Chapter 13
Chaotic Communications in Semiconductor Lasers

Chaotic data encoding or scrambling is a technology for overcoming the difficulties of the digital methods in secure communications. Using chaotic lasers as light sources, high-speed and broadband secure communications can be established. In this chapter, we discuss cryptographic applications of chaos in semiconductor lasers. The technique we treat in this chapter is an analog chaotic encryption and decryption. Messages to be sent are encoded into chaotic time series generated from a chaotic semiconductor laser and decoded by a chaotic laser with the same characteristics. The key for chaos communications is chaos synchronization, which we discussed in the previous chapter. In chaotic communications, a small message is embedded into a chaotic laser carrier and the total signal is sent to the receiver. Only the chaotic oscillation is reproduced based on chaos synchronization and a chaos-pass filtering effect in the receiver laser. By subtracting the receiver output from the transmission signal, the message is successfully decoded.

13.1 Message Encryption in a Chaotic Carrier and Its Decryption

13.1.1 Chaotic Communications

The development of efficient technologies for high-speed and massive data transmissions is an urgent subject in the rapidly growing information-oriented society. One of the important issues of information and communication networks is the security problem. In secure data transmissions, a message to be sent is usually encoded by computer software and the security of encoding is guaranteed by the complexity of the calculations necessary to decode the original message. However, the development of digital computer technology is so fast that the standard code for scrambling data in secure communication systems can be soon decoded by a fast computer. On the other hand, the enhancement of the complexity of calculations for encoding and decoding messages may lose real-time processing of data transmissions. In the meantime,

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the method of quantum computing has been developed as one of the candidates to decipher quickly encoded data in standard secure communication systems. As an alternative method, chaotic communications have been proposed for high-speed and broadband capabilities with hardware based secure communications (Kennedy et al. 2000; Dachselt and Schwarz 2001).

There are two techniques for chaos-based secure communications; one is digital encoding and the other is analog encryption. As examples of digital techniques, the method of code scrambling based on chaotic signal generations such as discrete-sequence (DS) optical code division multiple access (CDMA) is used for chaos communications. The method of chaos CDMA uses long life chaotic non-correlated data sequences embedded into the chaotic orbits as CDMA codes. It is verified that the generated codes have an advantage over the existing Gold series for the irregularity and non-correlation properties (Chen et al. 2001). Chaos induced by semiconductor lasers is also effective to generate a series of ultrafast physical random numbers suitable for broadband optical communications (Uchida et al. 2008). The related topic will be treated in the next chapter. Another example is the technique of secure chaos key generation instead of random numbers in ordinary secure communication systems (Uchida et al. 2003a). Another one is an analog technique. In this chapter, we are concerned with the analog method, since chaos in laser systems is best suited for analog data encryption and decryption by nature. In the analog technique, when a fraction of a chaotic signal from a transmitter is sent to a receiver, the two systems synchronize with each other under certain conditions, as discussed in the previous chapter. Not only the two system configurations but also the chaos parameters of the two systems must be the same for perfect chaos synchronization. The merits of the use of semiconductor lasers in chaotic communications are clear, since light is the carrier of modern basic communication channels and the generation of high-speed and broadband signals is easily attained compared with, e.g., nonlinear electronic circuits. Chaotic communications require special hardware to generate a chaotic signal and to realize synchronization. Even if one tries to decode messages by computing or guessing chaotic states from the signals obtained, it is very difficult to decode messages by available techniques without knowledge of the chaos keys because they are embedded into high-dimensional chaotic spaces (Ohtsubo 2002b).

First, we show the basic idea of analog chaos communications. Figure 13.1 shows the model for analog chaos communication systems. The basics of the technique is chaos synchronization between two nonlinear systems, transmitter and receiver, as already noted. A message with small amplitude (compared with the chaotic variations of the transmitter output) is embedded into a chaotic carrier in the transmitter. The chaotic carrier with the message is sent to the receiver through the communication channel. In the receiver, the system only synchronizes to the chaotic signal from the transmitter. Then, the message is decoded by subtracting the receiver output from the transmitted signal. If the amplitude of the message is small enough, we can achieve successful chaos synchronization even if the transmission signal includes the perturbation (message) to the chaotic signal. However, small signal approximation is not always necessary. For example, a message may be comparable with chaotic variations in the chaos modulation (CMO) technique as discussed later.

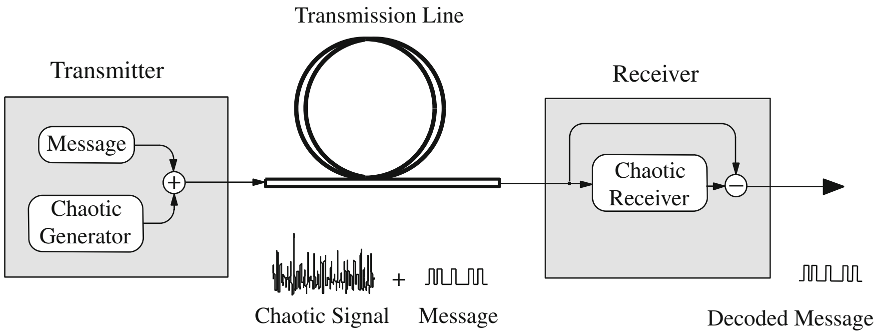

Fig. 13.1 General concept of chaos synchronization with analog data encryption

Chaotic data communications using laser systems are categorized into three classes depending on the techniques of message encoding and decoding (Ohtsubo 2002a,b; Liu et al. 2002b; Ohtsubo and Davis 2005). They are chaos masking (CMA), CMO, and chaos shift keying (CSK). Each method can be mathematically formulated by laser rate equations in transmitter and receiver lasers and the dynamic behaviors of the systems can be described by these rate equations. Even for optical communications, we can also generate chaotic oscillations from nonlinear electronic circuits and transmit a chaotic signal converted by optoelectronic devices through optical channels. However, the carrier of communications is light and laser chaos is best suited for such purposes. Therefore, many systems using chaos of various laser systems have been proposed for chaos synchronization and communications. In spite of existing work, we still need extensive studies about many subjects to put the systems into practical use, for example, the degree of security, the accuracy of synchronization for parameter mismatches between transmitter and receiver systems, robustness of communications, and other things.

13.1.2 Chaos Masking

Following the proposal of chaos synchronization in nonlinear systems, (Pecora and Carroll 1991; Carroll and Pecora 1991) pointed out the possibility of secure communications based on chaos synchronization. They used electronic circuits to realize Lorenz chaos (see Appendix A.4). In their method, a chaotic signal (variable x) in a transmitter system is sent to a receiver system as a synchronous signal. At the same time, a chaotic variable z in the transmitter with a small message m was sent to the receiver and chaos synchronization between the transmitter and receiver systems could be achieved. The receiver system consisted of a subsystem of the variables y and z. After subtracting the synchronized signal z' in the receiver from the transmitted signal $z+m$, the message was successfully decoded. The technique is essentially categorized into the method of CMA. However, two different channels were required

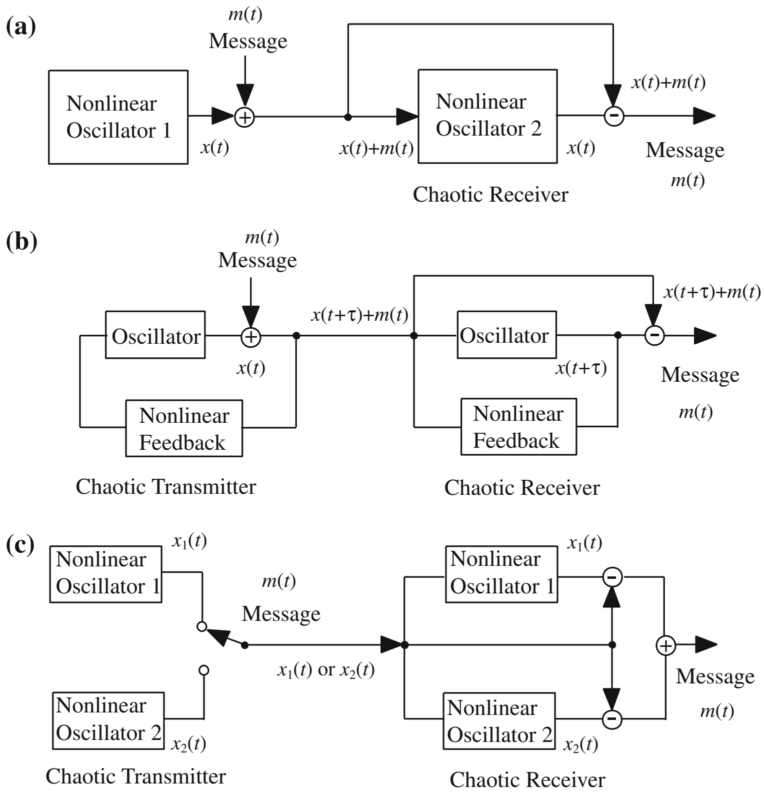

Fig. 13.2 General schemes of optical communications in analog chaotic systems. Models of **a** chaos masking (CMA), **b** chaos modulation (CMO), and **c** chaos shift keying (CSK)

for the data transmission in their method. Cuomo et al. (1993) proposed a method of chaotic communication using a single transmission channel for the same Lorenz system as that of Pecora and Carroll.

In a laser system, we cannot divide the system into subsystems as shown in Appendix A.4. Therefore, a fraction of a chaotic laser output power from a transmitter is sent to a receiver laser through a single communication channel. Figure 13.2 shows the general three schemes of optical communications in analog chaotic systems. Figure 13.2a shows the system of CMA, where a small message $m(t)$ is embedded into a chaotic carrier $x(t)$ in a transmitter and, then, the signal of $x(t) + m(t)$ is sent to a receiver. The receiver system is the same as that of the transmitter and the two systems are operating at the same parameter values. Only the chaotic signal of $x(t)$ is reproduced in the receiver system if the amplitude of the message is small enough. Then, the message $m(t)$ is decoded by subtracting the receiver output $x(t)$ from the transmission signal $x(t) + m(t)$. To hide a message into chaotic carriers securely (namely, mask the message) and to reproduce good quality of a decoded message, the amplitude of the message must be sufficiently small compared with the averaged chaotic carrier signal. Usually, the fraction is less than 1 % of the average chaotic power.

13.1.3 Chaos Modulation

In the method of CMO shown in Fig. 13.2b, both a chaotic carrier and a message conform a new chaotic oscillation in the nonlinear system (Liu et al. 2001a, 2002b; Ohtsubo and Davis 2005). Therefore, a message embedded into the chaotic carrier may not be small. The transmitter and receiver systems can be described by the equivalent mathematical differential equations. Therefore, complete chaos synchronization is achieved in the system and an excellent synchronous signal can be obtained in the receiver output. The method resembles CMA, however CMO is essentially a different technique from CMA. As shown in Fig. 13.2b, a message is mixed with a chaotic carrier in the nonlinear oscillator and the two signals conform a new chaotic state different from the original one. In CMO, a delayed feedback system is usually used as a chaotic generator. The new chaotic signal is given by $x(t+\tau) = f(x(t)+m(t))$ after the delay time τ, where f is the nonlinear function of the system. This new signal together with the message $x(t+\tau)+m(t+\tau)$ is sent to the receiver. Since the transmitter and the receiver are the same nonlinear systems, the chaotic oscillation $x(t+\tau) = f(x(t)+m(t))$ is exactly reproduced in the receiver system as chaos synchronization. By subtracting the synchronized chaotic signal $x(t+\tau)$ from the transmitted signal $x(t+\tau)+m(t+\tau)$, we can decode the message. Sometimes, the message is decoded by dividing the transmitted signal by the synchronized chaotic signal in the receiver. From the point-of-view of encoding and decoding message, the method of CMO has no restriction on the magnitude of the message as a secure communication, since both the chaotic carrier and the message conform new chaotic states in the nonlinear systems. However, in the optics case, the message is usually decoded as an intensity $S(t) = |E(t)|^2$. Therefore, the amplitude of the message must be small enough when we use the ordinary message decoding technique. Furthermore, the degree of security for data transmissions becomes worse when the signal level of a message is large. Therefore, the amplitude of a message in CMO should also be small.

