optical feedback is originally applied for the stabilization of laser oscillations, however, it sometimes induces instabilities in lasers. Before discussing instabilities, we present the theoretical background of grating feedback and stabilization of optical frequency. For a small feedback coefficient and also small detuning between the laser and grating frequencies, the complex field equation can be approximately written by a similar equation of conventional optical feedback as

$$\frac{\mathrm{d}E(t)}{\mathrm{d}t} = \frac{1}{2}(1 - i\alpha)G_n\{n(t) - n_{\text{th}}\}E(t) + \frac{\kappa_g}{\tau_{\text{in}}}E(t-\tau)\exp(-i\Delta\omega t + i\omega_g \tau) \tag{4.36}$$

where κ_g is the feedback coefficient from the grating mirror and $\Delta\omega$ is the angular frequency detuning given by $\Delta\omega = \omega_g - \omega_0$ (ω_g being the angular frequency of the grating feedback). However, in general, the optical feedback from the grating mirror is strong and the frequency detuning between the laser oscillation and the grating is as large as up to several nanometers in wavelength. Therefore, the approximation in (4.36) is only valid within a small range of grating feedback. To treat the dynamics of grating feedback in a strict sense, the relation of the phase between the complex fields for the forward and backward propagations as a multiple-reflection model must be taken into account (Pittoni et al. 2001). Instead, we here consider the static model of the grating feedback and some stable and unstable features of the dynamics are presented.

The effective reflectivity of the static model in grating feedback including the laser facet and the grating mirror with multiple reflections is calculated in the same manner as the conventional mirror in (4.25) and is given by (Binder et al. 1990; Genty et al. 2000)

$$r_{\mathrm{eff}} = |r_{\mathrm{eff}}| \exp(i\phi_r) = \frac{r_0 + r(\omega)\exp(i\omega\tau)}{1 + r_0 r(\omega)\exp(i\omega\tau)} \tag{4.37}$$

The above equation has the same form as (4.25), but the external reflectivity by the grating mirror is a function of the optical frequency $\nu = \omega/2\pi$. The condition of the laser oscillation can be written in the same form as (4.29) and the gain is also given by (4.30). We here apply the steady-state analysis and calculate the conditions for the phase and the gain. Putting the angular frequency of the laser oscillation as $\omega = \omega_g$, the phase condition in the presence of grating optical feedback reads as

$$2\eta\omega_g l/c + \phi_r = 2m\pi \tag{4.38}$$

where m is an integer and $2\eta\omega_g l/c = 2m'\pi$ is the oscillation condition for a solitary laser. From the relation $\Delta(\eta\omega_g) = \omega_{\mathrm{th}}\Delta\eta + (\omega_g - \omega_{\mathrm{th}})\eta$, the change in the round-trip phase $\Delta\phi_d$ compared to $2m\pi$ due to grating tuning is written by

$$\Delta\phi_d = \frac{2l}{c}\{\omega_{\mathrm{th}}\Delta\eta + (\omega_g - \omega_{\mathrm{th}})\eta\} + \phi_r \tag{4.39}$$

where $\Delta\eta$ is expanded by the carrier density and the angular frequency as

$$\Delta\eta = \frac{\partial\eta}{\partial n}(n - n_{\mathrm{th}}) + \frac{\partial\eta}{\partial\omega}(\omega_g - \omega_{\mathrm{th}}) \tag{4.40}$$

Using the definition of the refractive index in (3.40), i.e., $\eta_c = \eta - i\eta'$, together with the equalities

$$\frac{\partial\eta}{\partial n} = \alpha\frac{\partial\eta'}{\partial n} = -\frac{\alpha c}{2\omega_{\mathrm{th}}}\frac{\partial g}{\partial n} \tag{4.41}$$

the relation between the carrier density and the gain is written as

$$\frac{\partial\eta}{\partial n}(n - n_{\mathrm{th}}) = -\frac{\alpha c}{2\omega_{\mathrm{th}}}(g_{\mathrm{g}} - g_{\mathrm{th}}) \tag{4.42}$$

where g_{g} is the gain in the presence of grating feedback. Substituting (4.40)–(4.42) into (4.39) together with the relation in (3.8), the phase change reads as

$$\Delta\phi_d = -\alpha(g - g_{\rm th})l + \frac{2\eta_e l}{c}(\omega_g - \omega_{\rm th}) + \phi_r \quad (4.43)$$

Putting $\Delta\phi_d = 0$ for a possible solution for the laser oscillation and using the internal trip time of light $\tau_{\rm in} = 2\eta_e l/c$, we obtain

$$\omega_g - \omega_{\rm th} = \frac{1}{\tau_{\rm in}}\{\alpha(g_{\rm g} - g_{\rm th})l - \phi_r\} \quad (4.44)$$