13.1.4 Chaos Shift Keying

The signal shift keying technique, which is frequently used in ordinary communication systems, is also applicable to chaotic data transmissions. Figure 13.2c is an example of such system diagrams. In CSK, two chaotic states $x_1(t)$ and $x_2(t)$ are generated in a transmitter system. The switching itself to send either chaotic state is a message $m(t)$. In the receiver system, each state $x_1(t)$ or $x_2(t)$ is detected by the technique of chaos synchronization. Therefore, two sets of chaotic generators are usually prepared both for the transmitter and the receiver. However, the difference between two chaotic states in the CSK system must be very small, since the message can be easily estimated from the attractors when the difference of chaotic oscillations between the two states is too large. In chaos synchronization in nonlinear systems, the time required for the synchronization between receiver and transmitter is finite for the switching of chaotic states. Therefore, we must take into account the transient and finite response of signals for practical use of the systems.

13.1.5 Chaotic Data Communications in Laser Systems

Shortly after the proposal of chaos synchronization by Pecora and Carroll, Colet and Roy (1994) demonstrated chaos synchronization in laser systems using loss modulated solid-state lasers by numerical simulations and predicted the possibility of chaotic communications based on such nonlinear systems. They showed chaotic data transmission of a binary bit-sequence with a rate of 100 kbps in the system. VanWiggeren and Roy (1998a,b) demonstrated data transmission for secure communications based on CMO using laser systems. They proposed a ring fiber laser system with an optical feedback loop (delay loop) as a chaotic generator. A message to be transmitted was put into the feedback loop as a modulation, then a new chaotic oscillation was produced in the feedback system. They successfully demonstrated data transmission higher than a bit rate of 100 Mbps. Goedgebuer et al. (1998) and Larger et al. (1998) also reported chaotic data transmission based on optoelectronic feedback using wavelength-to-current conversion systems in semiconductor lasers. Their method was also categorized into CMO. Other CMO systems were also proposed by using laser systems (Luo et al. 2000; Abarbanel et al. 2001; Liu et al. 2001c; Tang et al. 2001). Tang and Liu (2001) experimentally demonstrated data transmission of a pseudo-random binary bit-sequence with a 2.5 Gbps non-return-to-zero (NRZ) signal corresponding to the OC-48 standard bit rate in optoelectronic feedback semiconductor laser systems.

After the demonstration of chaotic communications based on CMO, the method of CMA was widely studied theoretically and experimentally because of the ease of implementation in semiconductor laser systems (Mirasso et al. 1996; Sánchez-Díaz et al. 1999; Sivaprakasam and Shore 1999, 2000a,b; White and Moloney 1999; Jones et al. 2000; Rogister et al. 2001). Annovazzi-Lodi et al. (1996, 1997) proposed a method of CSK using semiconductor lasers with optical feedback. Also studies on chaotic communications based on CSK in various laser systems have been reported (Liu and Davis 2001; Davis et al. 2001; Mirasso et al. 2002). Almost all these systems used chaotic oscillations in class B lasers, such as solid-state lasers, fiber lasers, and semiconductor lasers. In chaotic laser communications, the effects of optical feedback, optical injection from a different laser, and optoelectronic feedback have been frequently used to generate chaotic signals.

Liu et al. (2002b) investigated three configurations of systems using semiconductor lasers (optical injection-locking, optical feedback, and optoelectronic feedback systems) and compared the performances of data transmissions for three different techniques (CMA, CMO, and CSK). As a result, the optoelectronic feedback system with CMO showed excellent performance for data transmissions. As discussed later, chaotic carrier frequency, which is closely related to the relaxation oscillation of the solitary laser, is an important measure of the capability of data transmissions and semiconductor lasers with high frequency response are indispensable as high-speed chaotic generators for that purpose. Distributed feedback (DFB) lasers of near infrared wavelength oscillations are frequently used for chaotic laser communications, since they are suitable for chaotic light sources of ordinary optical communication systems with high frequency response. Vertical-cavity surface-emitting lasers (VCSELs) are also promising devices for future semiconductor lasers and also chaotic lasers. Other lasers such as MQW lasers with visible oscillations may be used for chaotic light sources for short-range communications. However, we assume edge-emitting lasers as chaotic generators in the following. Even if device structures are different from each other, the system that is described by the same laser rate equations shows the same dynamics of chaotic oscillations as discussed earlier.

As stated in Chap. 12, there are two types of mechanisms of chaos synchronization; one is complete chaos synchronization and the other is synchronization of chaotic oscillation by optical injection locking and amplification. In a delay differential system, there is a solution for complete chaos synchronization where transmitter and receiver lasers can be described by mathematically identical forms of the equations. On the other hand, we can expect synchronization of chaotic oscillations based on the injection-locking phenomenon in nonlinear amplifying systems in the chaotic transmitter and receiver lasers. We can use both systems for secure communications based on chaos synchronization, although the degree of security is different in the two schemes.

13.2 Cryptographic Applications in Optical Feedback Systems

13.2.1 Chaotic Communications in Optical Feedback Systems

In this section, we focus on chaotic secure communications using systems of semiconductor lasers with optical feedback. Sánchez-Díaz et al. (1999) numerically studied chaotic communications based on CMA in the systems and demonstrated data transmissions of a bit rate of 4 Gbps. In their method, a direct modulation to the injection current in a transmitter semiconductor laser was used as the message encoding, therefore the technique was in principle CMO rather than CMA. However, it is assumed to be CMA as long as the modulation amplitude of a message is

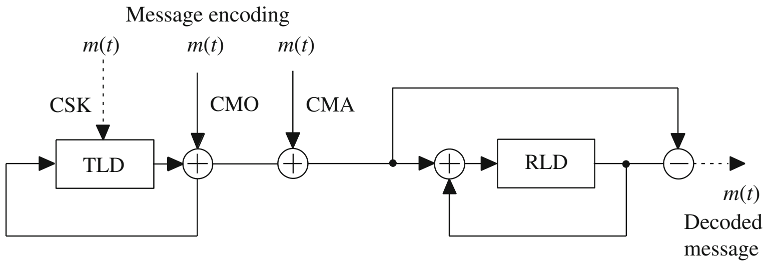

Fig. 13.3 Schematic diagram of chaotic communications in semiconductor lasers with optical feedback. $m(t)$ is a message signal to be embedded. The position for each message encoding scheme is shown in the figure. The *solid lines* are optical connections and the *broken lines* are electronic connections

very small. Annovazzi-Lodi et al. (1996) proposed a CSK system in semiconductor lasers with optical feedback using systems consisting of a single chaotic transmitter and two chaotic receivers. Mirasso et al. (2002) also proposed a CSK system using a single chaotic generator of a semiconductor laser with optical feedback both for a transmitter and a receiver. They numerically demonstrated data transmissions of a bit rate of 2 Gbps. The technique is called ON/OFF CSK. A lot of theoretical and numerical studies on chaotic data transmissions and communications have been reported in semiconductor lasers with optical feedback. However, only a few studies have been published for experimental data transmission of binary messages in optical feedback systems with bit rate over gigabit-per-second (Argyris et al. 2005). Also few theoretical and experimental studies have been reported for CMO in systems of semiconductor lasers with optical feedback (Liu et al. 2001c). A system of a semiconductor laser with optical feedback has a merit for its simplicity, especially in CMA and CSK. However, we need extra devices for the message modulation in CMO, such as electro-optic (EO) modulators.

Using a chaotic generator of a semiconductor laser with optical feedback, we formulate the rate equations for transmitter and receiver lasers. Figure 13.3 shows the schematic diagram of chaotic communications in semiconductor lasers with optical feedback. Embedding a message into the transmitter system, the transmitter can be modeled by the following coupled equations for the complex field E and the carrier density n, according to the configurations of unidirectionally coupled semiconductor lasers with optical feedback (Ohtsubo 2002a; Liu et al. 2002b):

$$\begin{aligned}\frac{\mathrm{d}E_{\mathrm{T}}(t)}{\mathrm{d}t} &= \frac{1}{2}(1 - i\alpha_{\mathrm{T}})G_{n,\mathrm{T}}\{n_{\mathrm{T}}(t) - n_{\mathrm{th,T}}\}E_{\mathrm{T}}(t) \\ &\quad + \frac{\kappa_{\mathrm{T}}}{\tau_{\mathrm{in,T}}}E_{\mathrm{T}}(t - \tau_{\mathrm{T}})\exp(i\omega_{0,\mathrm{T}}\tau_{\mathrm{T}}) + \eta_{\mathrm{CMO}}m_{\mathrm{CMO}}(t)\end{aligned} \tag{13.1}$$

$$\frac{dn_{\mathrm{T}}(t)}{dt} = \frac{J_{\mathrm{T}}}{ed}\{1 + \eta_{\mathrm{CSK}} m_{\mathrm{CSK}}(t)\} - \frac{n_{\mathrm{T}}(t)}{\tau_{\mathrm{s,T}}} - G_{n,\mathrm{T}}\{n_{\mathrm{T}}(t) - n_{0,\mathrm{T}}\}|E_{\mathrm{T}}(t)|^2 \tag{13.2}$$

whereas the receiver driven by the transmitted signal can be described by

$$\begin{aligned}\frac{dE_{\mathrm{R}}(t)}{dt} &= \frac{1}{2}G_{n,\mathrm{R}}(1 - i\alpha_{\mathrm{R}})\{n_{\mathrm{R}}(t) - n_{\mathrm{th,R}}\}E_{\mathrm{R}}(t)\\ &+ \frac{\kappa_{\mathrm{R}}}{\tau_{\mathrm{in,R}}}E_{\mathrm{R}}(t - \tau_{\mathrm{R}})\exp(i\omega_{0,\mathrm{R}}\tau_{\mathrm{R}})\\ &+ \frac{\kappa_{\mathrm{cp}}}{\tau_{\mathrm{in,R}}}E_{\mathrm{T}}(t - \tau_c)\exp(i\omega_{0,\mathrm{T}}\tau_c - i\Delta\omega t)\\ &+ \eta_{\mathrm{CMA}} m_{\mathrm{CMA}}(t - \tau_c) + \eta_{\mathrm{CMO}} m_{\mathrm{CMO}}(t - \tau_c)\end{aligned} \tag{13.3}$$

$$\frac{dn_{\mathrm{R}}(t)}{dt} = \frac{J_{\mathrm{R}}}{ed} - \frac{n_{\mathrm{R}}(t)}{\tau_{\mathrm{s,R}}} - G_{n,\mathrm{R}}\{n_{\mathrm{R}}(t) - n_{0,\mathrm{R}}\}|E_{\mathrm{R}}(t)|^2 \tag{13.4}$$

where $\Delta\omega = \omega_{0,\mathrm{T}} - \omega_{0,\mathrm{R}}$, κ_{cp} is the coupling coefficient of light from the transmitter to the receiver, $m_{\mathrm{CMA}}(t)$, $m_{\mathrm{CMO}}(t)$, and $m_{\mathrm{CSK}}(t)$ are the message sequences corresponding to CMA, CMO, and CSK systems, respectively, and η_{CMA}, η_{CMO}, and η_{CSK} are the actual modulation coefficients for each system. For example, when we consider a chaotic communication system with CMA, we put $\eta_{\mathrm{CMA}} \neq 0$, and $\eta_{\mathrm{CMO}} = \eta_{\mathrm{CSK}} = 0$. The modulation depth of the message in chaotic secure communications is generally very small so as not to disturb the chaotic attractors. In actual systems, the perturbation due to the message encoding is not for amplitude but for optical intensity, except for the case of injection current modulation. However, we can approximately assume amplitude perturbation as far as it is very small compared with the average chaotic amplitudes. Otherwise, we could formulate rate equations for intensity perturbations for the numerical simulations.

As can be easily recognized from (13.1)–(13.4), there is a condition where the rate equations in the transmitter are mathematically identical to those in the receiver in a CMO system. Thus, complete chaos synchronization is performed in this system. On the other hand, for the other cases (CMA and CSK systems), a message always behaves as a small perturbation to each chaotic system. Therefore, the modulation coefficients η_{CMA} and η_{CSK} must be small enough to achieve good chaos synchronization. They should be usually less than 1 % of the average of the chaotic fluctuations.

13.2.2 Chaos Masking in Optical Feedback Systems

In the following, we describe the particular technique for each modulation scheme in optical feedback systems. In CMA, a small message to be sent is added to a chaotic carrier from the transmitter laser. The system under consideration is the same as in Fig. 13.2a. In CMA, we set $\eta_{\mathrm{CMA}} \neq 0$ and $\eta_{\mathrm{CMO}} = \eta_{\mathrm{CSK}} = 0$ in (13.1)–(13.4). Since there is a message term on the right hand side of the receiver Eq. (13.3), complete chaos synchronization is not realized in this system in a strict sense. However, we can approximately observe complete chaos synchronization as long as the modulation depth of the message is small enough. The accuracy of chaos synchronization depends on the parameter mismatches for the chaos keys. For example, chaos synchronization with high accuracy is achieved when the total light input to the receiver laser both from the external reflector and the transmitter laser is almost equal to the amount of external feedback in the transmitter.

In spite of the presence of the perturbation for a message in a transmitted signal, only the chaotic carrier is reproduced in the receiver output in CMA. This phenomenon is known as chaos-pass-filtering. This fact is verified by numerical and experimental studies (Ohtsubo 2002b). The phenomenon of chaos-pass-filtering in nonlinear systems is not obvious and needs some explanation. The origin of chaos-pass filtering will be discussed in the following subsection. A small message is also considered as a perturbation for a chaotic system like noises in the system. It seems that noises are discriminated from intrinsic chaotic dynamics induced in the nonlinear system. As far as perturbation in a system is small, original chaotic dynamics is preserved in the transmitted signal. As is easily understood, the message in CMA is decoded by subtracting the receiver output from the transmitted signal of the chaotic carrier together with the message. The accuracy of the decoding becomes worse when the modulation depth of the message increases, though it depends on the synchronization schemes (complete or optical injection-locking regime). Further, the security of data transmissions is degraded with the increase of the modulation depth, since the message may be directly visible in the transmitted signal.

In CMA, a message is added to a chaotic carrier generated from a transmitter. For example, a chaotic carrier is modulated through an EO modulator, which is an intensity modulation to the chaotic carrier. However, an alternative method is frequently used for encoding a message. A message is simply added to the bias injection current and it is an easier way to modulate intensities in semiconductor lasers. Strictly speaking, it is a technique of CMO rather than CMA. However, it reduces to the method of CMA when the level of a message is small enough. Indeed, a lot of theoretical and experimental work has been published based on the same techniques of injection current modulation as message encoding in the system.

Before showing the results for data transmissions and decoding of messages in CMA, the unique phenomenon of chaos-pass-filtering is discussed. Figure 13.4 shows an experimental example for chaos synchronization when a message is embedded into the transmitter signal (Kusumoto and Ohtsubo 2002). The message is added to a chaotic carrier as an injection current modulation in the transmitter laser and it is a sinusoidal wave with a frequency of 1.5 GHz. The relaxation oscillation frequency of the solitary laser is about 4 GHz. The two chaotic waveforms look the same as shown in Fig. 13.4a and they are synchronized with each other in spite of the presence of the message. The synchronization is also confirmed by the correlation plot. Even in the presence of the message in the transmitter laser, the correlation coefficient

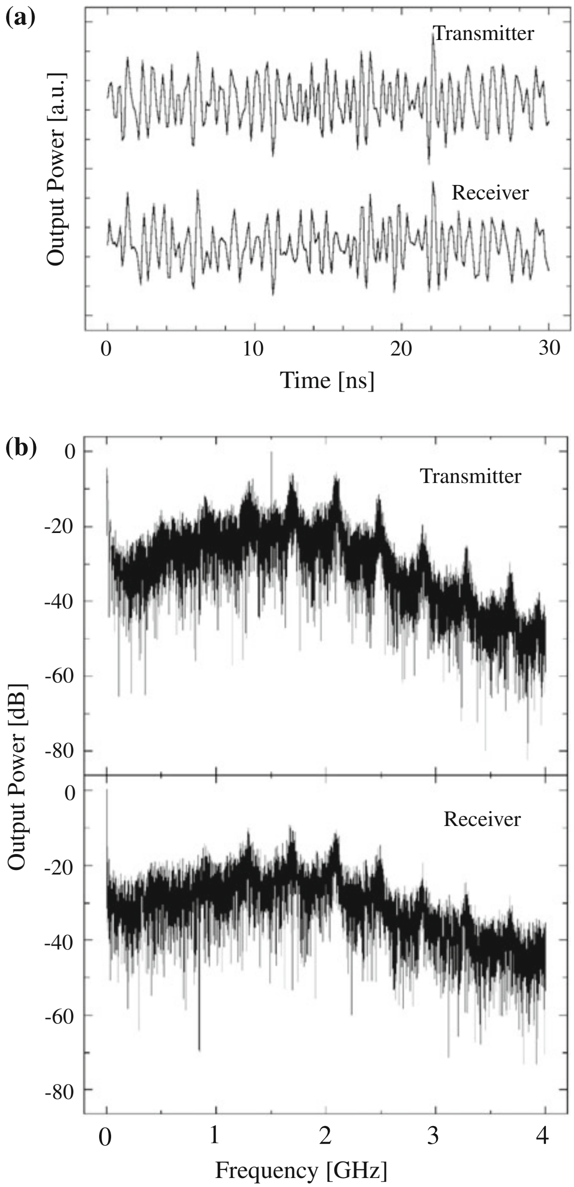

Fig. 13.4 Chaos-pass-filtering effect. **a** Waveforms of transmitted signal and receiver output in a closed-loop system of semiconductor lasers with optical feedback. A message of a sinusoidal wave of a frequency of 1.5 GHz with a modulation depth of −14 dB is included in the transmitter signal. **b** Corresponding rf spectra to **a**. The synchronization is based on an optical injection-locking scheme

between the two laser outputs is 0.86, which is high enough for message decoding in chaotic communications based on chaos synchronization. Figure 13.4b shows the rf spectra corresponding to the waveforms in Fig. 13.4a. Besides the broad spectral peaks of the external cavity mode and its higher harmonics, a sharp spectral peak for the message of 1.5 GHz is clearly visible in the transmitter spectrum. On the

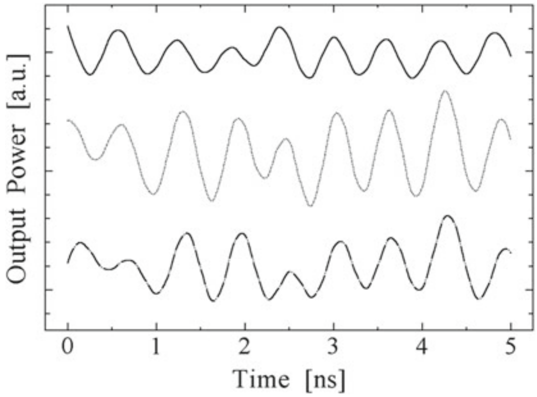

Fig. 13.5 Message decoding for signals in Fig. 13.4a with a narrow band-pass filter of ± 100 MHz centered at a message frequency of 1.5 GHz. Decoded message (*upper trace*) and transmitter (*middle trace*) and receiver (*bottom trace*) outputs

other hand, the spectrum bears resemblance to that of the transmitter but no distinct spectral component for the message is present in the receiver spectrum due to a chaos-pass-filtering effect. As a result, we can extract the message simply by subtracting the reproduced chaotic signal in the receiver laser from the transmitted signal, thus chaotic communication is realized. The receiver laser generates only the intrinsic chaotic oscillations the same as that from the transmitter if the message embedded into the chaotic carrier is small enough. The effect of the insensitivity for small external perturbations to chaos can be considered as a different phenomenon such as the sensitivity of chaos for initial conditions. Chaos seems to distinguish external perturbations and the system nonlinearity.

Looking at the spectrum of the transmitted signal in Fig. 13.4, the question may arise that the message may be extracted by filtering the waveform with a narrow band-pass filter at the message frequency. Figure 13.5 shows the filtered waveforms for the decoded message as well as the transmitted signal and the receiver output. The waveforms are the results for a narrow band-pass filter of ± 100 MHz centered at the message frequency of 1.5 GHz. The decoded message is a simple subtraction of the receiver output from that to the transmitter. The decoded message is reproduced as a good sinusoidal oscillation, which is almost the same signal as the original message. However, the filtered waveforms for the transmitted signal and the receiver output are not good harmonic signals and they are even not in-phase with the message signal. The degree of the security of communications must be evaluated for actual data of binary bit-sequences. However, the results obtained in Fig. 13.5 show some of evidence for the security in the present systems.

As an example of a signal transmission of binary data in CMA, numerical results by Sánchez-Díaz et al. (1999) are shown. They conducted data transmissions of pseudo-random bit-sequences of a 4 Gbps NRZ signal in a closed-loop system. The message is a small perturbation of 0.5 % of an averaged chaotic oscillation in the transmitter and it is fed to the injection current to the laser as a direct modulation. Since the perturbation is sufficiently small, the method is considered as CMA. They also assumed the nonlinear effect of signal transmissions through optical fibers between the transmitter and receiver systems. Figure 13.6 shows their results. In their method, the decoded signal is obtained by the comparison of the transmitted signal $E_{\mathrm{trans}}(t)$ ($\propto E_{\mathrm{T}}(t)$) with the decoded one $E_{\mathrm{R}}(t)$ as

$$m'(t) = \sqrt{\frac{|E_{\mathrm{trans}}(t)|^2}{|E_{\mathrm{R}}|^2}} - 1 \tag{13.5}$$

The fidelity of the chaotic signal after the transmission through the optical fiber may be degraded due to a nonlinear dispersion effect in the fiber. The data transmission in optical fiber is described by the following nonlinear Schrödinger equation (Sánchez-Díaz et al. 1999):

$$\begin{aligned} i\frac{\partial E(z,\mathrm{T})}{\partial z} = {} & -\frac{i}{2}\alpha_f E(z,T) + \frac{1}{2}\beta_2 \frac{\partial^2 E(z,T)}{\partial T^2} \\ & - \gamma_{\mathrm{non}}|E(z,T)|^2 E(z,T) \end{aligned} \tag{13.6}$$

where $E(z,T)$ is the slowly varying complex field, z is the propagation distance, and T is the time measured in the reference frame moving at the group velocity. γ_{non} is the nonlinear parameter that takes into account the optical Kerr effect, α_f is the fiber loss, and β_2 is the second-order dispersion parameter of optical fiber. They also used a low-pass Fabry–Perot filter with a bandwidth of 5 GHz to obtain the final decoded message of Fig. 13.6f, though a higher order Butterworth electronic filter is usually used. Excellent chaos synchronization was attained in spite of the effect of nonlinear optical fiber transmission through 50 km and the message was successfully reconstructed.