Then, the reduction of gain in the presence of grating feedback is given by

$$g_{\rm g} - g_{\rm th} = \frac{1}{l}\ln\frac{1}{r_0|r_{\rm eff}(\omega_g)|} \quad (4.45)$$

and the linewidth reduction factor is calculated as

$$F_g = \frac{{\rm d}\omega_{\rm th}}{{\rm d}\omega_g} = 1 + \frac{1}{\tau_{\rm in}}\frac{{\rm d}\phi_r}{{\rm d}\omega_g} - \frac{\alpha}{\tau_{\rm in}}\frac{\rm d}{{\rm d}\omega_g}\left\{\ln\frac{1}{|r_{\rm eff}(\omega_g)|}\right\} \quad (4.46)$$

The linewidth of a semiconductor laser with grating optical feedback is finally written as

$$\Delta\nu_g = \frac{\Delta\nu}{F_g^2} \quad (4.47)$$

where $\Delta\nu$ is again the linewidth of the solitary laser defined by (3.114). When the laser beam has a Gaussian profile and a certain diffraction order is selected by the grating as a feedback light, the reflectivity is explicitly given by

$$r(\omega_g) = r_g \exp\{-\frac{(\omega_g - \omega_G)^2}{\Delta\omega_G^2}\} \quad (4.48)$$

where ω_G is the selected angular frequency of the grating, r_g is its reflectivity, and $\Delta\omega_G$ is the width of the grating resolution at that angular frequency defined by $\Delta\omega_G = c\tan\theta/w_0$ (θ is the incidence angle of light onto the grating and $2w_0$ is the diameter of the Gaussian beam). The linewidth of a semiconductor laser is narrowed by a grating feedback under stable oscillation. However, it is again noted that the laser becomes unstable even by a grating feedback for a certain range of the feedback strength, either for small or strong grating feedback.

4.4 Phase-Conjugate Feedback

A semiconductor laser is frequently used as a light source of phase-conjugate optics (Pochi 1993). Or a phase-conjugate mirror is positively used to return light exactly into the active region in a semiconductor laser, since the light reflected from the phase-conjugate mirror is automatically fed back into the laser cavity due to the generation of the conjugate wave without any additional optical components in the external optical path. The phase-conjugate feedback induces instabilities in the laser oscillation and the dynamics of the laser are not always the same as those from the ordinary feedback reflector. The typical timescale in semiconductor lasers with optical feedback is of the order of a nanosecond, defined by the laser relaxation oscillation frequency. Therefore, typical effects of phase-conjugate feedback occur when the phase-conjugate mirrors respond as fast as this timescale. Such phase-conjugate mirrors are realized in quick-response Kerr media with large third-order susceptibility and also quick-response photorefractive mirrors of semiconductor materials (Agrawal and Klaus 1991; Agrawal and Gray 1992; Tartwijk et al. 1992; Langley and Shore 1994; Gray et al. 1993, 1994; Bochove et al. 1997). On the other hand, the dynamics for slow-response photorefractive mirrors, where the response is much slower than the time variations of the laser dynamics, are the same as those for ordinary plain reflection mirrors. For a slow-response photorefractive crystal, for example a $TiBaO_3$ crystal, the laser light automatically returns into the laser cavity, however, the mirror produces the same dynamics of optical feedback as an ordinary reflection mirror (Miltyeni et al. 1995; Liby and Statman 1996; Murakami and Ohtsubo 1999). Only the spatial phase-conjugate characteristic is effective in such optical feedback. In either case of fast or slow response phase-conjugate mirrors, phase-conjugate feedback can be also applied to control the quality of oscillations for semiconductor lasers (Gray et al. 1995; Kurz and Mukai 1996; Anderson 1999).

Figure 4.6 shows an optical setup for generating a phase-conjugate wave by four-wave mixing from a phase-conjugate mirror. We here assume that the phase-conjugate mirror responds much faster than the typical chaotic fluctuations of semiconductor lasers. The angular frequencies of the signal and pump beams at the phase-conjugate mirror are set to be ω_0 and ω_p, respectively, and the generated phase-conjugate wave has a frequency $\omega_c = 2\omega_p - \omega_0$. Therefore, we consider the angular frequency detuning $2\delta = 2(\omega_p - \omega_0)$ between the laser angular frequency and that of the feedback light. Thus, the equation of the complex field E for the semiconductor laser with phase-conjugate feedback is given by

$$\frac{\mathrm{d}E(t)}{\mathrm{d}t} = \frac{1}{2}(1-i\alpha)G_n\{n(t)-n_{\mathrm{th}}\}E(t)+\frac{\kappa}{\tau_{\mathrm{in}}}E^*(t-\tau)\exp\left\{-i2\delta\left(t-\frac{\tau}{2}\right)+i\phi_{\mathrm{PCM}}\right\} \tag{4.49}$$

where ϕ_{PCM} is the phase shift induced by the reflection at the phase-conjugate mirror. The final term in the above equation is the effect of phase-conjugate feedback. The rate equations for the field amplitude, the phase, and the carrier density are written as

$$\frac{\mathrm{d}A(t)}{\mathrm{d}t} = \frac{1}{2}G_n\{n(t)-n_{\mathrm{th}}\}A(t)+\frac{\kappa}{\tau_{\mathrm{in}}}A(t-\tau)\cos\theta(t) \tag{4.50}$$

$$\frac{\mathrm{d}\phi(t)}{\mathrm{d}t} = \frac{1}{2}\alpha G_n\{n(t)-n_{\mathrm{th}}\}-\frac{\kappa}{\tau_{\mathrm{in}}}\frac{A(t-\tau)}{A(t)}\sin\theta(t) \tag{4.51}$$