The quality of the reproduced chaotic signal and, accordingly, the decoded message in the receiver are degraded by the transmission through the nonlinear optical fiber. The system performance at the modulation of 2 Gbps is displayed in Fig. 13.7 for different propagation length in optical fiber (Sánchez-Díaz et al. 1999). Whether the message is included in the chaotic carrier or not, the synchronization becomes worse for a long fiber transmission and the quality of the decoding becomes worse accordingly. The degradation comes from both the dispersion and nonlinearities of optical fiber. In actual optical fiber transmission, there is a loss of light through the optical fiber. Therefore, an in-line amplifier is placed at a certain distance in practical optical fiber communications. This may cause further degradation of chaotic signals due to the enhancement of the nonlinear effects after the amplification. In-line amplifiers would aggravate the negative effect of the fiber nonlinearities on the synchronization.

13.2.3 Chaos Modulation in Optical Feedback Systems

In CMO, we put $\eta_{\mathrm{CMO}} \neq 0$ and $\eta_{\mathrm{CMA}} = \eta_{\mathrm{CSK}} = 0$ in (13.1)–(13.4). A message is added within the transmitter, for example in the feedback loop in Fig. 12.4a, and the

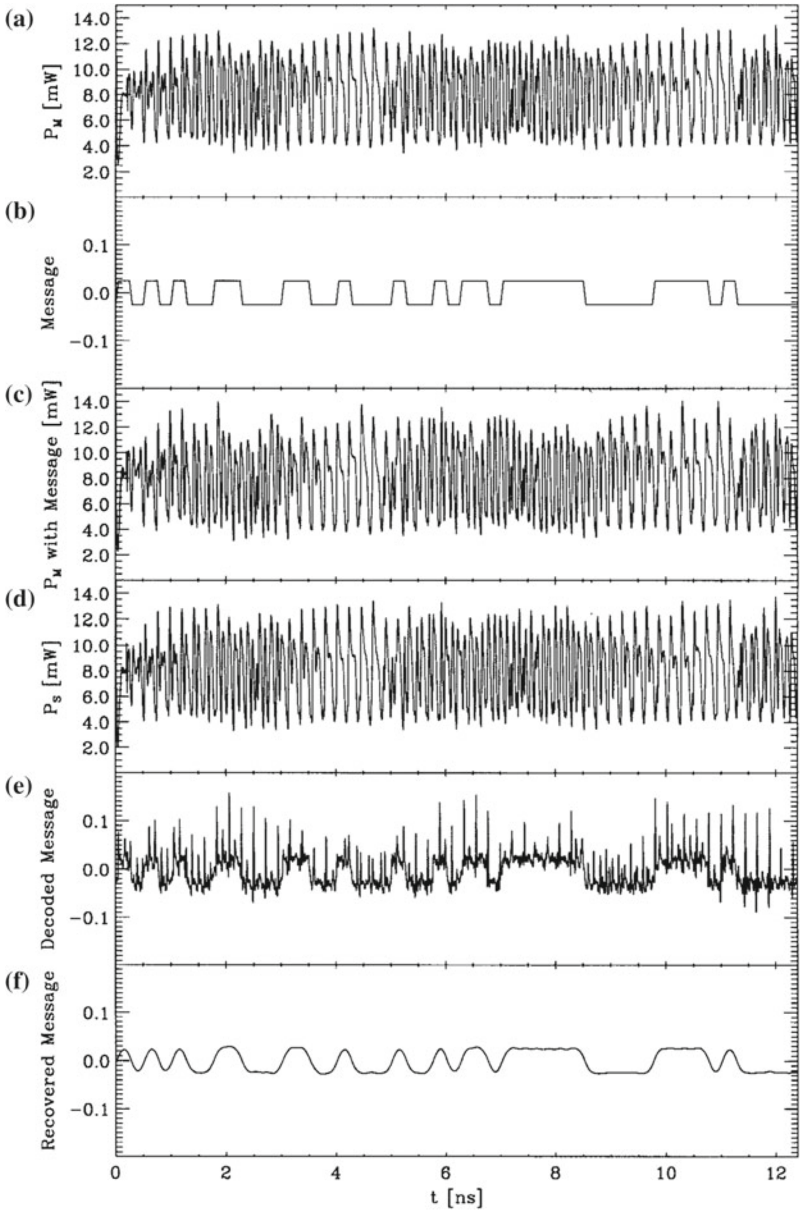

Fig. 13.6 Chaotic communications based on CMA in semiconductor lasers with optical feedback. **a** Chaotic oscillation from transmitter, **b** binary bit-sequence for data transmission, **c** chaotic waveform together with message, **d** synchronized chaotic oscillation in receiver after data transmission through a nonlinear optical fiber of 50 km, **e** decoded signal, and **f** low-pass filtered waveform for decoded signal [after Sánchez-Díaz et al. (1999); © 1999 IEEE]

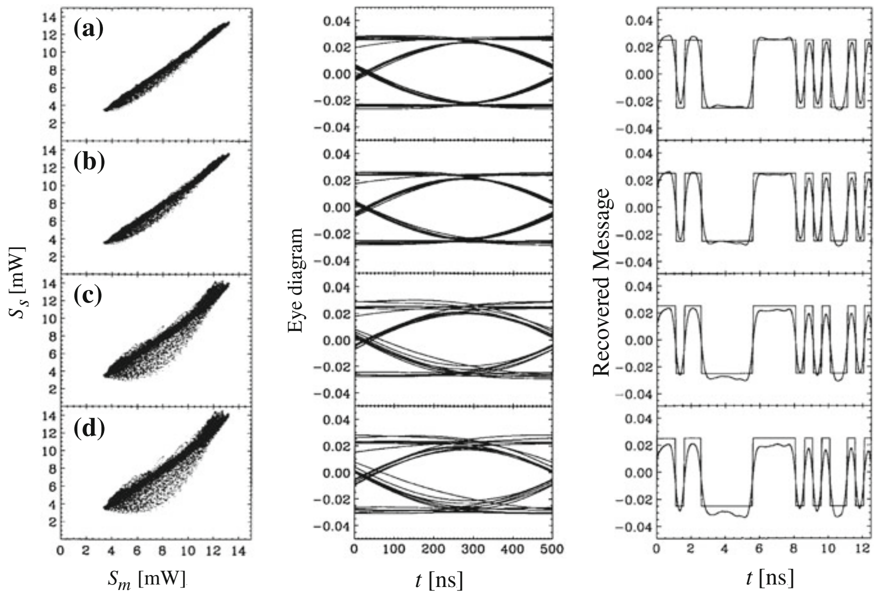

Fig. 13.7 Signal recovery for fiber transmissions of 50, 100, 150, and 200 km from **a** to **d**, respectively. The *left column* shows the correlation plots between the chaotic carrier in the transmitter without a message and the receiver output at synchronization. The *middle* shows the corresponding eye patterns. The *right* shows the decoded messages after filtering [after Sánchez-Díaz et al. (1999); © 1999 IEEE]

transmission signal of $E(t+\tau) = f(E(t) + m_{\mathrm{CMO}}(t))$ is generated and transmitted to the receiver. The original chaotic signal together with the message conforms a new chaotic signal and, therefore, the message may essentially have a large amplitude. However, care must be taken when embedding a large message signal into chaotic oscillations, since the message itself may explicitly appear in the transmission signal for the worst case. For this reason, a message with small amplitude is usually used in CMO. The method of CMO is approximately equal to CMA when an encoding message is small enough.

We present a numerical example of the CMO method. This scheme requires the achievement of complete chaos synchronization and hence is usually difficult to realize by experiment (Liu et al. 2002a). The possibility of transmission of a NRZ pseudo-random binary sequence at 2.5 Gbps using CMO was demonstrated by numerical simulation. Figure 13.8 shows the results (Liu et al. 2001c). In this example, it is assumed that an EO modulator is inserted into the optical feedback loop, and is used to modulate the feedback signal as $m_{\mathrm{CMO}}(t) = 1 + b_m$ for “1” and $m_{\mathrm{CMO}}(t) = 1 - b_m$ or “0”, where b_m ($b_m < 1$) is the modulation depth. Figure 13.8a shows the dependence of the synchronization error on the amplitude of the embedded signal b_m, and Fig. 13.8b is the dependence of the bit error rate on the frequency of the detuning between the transmitter and receiver lasers, both with noise added

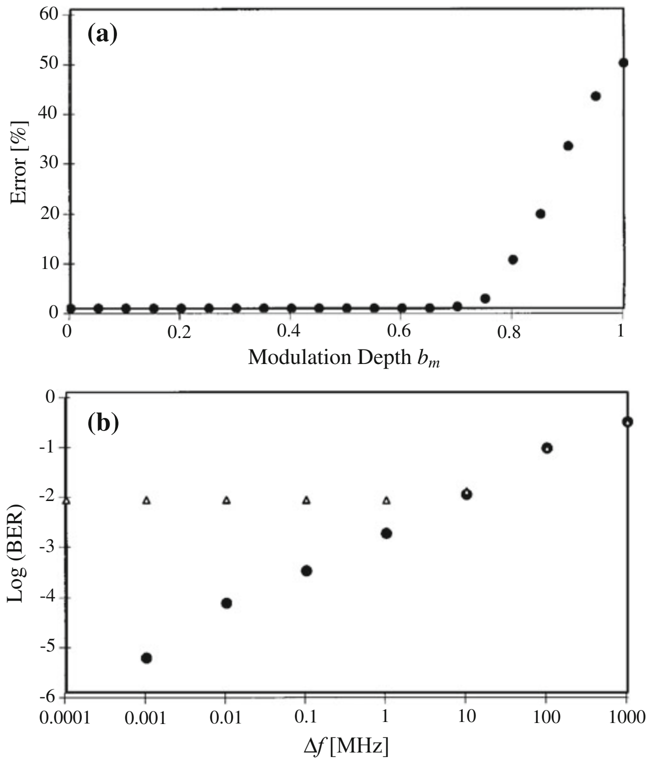

Fig. 13.8 Calculated synchronization error and bit error rate (BER) for normalized dimensionless bias injection currents of 0.67, normalized feedback and injection ratios of 0.01. **a** Synchronization error versus modulation amplitude b_m. **b** BER versus frequency detuning at a modulation factor of $b_m = 0.15$. *Black dots* no noise, *triangles* noise at SNR $= 40\,\text{dB}$

(triangles) and without noise (circles). The results show good synchronization for a wide range of relative modulation amplitudes and sensitive dependence of the BER on the detuning, which are strong features of the CMO method using complete synchronization.