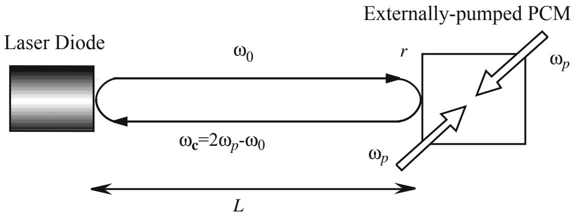

Fig. 4.6 Optical setup in a semiconductor laser with phase-conjugate optical feedback

$$\frac{\mathrm{d}n(t)}{\mathrm{d}t} = \frac{J}{ed} - \frac{n(t)}{\tau_s} - G_n\{n(t) - n_0\}A^2(t) \tag{4.52}$$

$$\theta(t) = 2\delta\left(t - \frac{\tau}{2}\right) + \phi(t) + \phi(t - \tau) + \phi_{\mathrm{PCM}} \tag{4.53}$$

Equations (4.50)–(4.52) are in the same form as (4.5)–(4.7), however, (4.53) is different from (4.8) even for zero detuning ($\delta = 0$). This makes the laser dynamics of phase-conjugate feedback different from those of an ordinary optical feedback reflector.

A typical feature of the dynamics in phase-conjugate feedback is the phase locking phenomenon. The steady-state solutions for the field, the phase, and the carrier density at zero detuning $\delta = 0$ are given by

$$A_s^2 = \frac{J/ed - n_s/\tau_s}{G_n(n_s - n_0)} \tag{4.54}$$

$$\phi_s = \frac{1}{2}\tan^{-1}(-\alpha) \tag{4.55}$$

$$n_s = n_{\mathrm{th}} - \frac{2\kappa\cos(2\phi_s)}{G_n} \tag{4.56}$$

Namely, the phase is locked to a certain value given by (4.55), while it changes depending on the time of the feedback loop in the conventional external reflector and it has multiple solutions for the laser oscillations (see (4.10)). The laser for ordinary optical feedback is very sensitive to short variations of the external mirror compatible with optical wavelength. However, the phase of the laser with phase-conjugate feedback does not show any change for such a small variation of the external mirror. Here, we discussed the case when the phase-conjugate mirror responds immediately after the arrival of the signal beam. The laser dynamics of semiconductor lasers with a finite response time in a phase-conjugate mirror have also been discussed (DeTienne et al. 1997; van der Graaf et al. 1998). For a finite response of a phase-conjugate mirror, the equation for the complex field is given by

$$\begin{aligned}\frac{\mathrm{d}E(t)}{\mathrm{d}t} &= \frac{1}{2}(1-i\alpha)G_n\{n(t)-n_{\mathrm{th}}\}E(t)\\ &\quad + \frac{\kappa}{\tau_{\mathrm{in}}}\exp\left\{-i2\delta\left(t-\frac{\tau}{2}\right)\right\}\int_{-\infty}^{t}E^*(t'-\tau)\exp\left\{-(1-i\delta t_m)\frac{(t-t')}{t_m}\right\}\mathrm{d}t'\end{aligned} \tag{4.57}$$

where t_m is the time that it takes the light to penetrate the phase-conjugate mirror. Above, we assumed a fast-response phase-conjugate mirror, but similar dynamics are obtained for a finite-response phase-conjugate mirror.

4.5 Incoherent Feedback

Coherent optical feedback effects are important in applications of semiconductor lasers. For a long external cavity when the feedback light has an incoherent coupling with the original light in the laser cavity, the rate equations in (4.5)–(4.7) are still applicable for investigating the laser dynamics. Even in incoherent optical feedback, a laser becomes unstable and shows instability and chaos in its output. For example, the dynamics of long external optical feedback from a reflector over the coherence length of a semiconductor laser is treated as those for incoherent schemes. Indeed, the coherence length of a semiconductor laser is usually several tens to a hundred meters, since the linewidth of the laser oscillations is around several mega hertz without any frequency stabilization. Also, polarization-rotated optical feedback under certain conditions is sometimes treated as a system of incoherent optical feedback. In these case, the returned laser field does not interfere with the inner oscillation field, but acts as the perturbation for carriers and has the coupling with them. Namely, the feedback term is introduced to the carrier density in the rate equations. Through this interaction, the laser shows instabilities.

The model is described by the following rate equations (Otsuka and Chern 1991):

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} = G_n\{n(t)-n_{\mathrm{th}}\}S(t) \tag{4.58}$$

$$\frac{\mathrm{d}n(t)}{\mathrm{d}t} = \frac{J}{ed} - \frac{n(t)}{\tau_s} - G_n\{n(t)-n_0\}\{S(t)+\kappa' S(t-\tau)\} \tag{4.59}$$

where κ' is the feedback coefficient coupled with the carrier density and τ has the same definition as before (the round-trip time of light in the external cavity). We do not have to consider the phase, since the phenomena come from the incoherent origin. The rate equations are only written by two differential equations, however, they are coupled with each other by the delay differential term. Thus, we can expect instabilities and chaos in semiconductor lasers. One of the typical features in incoherent optical feedback is sustained pulsations in the laser output. The gain saturation term discussed in Sect. 3.3.4 must be taken into account for such pulsations. In incoherent optical feedback in semiconductor lasers, we obtain not only irregular or chaotic pulsations in the laser output but also regular pulsings (such as period-1 oscillations) with high-speed oscillations as fast as picoseconds (Otsuka and Chern 1991). Those regular fast pulsing oscillations are important for the application of light sources in high-speed optical communications.

4.6 Polarization-Rotated Optical Feedback

Generally, narrow-stripe edge-emitting semiconductor laser oscillates at a transverse electric mode (TE mode). On the other hand, the counter polarization mode, i.e., transverse magnetic mode (TM mode), is not a lasing mode and is scarcely excited. The optical gain of a TM mode is slightly less than that of a TE mode and the laser preferredly oscillates at the TE mode due to the nonlinear effect of lasing. However, the TM mode starts to oscillate when the TE mode is coherently coupled to the TM mode through the polarization-rotated optical feedback. We consider here the case of strong optical feedback from a crossed-polarization component, where the orthogonal-polarization component becomes the lasing mode. Figure 4.7 shows two examples of single path systems with orthogonal-polarization optical feedback. Figure 4.7a is a ring-loop model for orthogonal-polarization optical feedback, by which we can avoid multiple-reflection scheme within optical feedback loop. The main oscillated TE mode from a narrow-stripe edge-emitting laser goes through a polarization beam splitter and is converted into a TM mode by $\lambda/4$ and $\lambda/2$ wave-plates. Figure 4.7b is another example of orthogonal-polarization feedback systems. The TE polarized beam enters a Faraday rotator (RT), whose input polarizer is removed, and the beam's polarization rotates 45°. The beam reflected by the feedback mirror is reinjected to the rotator, and this creates an orthogonal polarized beam to the laser oscillation mode (i.e., TM mode). In this configuration, the reflected vertical beam from the laser facet is once passed through the rotator, but it is blocked by the polarizer (PL). Thus, a single feedback loop is guaranteed in this setup. For both systems, the effect of orthogonal-polarization feedback can be described by the same rate equations. For a strong crossed-polarizing optical feedback (say, for example, 10 times larger than ordinary parallel-polarization optical feedback to induce chaotic oscillations), the TM oscillation merges in the laser output power besides the TE oscillation mode. In this situation, we can observe quite different dynamics compared with ordinary parallel-polarizing optical feedback and the detail of the dynamics will be discussed in Chap. 5 (Heil et al. 2003).

For the crossed-polarization scheme with strong optical feedback, we must use a coherent model for the laser oscillations, since both the amplitudes of TE- and TM-modes are time-dependent functions and coherently couple with each other. Then the rate equations of crossed-polarization feedback system are written as

$$\frac{\mathrm{d}A_{\mathrm{TE}}(t)}{\mathrm{d}t} = \frac{1}{2}G_{n,\mathrm{TE}}\{n(t) - n_{\mathrm{th,TE}}\}A_{\mathrm{TE}}(t) \tag{4.60}$$

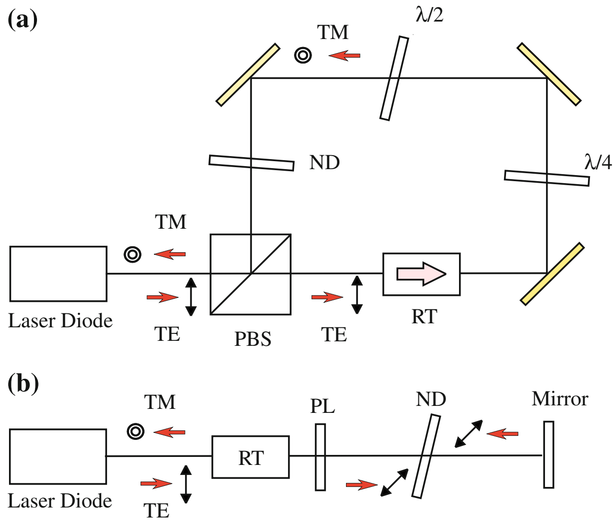

Fig. 4.7 Optical setups of orthogonal polarization feedback in semiconductor laser. **a** Ring-loop feedback system. PBS: polarization beam splitter, RT: Faraday rotator, $\lambda/4$: $\lambda/4$ waveplate, $\lambda/2$: $\lambda/2$ waveplate, ND: neutral density filter. **b** Single-pass feedback system. PL: polarizer