13.2.4 Chaos Shift Keying in Optical Feedback Systems

A set of chaotic generators is usually required both for transmitter and receiver systems in CSK. Indeed, chaotic signals from two transmitters are switched according to the binary value of 0 or 1 of a message sequence and they are sent to the two receivers. The receivers synchronize to the corresponding transmitters and the decoding is done by the synchronization. Annovazzi-Lodi et al. (1997) used a single transmitter consisting of a semiconductor laser with optical feedback in CSK instead of two transmitters. A NRZ binary message is put into the injection current of the laser

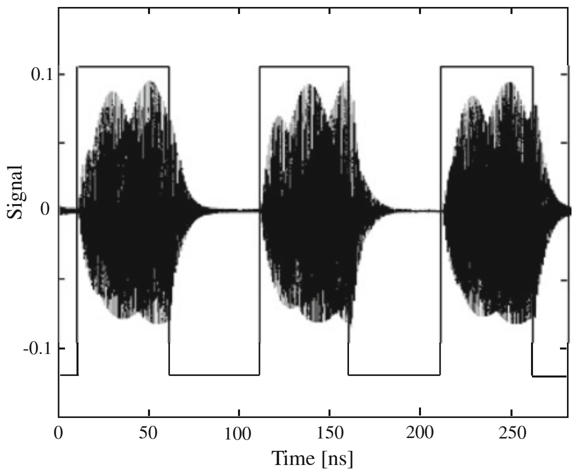

Fig. 13.9 Square message to be transmitted in a CSK system and error signal from one of the receiver outputs. The system has a single transmitter of a semiconductor laser with optical feedback and the receiver consists of two optical feedback systems [after Annovazzi-Lodi et al. (1997); © 1997 IEEE]

at a certain bias point. According to the message, two chaotic states are generated and they are sent to the receiver. The difference between the two chaotic attractors must be sufficiently small not to be distinguished easily for secure communications. The receiver is a set of chaotic generators and they have the same characteristics except for the injection currents. The injection currents are set at either the high or low value of the binary message. Then, each receiver synchronizes with the corresponding chaotic state and synchronous and asynchronous (chaotic bursts) signals are obtained for the time sequence from comparison between the transmitted signal and the receiver output. Figure 13.9 shows one of the receiver outputs in the CSK system. The square waveform is a message to be transmitted and the error signal is the receiver output. The other receiver laser outputs the compensating signal to the waveform in Fig. 13.9. Then, the message is decoded from these two signals.

When two nonlinear chaotic systems synchronize with each other, the receiver does not respond immediately after it receives a chaotic signal from the transmitter. Usually, the receiver outputs the synchronous signal after a certain transient time. In a CSK system, the chaotic oscillation switches from one state to the other according to the ON/OFF signals. Therefore, we must consider the synchronization recovery time after the switching of signals. This limits the efficiency of the possible bit rate of the data transmission. This synchronization recovery time depends on each system configuration (open- or close-loop system) and the device and system parameters. The typical frequency of a chaotic carrier in semiconductor lasers with optical feedback is of the order of the relaxation oscillation frequency at the solitary oscillation and a message frequency must be less than this. The time required for the synchronization in CSK is from nano-second to several nano-seconds depending on the system parameters (Vicente et al. 2002). On the other hand, it is possible for CMA and CMO methods to achieve higher data transmissions than the relaxation oscillation frequency, since chaotic oscillations in the systems have broadband characteristics over the relaxation oscillations. In the CSK discussed here, the transmitter laser is directly modulated by a message through the injection current, so that it looks like a method of CMO. However, the original chaotic attractor is not affected by the modulation. It is distinguished from CMO as long as the amplitude of the message is small (less than 1 % of the averaged chaotic oscillation). The synchronization recovery time will be discussed in Sect. 13.9.

13.2.5 Chaotic Communications in Incoherent Optical Feedback Systems

Incoherent optical feedback systems are also used for chaotic communications based on chaos synchronization as discussed in the previous chapter. In the systems, the laser output power from the transmitter is coupled with the carrier density of the receiver laser. Therefore, we do not need to consider the optical phase and tune the optical frequencies between the transmitter and receiver lasers. As a result, we can easily achieve chaos synchronization. However, the grade of the security in the communications is deteriorated, since one of the keys for secure chaotic communications is eliminated. Nevertheless, the output generated by a semiconductor laser with incoherent optical feedback is a higher dimensional chaos and the system has enough complexity for secure communications. It is still not easy to reproduce the transmitter chaos in the receiver without knowing the chaos parameters in the system. Rogister et al. (2001) conducted chaotic data transmission based on CSK using incoherent optical feedback systems by numerical simulations. The synchronization system is the same as in Sect. 12.3.5. They succeeded in the data transmission of 250 Mbps pseudo-random-bit sequences with excellent quality using ON/OFF CSK. The level of the message was only 0.3 % of the bias injection level and the secure communication was achieved by hiding the data behind the chaotic signal.

13.2.6 Chaos Pass Filtering Effects

The effect of chaos-pass filtering is essential to attain successful secure communications by hiding messages behind chaotic signals in chaos synchronization systems. In accordance with the effect, the chaotic signal, which is transmitted from the transmitter laser, is only reproduced by the receiver laser even if the transmitted signal includes a message. Thus, the message is correctly decoded by subtracting the chaos signal of the receiver laser from the transmitted signal under appropriate conditions for the synchronization system. However, the above expression of ‘chaotic signal is only reproduced’ may not be correct. As will be discussed later, the effect of the chaos-pass filtering is that the main component of the chaotic signal from the transmitter laser is closely copied in the receiver laser and the chaos transmittance is usually equal to or larger than unity, while the transmittance of the message, whose

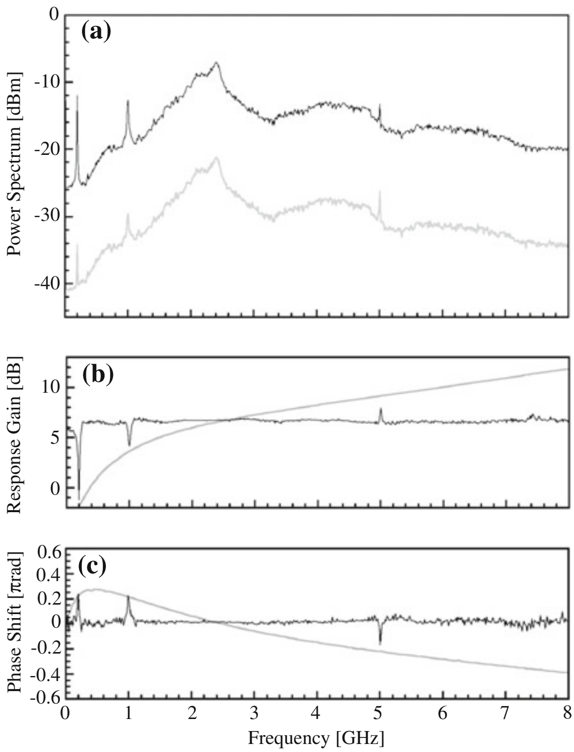

Fig. 13.10 Transfer functions in a chaos synchronization system. The injection ratio from the transmitter to the receiver laser is 50 %. The transmitting signal from the transmitter is a chaotic signal together with three sinusoidal messages with modulation frequencies of 0.2, 1.0, and 5.0 GHz. The modulation indices are 0.05, 0.05, and 0.10, respectively. **a** Power spectra for transmitter signal and receiver output. *Solid line* transmitter, and *gray line* receiver. The receiver spectrum is vertically shifted by −15 dB. **b** Transfer function from the transmitter to the receiver. *Solid line* transfer gain, and *gray line* transfer function for sinusoidal modulation only (modulation index is 0.05). **c** Phase shift between the transmitter signal and the receiver output. *Solid line* phase shift between the transmitter signal and the receiver output, and *gray line* phase shift for sinusoidal modulation only (modulation index is 0.05) [after Murakami and Shore (2005); © 2005 APS]

frequency component is less than the relaxation oscillation, is much less than unity. Several studies for the effects of chaos-pass filtering have been reported both theoretically and experimentally (Uchida et al. 2003b; Paul et al. 2004; Murakami and Shore 2005, 2006). Especially, chaos-pass filtering plays a crucial role in chaotic masking systems. In the following, we discuss the effect of chaos-pass filtering in semiconductor lasers with optical feedback.

Consider a CMA system in semiconductor laser with optical feedback described by (13.1)–(13.4) and assume three different sinusoidal modulations for the injection current J_T in the transmitter laser as messages. Figure 13.10 shows the calculated transfer function between the transmitted signal and the receiver output. The chaos synchronization system assumed here is a type of open-loop and the synchronization is generalized one. To show the effect of chaos-pass filtering explicitly, the ratio of the optical injection from the transmitter to the receiver laser is as large as 50 %. A transmission rate of several percents from the transmitter to the receiver is sufficient to attain chaos synchronization in the laser system and, indeed, such a small rate is usually used in a synchronization system using chaotic semiconductor lasers. The parameters of the solitary laser without optical feedback are $J = 1.3J_{\mathrm{th}}$ and $\nu_{\mathrm{R}} = 2.43\,\mathrm{GHz}$. The optical delay in the transmitter laser is set to be $\tau = 1\,\mathrm{ns}$ and the frequency detuning between the transmitter and receiver lasers is assumed to be $-0.1\,\mathrm{GHz}$. Figure 13.10a shows the power spectra for the transmitter signal including the messages and the synchronized receiver output. The power spectrum of the receiver laser is vertically shifted by $-15\,\mathrm{dB}$ to show clearly the difference. The frequency component of the chaotic signal is concentrated from 1 to 10 GHz. In the both spectra, embedded spectral peaks of sinusoidal signals are clearly seen. From the closer look at the two spectra, the spectra in the lower frequency components in the receiver are largely suppressed. On the other hand, the signal higher than the relaxation oscillation frequency is much enhanced in the receiver side.

Figure 13.10b shows the response ratio between the transmitter signal and receiver output. Since the injection rate from the transmitter to the receiver is strong as much as 50 % in this case, the gain of the chaotic signal component in the receiver laser is much lager than 0 dB. As for the message components, it is seen from the figure that the transmission rates for the lower frequency components below the solitary relaxation oscillation frequency are greatly decreased compared with the transmission gain of the chaotic signal, while the transmission rate for the higher frequency component is larger than unity. The gray line in the figure shows the transfer function from the transmitter to the receiver laser for only sinusoidal modulation with a modulation index of 5 % to the solitary transmitter laser. The crossover point of the chaotic gain and the sinusoidal modulation response is exactly equal to the relaxation oscillation frequency of the semiconductor laser at solitary oscillation. Figure 13.10c shows the phase shift between the transmitter signal and the receiver laser. The solid line represents the phase shift, which is calculated from the Fourier components of the transmitter signal and the receiver output. There is no phase shift for the chaotic transmittance, while the sinusoidal signals with lower frequency components have a positive phase shift and the higher frequency component has a negative shift. The gray line shows the phase shift for only sinusoidal modulation with a modulation index of 5 % to the solitary transmitter laser, which corresponds to the gray line in Fig. 13.10b. Again, the crossover point of the chaotic phase shift and the phase shift for the sinusoidal modulation is exactly equal to the relaxation oscillation frequency. It is noted that the phase shift becomes $-\pi$ at the resonant frequency by the strong optical injection from the transmitter to the receiver laser, when only a sinusoidal modulation is transmitted (though the resonant frequency is 12.7 GHz in this case and the frequency is out-of-scope in this graph).

As discussed above, the frequency components of chaotic signals concentrate at and around the relaxation oscillation frequency and they typically range from 1 to 10 GHz in chaotic semiconductor lasers. Another interesting point is that the transfer function of a chaotic signal from the transmitter to the receiver is almost constant for all the frequency range. On the other hand, the transfer function of a sinusoidal signal has a frequency dependence. Namely, a sinusoidal signal has a small response gain for lower frequency less than the relaxation oscillation, while a sinusoidal signal with higher frequency has a larger response gain than 0 dB. Thus, the difference for the response gains between chaotic and sinusoidal signals is the origin of the chaos-pass filtering effects. As another issue, the direct injection current modulation for the transmitter laser as a message encryption deteriorates the performance of chaos synchronization. Therefore, an external modulation for chaotic signals using an EO modulator is frequently used. Uchida et al. (2003b) studied the effects of chaos-pass filtering in a CMA system using semiconductor lasers with optical feedback, and obtained similar results for the response between the transmitter and the receiver as discussed here. It is derived from the above discussion that chaotic carrier frequency, which is limited by the resonant oscillation frequency of semiconductor laser, must be much greater than a main message frequency to attain higher data-bit transmission in chaotic semiconductor laser systems. Thus, the use of semiconductor lasers, which have high modulation bandwidth, is essential for massive chaotic secure communications.