$$\frac{\mathrm{d}\phi_{\mathrm{TE}}(t)}{\mathrm{d}t} = \frac{1}{2}\alpha G_{n,\mathrm{TE}}\{n(t) - n_{\mathrm{th,TE}}\} \tag{4.61}$$

$$\frac{\mathrm{d}A_{\mathrm{TM}}(t)}{\mathrm{d}t} = \frac{1}{2}G_{n,\mathrm{TM}}\{n(t) - n_{\mathrm{th,TM}}\}A_{\mathrm{TM}}(t) + \frac{\kappa}{\tau_{\mathrm{in}}}A_{\mathrm{TE}}(t-\tau)\cos\theta(t) \tag{4.62}$$

$$\frac{\mathrm{d}\phi_{\mathrm{TM}}(t)}{\mathrm{d}t} = \frac{1}{2}\alpha G_{n,\mathrm{TM}}\{n(t) - n_{\mathrm{th,TM}}\} - \frac{\kappa}{\tau_{\mathrm{in}}}\frac{A_{\mathrm{TE}}(t-\tau)}{A_{\mathrm{TM}}(t)}\sin\theta(t) \tag{4.63}$$

$$\frac{\mathrm{d}n(t)}{\mathrm{d}t} = \frac{J}{ed} - \frac{n(t)}{\tau_s} - \{n(t) - n_0\}\{G_{n,\mathrm{TE}}A^2_{\mathrm{TE}}(t) + G_{n,\mathrm{TM}}A^2_{\mathrm{TM}}(t)\} \tag{4.64}$$

$$\theta(t) = \omega_0\tau + \phi_{\mathrm{TM}}(t) - \phi_{\mathrm{TE}}(t-\tau) \tag{4.65}$$

where the subscripts TE and TM represent the variables and parameters for TE- and TM-modes. The gain G_n and the carrier density at threshold n_{th} has different values for the TE- and TM-modes in a strict sense. When an optical feedback is small, the terms for the TM-mode in (4.62) and (4.63) is eliminated and we can put $A^2_{\mathrm{TM}}(t) \propto A^2_{\mathrm{TE}}(t-\tau)$. Then replacing Eq. (4.60) for the photon number, the relations of (4.58) and (4.59) hold. Crossed-polarization optical feedback plays an important role in VCSELs as will be discussed in Chap. 8. In VCSELs, typical polarization dynamics are observable even for a small amount of optical feedback with crossed-polarization.

4.7 Filtered Feedback

We have discussed several optical feedback schemes and formulated the equations for the models. We can consider systematic treatments for these models (Yousefi and Lenstra 1999; Lenstra et al. 2005; Green and Krauskopf 2006). We here formulate the preceding optical feedback models. Also, the formulation can be extended to other feedback models such as optoelectronic feedback models, which will be discussed in Chap. 7. Through the introduction of systematic descriptions, we can give rise to a good perspective for universal understanding of the dynamics in feedback phenomena in semiconductor lasers, i.e., coherent and incoherent optical feedback, phase-conjugate feedback, grating feedback, etc. Figure 4.8 shows the notation of the system for filtered feedback. Assuming that the laser field E and the feedback function given by an external device F are slowly time-dependent amplitudes, the filtered feedback system is written as

$$\frac{\mathrm{d}E(t)}{\mathrm{d}t} = \frac{1}{2}(1-i\alpha)G_n\{n(t)-n_{\mathrm{th}}\}E(t) + \frac{\kappa_{\mathrm{feedback}}}{\tau_{\mathrm{in}}}F(t) \tag{4.66}$$

We assume that the emitted laser field is $E(t)\mathrm{e}^{-\mathrm{i}\omega_0 t} + c.c.$ and the feedback field $F(t)\mathrm{e}^{-\mathrm{i}\omega_0 t} + c.c.$. For a linearly responding device, the function is given by

$$F(t) = \int_{-\infty}^{t} r(t'-t)E(t')\mathrm{d}t \tag{4.67}$$

where $r(t)$ represents the response function of the external devices. It is noted that, in a case of phase-conjugate optical feedback, E in (4.67) must be replaced by E^*. The carrier density equation remains the same and is given as

$$\frac{dn(t)}{\mathrm{d}t} = \frac{J}{ed} - \frac{n(t)}{\tau_s} - G_n\{n(t)-n_0\}\,|E(t)|^2 \tag{4.68}$$

For simplicity, the response function is assumed to be given by a simple Lorentzian frequency filter. Indeed, the spectral form of the transfer function induced by optical feedback from a grating or Fabry-Perot filter can be given by a Lorentzian shape as will be discussed in Chap. 5. From the Fourier transform relation, the time-dependent response function is given as

$$r(t) = \Lambda\exp\{-\Lambda|t| - i(\omega_{\mathrm{c}}-\omega_0)t\} \tag{4.69}$$

where ω_{c} is the central frequency of the Lorentz spectrum and Λ is the half-width at half-maximum (HWHM) of the spectrum. Under this assumption, one obtains the differential equation for the feedback as

$$\frac{\mathrm{d}F(t)}{\mathrm{d}t} = \Lambda E(t-\tau)\exp(i\omega_0\tau) - \{\Lambda + i(\omega_{\mathrm{c}}-\omega_0)\}F(t) \tag{4.70}$$

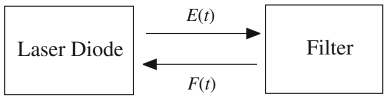

Fig. 4.8 Notation of filtered feedback

In general, the response does not always have a Lorentzian spectral function in coherent optical feedback. However, a general response function can be expanded by a linear superposition of Lorentz functions and one can generally decompose the response function as a sum of exponential functions of the same type of the equation $r(t) = \Lambda \exp\{-\Lambda|t| - i(\omega_c - \omega_0)t\}$.

From the above discussions, we can figure out general descriptions for the systems with filtered optical feedback. In the following, we will study the explicit forms of the feedback function for some limiting cases. In a conventional optical feedback without frequency filter (usual plane mirror feedback), Λ is assumed to be infinity. In this limit, the differential equation is simply reduced as

$$F(t) = E(t - \tau)\exp(i\omega_0\tau). \tag{4.71}$$

The expression, of course, is the same as the extra term added to the field equation of a semiconductor laser with optical feedback in (4.2). For a very narrow filter case, i.e., $\Lambda \to 0$, (one of such examples is optical injection from a different laser), the feedback function is easily calculated as

$$F(t) = E_{\mathrm{inj}}(t)\exp\{-i(\omega_m - \omega_0)t\}. \tag{4.72}$$

Injection-locking instability will be discussed in Chap. 6. The third example is optical feedback from a four-wave mixing phase-conjugate mirror with finite time response time and where the feedback field is detuned from the solitary laser, which was discussed in Sect. 4.4. In a four-wave mixing phase-conjugate optical feedback, the differential equation of the response function is modified

$$\frac{\mathrm{d}F(t)}{\mathrm{d}t} = \Lambda E^*(t - \tau)\exp\left\{-2i\delta(t - \frac{\tau}{2})\right\} - (\Lambda + i\delta)F(t), \tag{4.73}$$

where δ is the detuning of the angular frequency between the four-wave mixing pump beam ω_p and the reference frequency ω_0, i.e., $\delta = \omega_p - \omega_0$. A system with optoelectronic feedback is also written by the same feedback function as discussed here, and the dynamics of such systems will be treated in Chap. 7.

References

Agrawal GP (1984) Line narrowing in a single mode injection laser due to external optical feedback. IEEE J Quantum Electron 20:468–471

Agrawal GP, Dutta NK (1993) Semiconductor lasers, 2nd edn. Van Nostrand Reinhold, New York

Agrawal GP, Gray GR (1992) Effect of phase-conjugate feedback on the noise characteristics of semiconductor lasers. Phys Rev A 46:5890–5898

Agrawal GP, Klaus JT (1991) Effect of phase-conjugate feedback on semiconductor laser dynamics. Opt Lett 16:1325–1327

Acket GA, Lenstra D, den Boef J, Verbeek BH (1984) The influence of feedback intensity on longitudinal mode properties and optical noise in index-guided semiconductor lasers. IEEE J Quantum Electron 20:1163–1169

Anderson OK, Fischer APA, Lane LC, Louvergneaux E, Stolte S, Lenstra D (1999) Experimental stability diagram of a diode laser subject to weak phase-conjugate feedback from a Rubidium vapor cell. IEEE J Quantum Electron 35:577–582

Binder JO, Cormack D, Somani A (1990) Intermodal tuning characteristics of an INGaAsP laser with optical feedback from an external grating reflector. IEEE J Quantum Electron 26:1191–1199

Bochove E (1997) Theory of a semiconductor laser with phase-conjugate optical feedback. Phys Rev A 55:3891–3899

Cho Y, Umeda M (1986) Observation of chaos in a semiconductor laser with delayed feedback. Opt Commun 59:131–136

Cohen JS, Dentine RR, Verbeek BH (1988) The effect of optical feedback on the relaxation oscillation in semiconductor lasers. IEEE J Quantum Electron 24:1989–1995

DeTienne DH, Gray GR, Agrawal GP, Lenstra D (1997) Semiconductor laser dynamics for feedback from a finite-penetration-depth phase-conjugate mirror. IEEE J Quantum Electron 33:838–844

Fleming MW, Mooradian A (1981a) Fundamental line broadening of single-mode (AaAl)As diode lasers. Appl Phys Lett 38:511–513

Fleming MW, Mooradian A (1981b) Spectral characteristics of external-cavity controlled semiconductor lasers. IEEE J Quantum Electron 17:44–59