13.3 Cryptographic Applications in Optical Injection Systems

We can apply the systems of semiconductor lasers subjected to optical injection discussed in Sect. 12.4 to chaotic communications. Figure 13.11 shows the schematic diagram of the chaotic communications. Message encoding and decoding schemes are the same as the systems of optical feedback in Fig. 13.3. For CMA, CMO, and CSK, the equations for the transmitter laser are written by

$$\begin{aligned}\frac{\mathrm{d}E_{\mathrm{T}}(t)}{\mathrm{d}t} &= \frac{1}{2}(1-i\alpha_{\mathrm{T}})G_{n,\mathrm{T}}\{n_{\mathrm{T}}(t)-n_{\mathrm{th,T}}\}E_{\mathrm{T}}(t) \\ &\quad + \frac{\kappa_{\mathrm{inj,T}}}{\tau_{\mathrm{in.T}}}\{1+\eta_{\mathrm{CMO}}m_{\mathrm{CMO}}(t)\}E_{\mathrm{T}}(t)\exp(-i\Delta\omega_{\mathrm{T}}t)\end{aligned} \tag{13.7}$$

$$\begin{aligned}\frac{\mathrm{d}n_{\mathrm{T}}(t)}{\mathrm{d}t} &= \frac{J_{\mathrm{T}}}{ed}\{1+\eta_{\mathrm{CSK}}m_{\mathrm{CSK}}(t)\} - \frac{n_{\mathrm{R}}(t)}{\tau_{\mathrm{s,R}}} \\ &\quad - G_{n,\mathrm{T}}\{n_{\mathrm{T}}(t)-n_{0,\mathrm{T}}\}|E_{\mathrm{T}}(t)|^2\end{aligned} \tag{13.8}$$

where $\Delta\omega = \omega_{\mathrm{inj,T}} - \omega_{0,\mathrm{T}}$ is the frequency detuning between the injection laser and the transmitter laser. Whereas the receiver driven by the transmitted signal can be described by

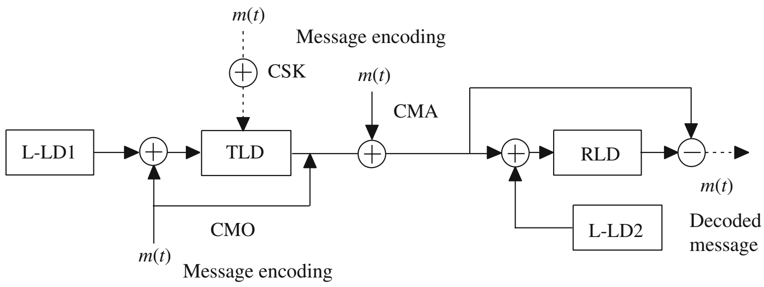

Fig. 13.11 Schematic diagram of chaotic communications in semiconductor lasers with optical injections. $m(t)$ is a message signal to be embedded. The position for each message encoding scheme is shown in the figure. The *solid lines* are optical connections and the *broken lines* are electronic connections

$$\begin{aligned}\frac{\mathrm{d}E_{\mathrm{R}}(t)}{\mathrm{d}t} &= \frac{1}{2}(1-i\alpha_{\mathrm{R}})G_{n,\mathrm{R}}\{n_{\mathrm{R}}(t)-n_{\mathrm{th,R}}\}E_{\mathrm{R}}(t) \\ &+ \frac{\kappa_{\mathrm{inj,R}}}{\tau_{\mathrm{in.R}}}E_{\mathrm{R}}(t)\exp(-i\Delta\omega_{\mathrm{R}}t) \\ &+ \frac{\kappa_{\mathrm{cp}}}{\tau_{\mathrm{in.R}}}\{1+\eta_{\mathrm{CMO}}m_{\mathrm{CMO}}(t-\tau_c)\}E_{\mathrm{T}}(t-\tau_c)\exp(-i\Delta\omega_{\mathrm{T,R}}t+i\omega_{0,\mathrm{T}}\tau_c) \\ &+ \eta_{\mathrm{CMA}}m_{\mathrm{CMA}}(t-\tau_c)\end{aligned} \tag{13.9}$$

$$\frac{\mathrm{d}n_{\mathrm{R}}(t)}{\mathrm{d}t} = \frac{J_{\mathrm{R}}}{ed} - \frac{n_{\mathrm{R}}(t)}{\tau_{\mathrm{s,R}}} - G_{n,\mathrm{R}}\{n_{\mathrm{R}}(t)-n_{0,\mathrm{R}}\}|E_{\mathrm{R}}(t)|^2 \tag{13.10}$$

where $\Delta\omega_{\mathrm{T,R}} = \omega_{0,\mathrm{T}} - \omega_{0,\mathrm{R}}$ is the angular frequency detuning between the transmitter and receiver lasers. Optical modulations of messages (CMA and CMO) are assumed to be applied to the complex fields, however EO modulators are usually used as intensity modulations. We need some modifications of the above rate equations for the intensity modulations. Nevertheless, (13.7)–(13.10) are again a good approximation for the optical injection systems as far as the modulation amplitude is small enough.

Figure 13.12 presents a numerical example of chaotic communications in a system of semiconductor lasers subjected to optical injection (Liu et al. 2001a). The figure is a message transmission of a 2.5 Gbps signal based on CMO. In the transmitter laser, a chaotic carrier is generated at the appropriate injection ratio and frequency detuning in the optical injection configuration. A binary message is encoded into the chaotic carrier. There are two schemes of chaotic modulations to the optical field in CMO; additive modulation and multiplicative modulation. This case is for additive modulation. After subtracting the receiver output from the transmitted signal and low-pass filtering it (bottom of the figure), they obtained a decoded message with good quality. The system of optical injection is also phase sensitive like the system of coherent optical feedback (Heil et al. 2003a). We must pay attention to the optical phase to achieve good quality of communications.

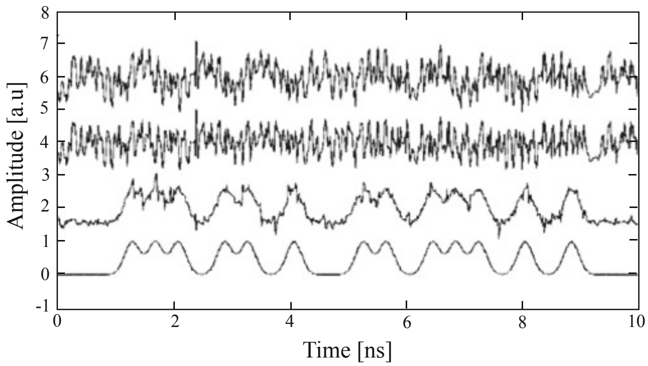

Fig. 13.12 Numerical example of message transmission in semiconductor lasers with optical injection in CMO. The system is an open-loop and the message is a 2.5 Gbps signal. From *top* to *bottom* transmitted signal including message, receiver output, decoded signal by subtraction of receiver output from transmitted signal, and low-pass filtered signal for decoded message [after Liu et al. (2001a); © 2001 IEEE]

13.4 Cryptographic Applications in Optoelectronic Systems

We here describe chaotic communications in semiconductor lasers with optoelectronic feedback systems. The system is incoherent coupling and the light from the laser is once detected by a photodetector. Then, the photocurrent is fed back into the bias injection current of the laser. Therefore, noises originating from photons are averaged out due to slow response of the carrier lifetime and high performance for synchronization between the transmitter and receiver lasers can be expected. A message is embedded into fast chaotic pulsation oscillations from sub-nanosecond to -picosecond and high-speed chaotic data transmissions are also expected due to the availability of high-speed electronic circuits. Figure 13.13 shows a model of chaotic communication systems in semiconductor lasers with optoelectronic feedback. The system can be described by only the photon number and the carrier density. A message is directly added to the laser intensity or the bias injection current, which is not the case for the systems of optical feedback and optical injection. The following formulation can be widely used in real optoelectronic systems without modification. For the transmitter, the rate equations read

$$\frac{\mathrm{d}S_{\mathrm{T}}(t)}{\mathrm{d}t} = G_{n,\mathrm{T}}\{n_{\mathrm{T}}(t) - n_{\mathrm{th,T}}\}S_{\mathrm{T}}(t) + R_{\mathrm{sp,T}} \tag{13.11}$$

$$\begin{aligned}\frac{\mathrm{d}n_{\mathrm{T}}(t)}{\mathrm{d}t} &= \frac{J_{\mathrm{T}}}{ed}\{1 + \eta_{\mathrm{CSK}} m_{\mathrm{CSK}}(t)\}\{1 + s_{\mathrm{T}}(t)\} \\ &\quad - \frac{n_{\mathrm{T}}(t)}{\tau_{\mathrm{s,T}}} - G_{n,\mathrm{T}}\{n_{\mathrm{T}}(t) - n_{0,\mathrm{T}}\}S_{\mathrm{T}}(t)\end{aligned} \tag{13.12}$$

$$s_{\mathrm{T}}(t) = \xi_{\mathrm{T}} S_{\mathrm{T}}(t - \tau_{\mathrm{T}}) + \eta_{\mathrm{CMO}} m_{\mathrm{CMO}}(t - \tau_{\mathrm{T}}) \tag{13.13}$$

When the system response is continuous, $s_{\mathrm{T}}(t)$ is replaced by a continuous response function in the same manner as in (12.68) as

$$y_{\mathrm{T}}(t) = \int_{-\infty}^{t} f(t' - t) s_{\mathrm{T}}(t') \mathrm{d}t' \tag{13.14}$$

whereas the receiver driven by the transmitted signal can be described by

$$\frac{\mathrm{d}S_{\mathrm{R}}(t)}{\mathrm{d}t} = G_{n,\mathrm{R}}\{n_{\mathrm{R}}(t) - n_{\mathrm{th,R}}\} S_{\mathrm{R}}(t) + R_{\mathrm{sp,R}} \tag{13.15}$$

$$\frac{dn_{\mathrm{R}}(t)}{\mathrm{d}t} = \frac{J_{\mathrm{R}}}{ed}\{1 + s_{\mathrm{R}}(t)\} - \frac{n_{\mathrm{R}}(t)}{\tau_{\mathrm{s,R}}} - G_{n,\mathrm{R}}\{n_{\mathrm{R}}(t) - n_{0,\mathrm{R}}\} S_{\mathrm{R}}(t) \tag{13.16}$$

$$s_{\mathrm{R}}(t) = \xi_{\mathrm{R}} S_{\mathrm{R}}(t - \tau_{\mathrm{R}}) + \xi_{\mathrm{cp}} S_{\mathrm{T}}(t - \tau_c) + \eta_{\mathrm{CMO}} m_{\mathrm{CMO}}(t - \tau_c) \tag{13.17}$$

Similarly, $s_{\mathrm{R}}(t)$ is replaced by a continuous response function for a finite response of the electronic feedback circuits.

Figure 13.14 shows the experimental results of chaotic communications in semiconductor lasers with optoelectronic feedback in CMO. The system is open-loop. A message is a pseudo-random signal of an NRZ pulse sequence. The message is embedded into the bias injection current with additive modulation. As discussed in Sect. 12.5, open-loop configuration shows better quality of chaos synchronization and modulation (Tang and Liu 2001; Liu et al. 2002b). Figure 13.14a shows the plot of signals for data transmission and decoding. The decoded signal (the third signal from the top) well reproduces the original message above the threshold level (dotted line). From the time lag, it is recognized that the synchronization is a complete type, although the time lag is compensated to compare the waveforms. Since chaotic signals generated in semiconductor lasers with an optoelectronic feedback system are pulse-like irregular oscillations, additive modulation is suited for CMO rather than multiplicative modulation. Figure 13.14b shows the eye pattern for the decoded message. A good quality of opened eyes is obtained.