Fischer I, van Tartwijk GHM, Levine AM, Elsäßer W, Göbel EO, Lenstra D (1996) Fast pulsing and chaotic itinerancy with a drift in the coherence collapse of semiconductor lasers. Phys Rev Lett 76:220–223

Fravre F (1987) Theoretical analysis of external optical feedback on DFB semiconductor lasers. IEEE J Quantum Electron 23:81–88

Gavrielides A, Kovanis V, Erneux T (1997) Analytical stability boundaries for a semiconductor laser subject to optical injection. Opt Commun 136:253–256

Genty G, Gröhn A, Talvitie H, Kaivola M, Ludvigsen H (2000) Analysis of the linewidth of a grating-feedback GaAlAs laser. IEEE J Quantum Electron 36:1193–1198

Glas P, Müller R, Klehr A (1983) Bistability, self-sustained oscillations, and irregular operation of a GaAs laser coupled to an external resonator. Opt Commun 47:297–301

Goldberg L, Taylor HF, Dandrige A, Weller JF, Miles RO (1982) Coherence collapse in single mode semiconductor lasers due to optical feedback. IEEE J Quantum Electron 18:555–679

Gray GR, Ryan AT, Agrawal GP, Gage EC (1993) Control of optical-feedback-induced laser intensity noise in optical data recording. Opt Eng 32:739–745

Gray GR, Huang D, Agrawal GP (1994) Chaotic dynamics of semiconductor lasers with phase-conjugate feedback. Phys Rev A 49:2096–2105

Gray GR, DeTienne DH, Agrawal GP (1995) Mode locking in semiconductor lasers by phas-conjugate optical feedback. Opt Lett 20:1295–1297

Green K, Krauskopf B (2006) Mode structure of a semiconductor laser subject to filtered optical feedback. Opt Commun 258:243–255

Henry CH (1986) Theory of spontaneous emission noise in open resonators and its applications to lasers and optical amplifiers. J Lightwave Technol 4:288–297

Helms J, Petermann K (1990) A simple analytic expression for the stable operation range of laser diode with optical feedback. IEEE J Quantum Electron 26:833–836

Heil T, Uchida A, Davis P, Aida T (2003) TE-TM dynamics in a semiconductor laser subject to polarization-rotated optical feedback. Phys Rev A 68: 033811-1–8

Katagiri Y, Hara S (1994) Increased spatial frequency in interferential undulations of coupled cavity lasers. Appl Opt 33:5564–5570

Kakiuchida H, Ohtsubo J (1994) Characteristics of a semiconductor laser with external feedback. IEEE J Quantum Electron 30:2087–2097

Kikuchi K, Okoshi T (1982) Simple formula giving spectrum-narrowing ratio of semiconductor-laser output obtained by optical feedback. Electron Lett 18:10–12

Koelink MH, Slot M, de Mul FFM, Grever R, Graaff AC, Dasse M, Aaroudse JG (1992) Laser Doppler velocimeter based on the self-mixing effect in a fiber-coupled semiconductor laser. Theory Appl Opt 31:3401–3408

Kürz P, Mukai T (1996) Frequency stabilization of semiconductor laser by external phase-conjugate feedback. Opt Lett 21:1369–1371

Lang R, Kobayashi K (1980) External optical feedback effects on semiconductor injection properties. IEEE J Quantum Electron 16:347–355

Langley LN, Shore KA (1994) Intensity noise and linewidth characteristics of laser diodes with phase conjugate optical feedback. IEE Proc Optoelectron 141:103–108

Lenstra D, van Vaalen M, Jaskorzynska B (1984) On the theory of a single-mode laser with weak optical feedback. Physica 125C:255–264

Lenstra L, Verbeek BH, den Boef AJ (1985) Coherence collapse in single-mode semiconductor lasers due to optical feedback. IEEE J Quantum Electron 21:674–679

Lenstra D, Vemuri G, Yousefi M (2005) Generalized optical feedback. In: Kane DM, Shore KA (eds) Unlocking dynamical diversity—Optical feedback effects on semiconductor lasers. Chap 3. Wiley, Chichester

Levine AM, van Tartwijk GHM, Lenstra D, Erneux T (1995) Diode lasers with optical feedback: stability of the maximum gain mode. Phys Rev A 52:R3436–R3439

Liby BW, Statman D (1996) Controlling the linewidth of a semiconductor laser with photorefractive phase conjugate feedback. IEEE J Quantum Electron 32:835–838

Lin C, Burrus CA Jr, Coldren LA (1984) Characteristics of single-longitudinal mode selection in short-coupled-cavity (SCC) injection lasers. J Lightwave Technol 2:544–549

Mils RO, Dandrigdge A, Tveten B, Taylor HF, Tgiallorenzi TG (1980) Feedback induced line broadening in cw channel-substrate planar laser diodes. Appl Phys Lett 37:990–992