13.5 Chaotic Communications in Mutually Coupled Semiconductor Lasers

Mutually coupled two chaotic systems can be used for secure communications (Klein et al. 2006; Vicente et al. 2007). Although it is not easy to perform directly secure data transmissions and communications, this scheme, for example, allows one to negotiate a key through a public channel as is discussed in the followings. We here consider

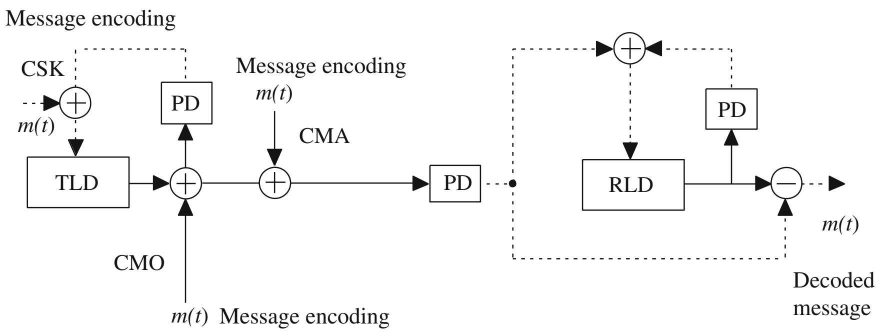

Fig. 13.13 Schematic diagram of chaotic communications in semiconductor lasers with optoelectronic feedback. $m(t)$ is a message signal to be embedded. The position for each message encoding scheme is shown in the figure. The *solid lines* are optical connections and the *broken lines* are electronic connections

a mutual coupling system described in Fig. 12.4c. The two lasers simultaneously transmit respective data through a single transmission channel. Vicente et al. (2007) proposed an asymmetry configuration of the mutual coupling system. In their system, the two lasers have different optical feedback lengths of external reflectors. The two lasers even synchronize developing a leader-laggard-type dynamics with one laser following the other by the coupling delay. This symmetry breaking complicates the simultaneous transmission of information in both directions. The type of chaos synchronization in this system is complete in spite of the asymmetric configuration and, moreover, the two lasers show a leader-laggard dynamics. Messages of "0" and "1" are embedded to the lasers through the injection currents and the messages are simultaneously transmitted to one laser to the other and vice versa. Based on chaos synchronization in the system, they numerically obtained the difference of the two messages. The proposed scheme is the system of simultaneously exchange information between the two lasers by using a single communication channel.

Figure 13.15 shows a result of data transmissions through mutual chaotic communications (Vicente et al. 2007). Figure 13.15a is a pseudo-random digital message of 1 Gbps embedded into the injection current in one of the lasers, $m_1(t)$. The amplitude of the message is $0.12 J_{\mathrm{th}}$. Figure 13.15b is the message in the counterpart laser, $m_2(t)$. Figure 13.15c is the expected subtraction of messages $\Delta m(t) = m_1(t) - m_2(t - \Delta\tau)$ with a given time lag ($\Delta\tau = 1$ ns in this case). Figure 13.15d is the calculated intensity difference ΔS after filtering with a fifth-order Butterworth filter with a cutoff frequency of 0.8 GHz. Thus, one can successfully obtained the difference of the embedded messages based on the mutual coupling system under appropriate synchronization conditions. In this system, the maximum encoding rate depends on the inverse of the resynchronization time after a bit arrives at one of the lasers. In the numerical conditions, it is $\sim$0.3 ns. Consequently, a maximum bit rate of about 3 Gbps could be achieved.

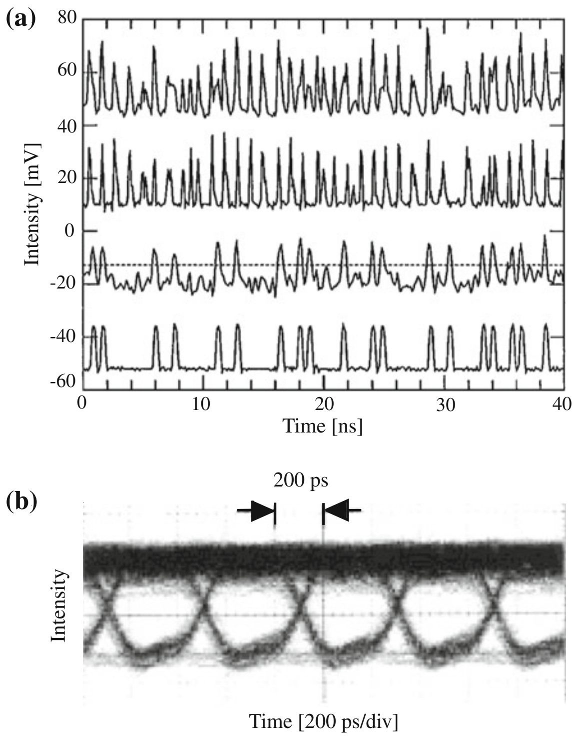

Fig. 13.14 Experimental example of message transmission in semiconductor lasers with optoelectronic feedback in CMO. The system is an open-loop using DFB lasers at the wavelength of 1.30 μm and the message is a 2.5 Gbps signal. **a** From *top* to *bottom* transmitted signal including message, receiver output, decoded filtered signal, and original message [after Tang and Liu (2001); © 2001 OSA]. **b** Eye pattern of the decoded signal [after Liu et al. (2002b); © 2002 IEEE]

Next, we must discuss the security aspects of data transmissions in the mutual coupling system, since both output powers from the two lasers can be accessible from the same communication channel. Therefore, an eavesdropper could easily monitor the difference of the messages. For the detected difference, a level of 1 in the message difference would clearly indicate that at the proper time the bit associated with one laser was a "1," while the one sent by the other laser was a "0." A similar argument holds when the message difference is -1. Only when the message difference is zero, which is the case for the coding of the same bit in both lasers, the eavesdropper has no clue as to which are the bits that are being sent. Thus, the mutual coupling configuration could be used to simultaneously negotiate a key for secret data transmissions (Mislovaty et al. 2003). The sender and the receiver can agree to discard those bits that are different from each other while accepting that the key that is formed by the first N bits that coincide with each other. Thus, a key is encrypted with the same level of security as in a unidirectional chaos communication scheme.

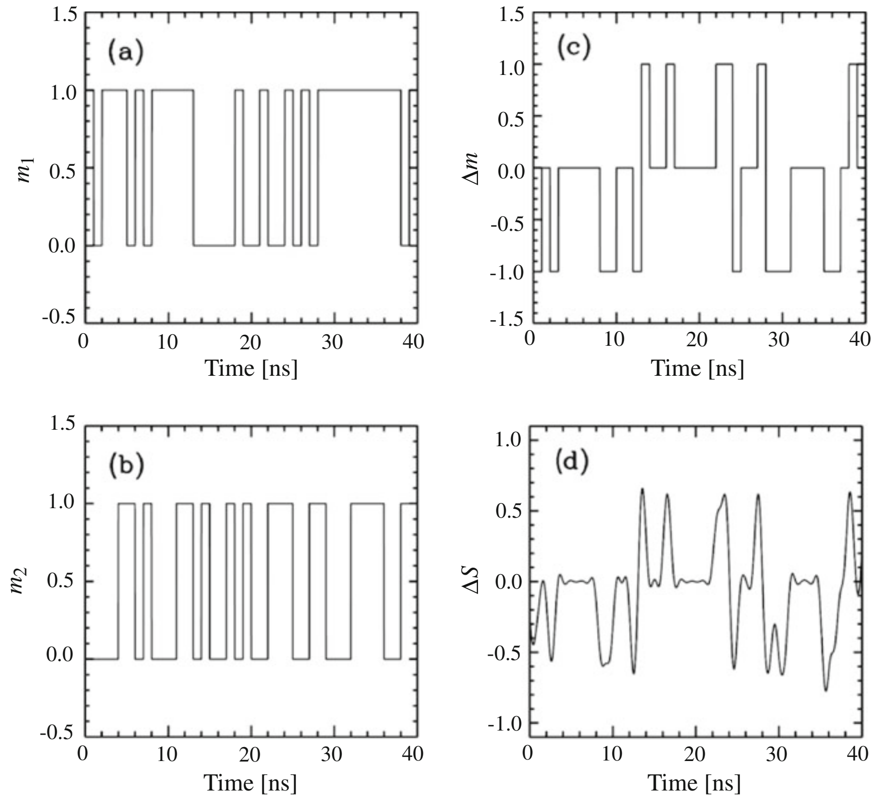

Fig. 13.15 Messages and decryption in mutual coupling semiconductor lasers. Messages from **a** laser 1 (m_1) and **b** laser 2 (m_2). **c** Expected subtraction of messages with time compensation. **d** Decrypted signal (ΔS) through a fifth-order Butterworth filter with a cutoff frequency of 0.8 GHz [after Vicente et al. (2007); © 2007 OSA]

13.6 Chaotic Communications in Drive-Response Systems

In Sect. 12.8, we discussed the enhancement of the correlation in chaos synchronization introducing a third laser as a common drive chaotic system to twin chaotic lasers (transmitter and receiver lasers). We can expect high performance of chaotic communications based on this method. In the following, we present numerical examples of chaos synchronization in common chaos drive system to transmitter and receiver systems. Annovazzi-Lodi et al. (2008, 2010a,b) proposed systems of optical injection discussed in Sect. 12.8 and optoelectronic driving. Here, we show an example of optoelectronic driving system, in which the drive chaotic signal is once detected by photodetectors and the photocurrents are fed to transmitter and receiver lasers as a common chaotic driving signals (Annovazzi-Lodi et al. 2008). We assume chaotic systems of semiconductor lasers with optical feedback both for the driving chaotic

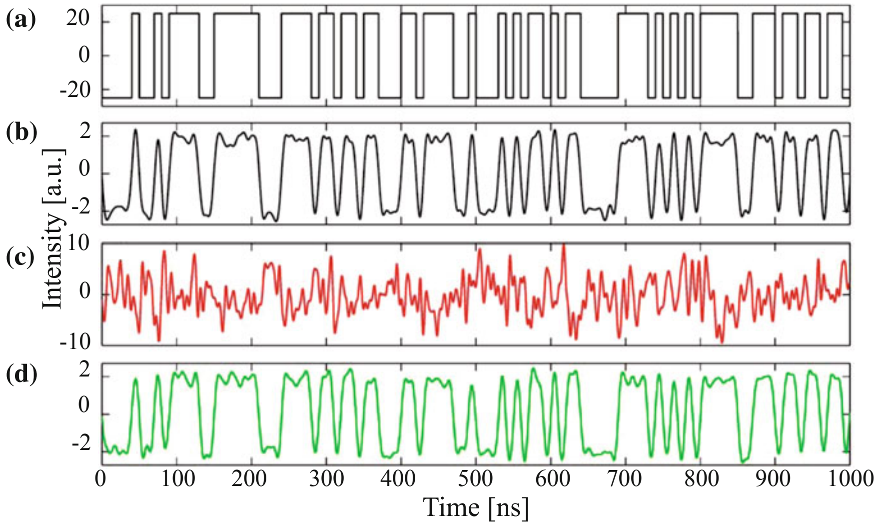

Fig. 13.16 Simulated message transmission in a Drive-Response systems. **a** Original message, **b** transmitter light output without chaos and off drive signal, **c** transmission signal of chaotic oscillation plus hidden message, and **d** decrypted message with drive signal. The signal, which is transmitted on a 4.65 GHz carrier, is 100 Mbps. The Langevin noise terms of the lasers and shot noises terms of photodetectors are taken into account [after Annovazzi-Lodi et al. (2008); © 2008 IEEE]

system and the twin response systems. The device parameters and the operating conditions may be different between the driving system and the response systems. On the other hand, in the response systems, the transmitter and the receiver should have almost the same device parameters and operating conditions to obtain good chaos synchronization between them. As a result, the quality of chaos synchronization between the drive and response systems may not be high, while a good correlation between the transmitter and receiver outputs can be attained due to the common chaotic driving. A message embedding into the transmitter chaos is sent to the receiver. Thus, the injection current to the laser is given by $J_{\mathrm{T}}(t) = J_{\mathrm{T}0} + J_D(t) + J_M(t)$, where $J_{\mathrm{T}0}$ is the constant bias injection current, $J_D(t)$ is the chaotic current from the driver, and $J_M(t)$ is the current corresponding to the message. The transmitted signal is once detected by a photodetector. The injection current to the receiver laser is given by $J_{\mathrm{R}}(t) = J_{\mathrm{R}0} + J_D(t)$, where $J_{\mathrm{R}0}$ is the constant bias injection current. Then, the output from the receiver laser is subtracted from the transmitted signal and, finally, the decrypted message can be obtained.