Mørk J, Tromborg B, Christiansen PL (1988) Bistability and low-frequency fluctuations in semiconductor lasers with optical feedback: a theoretical analysis. IEEE J Quantum Electron 24:123–133

Mørk J, Mark J, Tromborg B (1990a) Route to chaos and competition between relaxation oscillations for a semiconductor laser with optical feedback. Phys Rev Lett 65:1999–2002

Mørk J, Semkow M, Tromborg B (1990b) Measurement and theory of mode hopping in external cavity laser. Electron Lett 26:284–285

Mørk J, Tromborg B, Mark J (1992) Chaos in semiconductor lasers with optical feedback: theory and experiment. IEEE J Quantum Electron 28:93–108

Miltyeni E, Ziegler MO, Hofmann M, Sacher J, Elsäßer W, Göbel EO, MacFarlane DL (1995) Long-term stable mode locking of a visible diode laser with phase-conjugate feedback. Opt Lett 20:734–736

Murakami A, Ohtsubo J (1998) Dynamics and linear stability analysis in semiconductor lasers with phase-conjugate feedback. IEEE J Quantum Electron 34:1979–1986

Murakami A, Ohtsubo J (1999) Dynamics of semiconductor lasers with optical feedback from photorefractive phase conjugate mirror. Opt Rev 6:350–364

Murakami A, Ohtsubo J, Liu Y (1997) Stability analysis of semiconductor laser with phase-conjugate feedback. IEEE J Quantum Electron 33:1825–1831

Ohtsubo J (2002) Chaotic dynamics in semiconductor lasers with optical feedback. In: Wolf E (ed) Progress in optics, Chap 1, vol 44, North-Holland, Amsterdam

Osmundsen JH, Gade N (1983) Influence of optical feedback on laser frequency spectrum and threshold conditions. IEEE J Quantum Electron 19:465–469

Otsuka K, Chern JL (1991) High-speed picosecond pulse generation in semiconductor lasers with incoherent optical feedback. Opt Lett 16:1759–1761

Petermann K (1988) Laser diode modulation and noise. Kluwer Academic, Dordrecht

Pittoni F, Gioannini M, Montrosset I (2001) Time-domain analysis of fiber grating semiconductor laser operation in active mode-locking regime. IEEE J Select Topics Quantum Electron 7:280–286
Pochi Y (1993) Introduction to photorefractive nonlinear optics. Wiley, New York
Press WH, Flannery BP, Teukolsky SA, Vetterling WT (1986) Numerical recipes: the art of scientific computing. Cambridge University Press, Cambridge
Risch C, Voumard C (1977) Self-pulsation in the output intensity and spectrum of GaAs-AlGaAs cw diode lasers coupled to a frequency-selective external optical cavity. J Appl Phys 48:2083–2085
Sacher J, Elsäßer W, Göbel EO (1989) Intermittency in the coherence collapse of a semiconductor laser with external feedback. Phys Rev Lett 63:2224–2227
Tamburrini M, Spano P, Piazolla S (1983) Influence of an external cavity on semiconductor laser phase noise. Appl Phys Lett 43:410–412
Tkach RW, Chraplyvy AR (1985) Line broadening and mode splitting due to weak feedback in single frequency $1.5 - \mu$m lasers. Electron Lett 21:1081–1083
Tkach RW, Chraplyvy AR (1986) Regimes of feedback effects in $1.5 - \mu$m distributed feedback lasers. J Lightwave Technol 4:1655–1661
Tromborg B, Osmundsen JH, Olesen H (1984) Stability analysis for a semiconductor laser in an external cavity. IEEE J Quantum Electron 20:1023–1032
Tromborg B, Olssen H, Pan X, Saito S (1987) Transmission line description of optical feedback and injection locking for Fabry-Perot and DFB lasers. IEEE J Quantum Electron 23:1875–1889
van Tartwijk GHM, Lenstra D (1995) Semiconductor lasers with optical injection and feedback. Quantum Semiclass Opt 7:87–148
van Tartwijk GHM, van der Linden HJC, Lenstra D (1992) Theory of a diode laser with phase-conjugate feedback. Opt Lett 17:1590–1592
van der Graaf WA, Pesquera L, Lenstra D (1998) Stability of a diode laser with phase-conjugate feedback. Opt Lett 23:256–258
Voumard C (1977) External-cavity-controlled 32 MHz narrow-band cw GaAlAs-diode lasers. Opt Lett 1:61–63
Ye SY, Ohtsubo J (1998) Experimental investigation of stability enhancement in semiconductor lasers with optical feedback. Opt Rev 5:280–284
Yousefi M, Lenstra D (1999) Dynamical behavior of semiconductor laser with filtered external optical feedback. IEEE J Quantum Electron 35:970–976
Zah CE, Osinski JS, Menocal SG, Tabatabaie N, Lee TP, Dentai AG, Burrus CA (1987) Wide-bandwidth and high-power 1.3μm InGaAsP buried crescent lasers with semi-insulating Fe-doped InP current blocking layers. Electron Lett 23:52–53