Figure 13.16 is a result of numerical simulation (Annovazzi-Lodi et al. 2008). The delay times of optical feedback both in the drive and response systems are set to be the same, however the characteristics of the drive laser are fairly different from those of the response lasers. To represent the actual conditions of experiments, Langevin noises of the lasers, shot noise of the detectors, and Johnson noise of the 50 Ω termination resistance are taken into account in the numerical simulations. Under the conditions, they obtained the correlation coefficient of 0.68 between the drive and response outputs, while it is as high as 0.98 between the transmitter and receiver lasers under the common chaos driving. The message shown in Fig. 13.16a is a pseudo-random NRZ digital signal at 100 Mbps modulated by the carrier in amplitude with 100 % modulation depth. The level of the embedded message is 2.86 % of the bias injection current of the transmitter laser. Figure 13.16b is the output power from the transmitter laser without optical feedback and in the absence of chaotic driving signal from the third laser. Since various noise sources are taken into account, the observed output power from the laser would not be like the trace in Fig. 13.16a but in Fig. 13.16b in real experiment. Figure 13.16c shows the transmission signal including the transmitter chaos and the message and Fig. 13.16d is the decrypted message through a band-pass filter with a 400 MHz bandwidth around the carrier frequency. The decrypted message is almost equal to the time trace in Fig. 13.16b, thus a high quality data reconstruction with secure chaotic encryption is attained by the method of drive and response configurations. The system requires an extra communication channels from the driver to the response systems. Annovazzi-Lodi et al. proposed an application of the system for free-space broadcasting of the driving chaotic signal and communications using a diffuser. Therefore, the method may be useful as local signal scrambler in a small room communications. They also discussed the effects of parameter mismatches for chaos synchronization and the quality of message reconstructions in the systems.

13.7 Performance of Chaotic Communications

We have discussed chaotic communications for three message encryption schemes (CMA, CMO, and CSK) in three different systems of semiconductor lasers (optical feedback, optical injection, and optoelectronic feedback systems). Each scheme in each system has merit and demerit for chaotic data transmissions. Liu et al. (2002b) numerically compared the performances of chaos communications for these methods. Figure 13.17 shows the results of the comparison of the performance for the three systems of (a) optical feedback, (b) optical injection, and (c) optoelectronic feedback. A message is a 10 Gbps pseudo-random pulse sequence. The relaxation oscillation frequency of the laser assumed is set to be 12 GHz at the operating bias injection current. Open-loop configurations are assumed for all the systems. For CMA and CMO in optical feedback and optical injection systems, the ratio of the amplitude for the encoding message to that of the chaotic amplitude is set at 0.05. For the CSK system, the injection current is modulated as an ON/OFF modulation and the corresponding modulation index to the optical field is taken to be the same as those in CMA and CMO. At the modulation index of 0.05, signal-to-noise ratio (SNR) of 30 dB has the channel noise at a level of an equivalent laser linewidth of $\Delta\nu = 0.66$ MHz. On the other hand, the modulation index is assumed to be 0.2 in the optoelectronic feedback system.

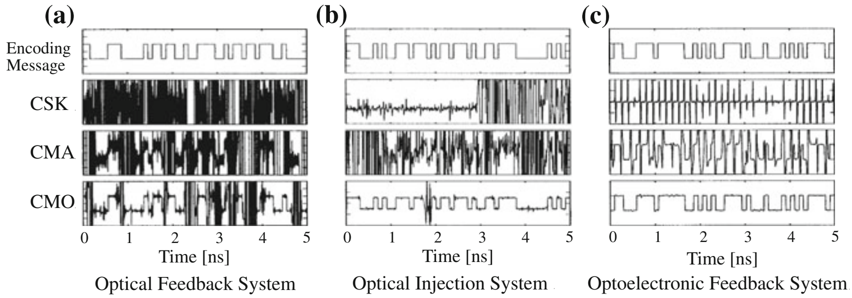

Fig. 13.17 Comparison of performance for chaotic communications in semiconductor laser systems at a data transmission rate of 10 Gbps. **a** Optical feedback system, **b** optical injection system, and **c** optoelectronic feedback system. All the systems are open-loop. From *top* to *bottom* in each figure: encrypted message, decoded signal in CSK, decoded signal in CMA, and decoded message in CMO [after Liu et al. (2002b); © 2002 IEEE]

A finite response time is required for chaos synchronization in a receiver system when each message is transmitted. In CSK, the chaotic state is always switched in accordance with the binary message and the receiver cannot follow the ON/OFF switching of the chaotic states. Thus, the performance of synchronization becomes worse and one cannot recover the message for the worst case. The basis of chaotic communications is that a message signal attached to the chaotic carrier, which is very small in comparison with the size of the trajectory as a chaotic attractor, will be averaged out and has almost no effect on the duplication of the chaotic trajectory. The whole chaotic carrier waveform can then be reproduced very precisely through all the local predictor functions. However, as already discussed, a message is essentially a perturbation for the chaotic attractor of a transmitter output in CMA and CSK even when it is small, thus the synchronization deviation increases for large message amplitude. On the other hand, for CMO, the symmetry of the transmitter and receiver systems is preserved even if a message is embedded into the transmitter. Therefore, the system of CMO is robust for synchronization deviation compared with CMA and CSK. As a result, we can successfully recover the message with good quality for CMO in the system of optoelectronic feedback as shown in Fig. 13.17c. The synchronization is interrupted by synchronization deviation and bursts at the high data-transmission rate of 10 Gbps even for CMO in the systems of optical feedback and optical injection, and one cannot obtain the original messages. In all systems, the messages are not recovered in the CSK and CMA schemes in Fig. 13.17. For a higher data-transmission rate, a system of optoelectronic feedback with CMO is best suited for chaotic communications.

The reason why the system of optoelectronic feedback is better than those of optical feedback and optical injection for chaos synchronization has been discussed in Sect. 12.5.1. The carrier lifetime plays an important role in the system of optoelectronic feedback, while the photon lifetime is crucial for the other systems. The photon noises are averaged out in the system due to a slow response of the carrier and, as a result, the system is robust for photon noises. As far as the response of the electronic circuits can follow the chaotic signal, we can expect good chaos synchronization and faithful message decoding. The measure of the performance for a communication system is the BER for the decoded message as a function of the SNR in the transmission channel. The SNR is defined as (Liu et al. 2002b)

$$\mathrm{SNR} = 10 \log \frac{S_m}{\sigma_n^2} \tag{13.18}$$

where S_m is the power of the transmitted message, and $\sigma_n^2 = N_0/2T_b$ is the variance of the channel noise with $N_0/2$ being the power spectral density of the channel noise and T_b being the bit duration. The channel SNR is a function of the channel noise, which is taken to additive white Gaussian noise, and the bit energy of the transmitted message, which depends on the modulation index of the message. BER arises from synchronization errors and bursts induced by channel errors and spontaneous emission noises in the lasers. The synchronization error σ_{error} has already been defined in (12.36).

Simply good quality of synchronization does not guarantee good retrieval of a message signal due to the sensitivity of the synchronized trace to any perturbation, including the perturbation caused by the intrinsic noise of the transmitter and that of the receiver. If some perturbation temporarily desynchronizes the synchronized transmitter and receiver for a period-of-time, the message signal within this period cannot be recovered. Therefore, the robustness of synchronization has to be considered in chaotic communications. Desynchronization could happen if the synchronized trace has any positive conditional Lyapunov exponent for a period-of-time while any perturbation is acting on this synchronized trace. It depends on the value of the positive conditional Lyapunov exponent and the strength of the perturbation during that period of time.

The synchronization error are categorized into two origins of synchronization deviation, which is associated with the accuracy of synchronization and desynchronization bursts, which is related to the robustness of synchronization in the system. The correlation coefficient of chaos synchronization between the transmitter and receiver lasers is usually not close to unity, although the two chaotic signals are similar. The deterioration of the correlation coefficient corresponds to the synchronization error. On the other hand, the chaotic output in the receiver completely differs from the transmitter signal at the occurrence of desynchronization bursts and the transmitter and receiver outputs have no correlation. Desynchronization burst is observed at the marginal region of the allowed parameter mismatches for the synchronization. The occurrence of desynchronization burst depends on the combination of chaotic parameters in the systems. It is an essential phenomenon in chaos synchronization systems with parameter mismatches. Even when the parameters of the transmitter and receiver systems are equal, desynchronization burst may suddenly occur by noises in the electronic circuits. The bits error caused by synchronization deviation is measured by the concept of synchronized bit error rate (SBER)

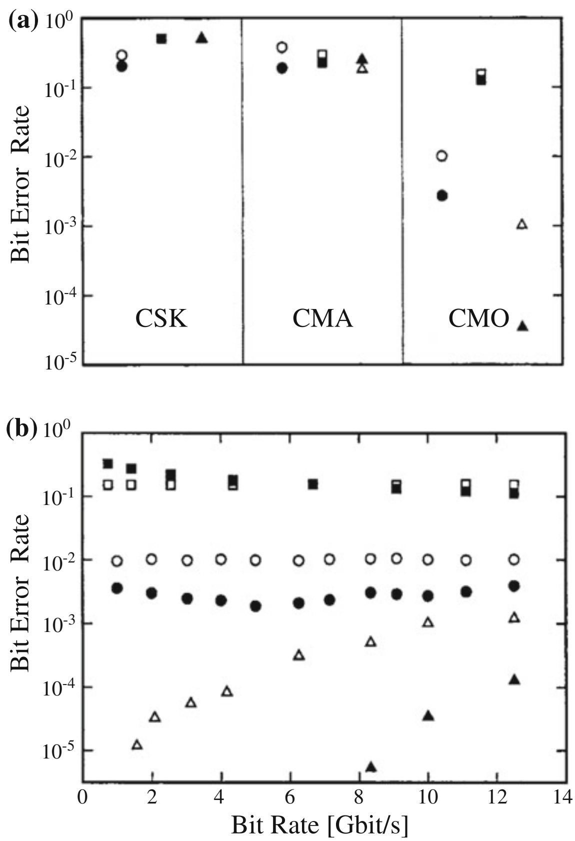

Fig. 13.18 Performance of BER. **a** BERs for optical injection system (marked as *circles*), optical feedback system (marked as *squares*), and optoelectronic feedback system (marked as *triangles*) of three different encryption schemes. The *solid symbols* mark the BER after the filtering, and the *open symbols* mark the BER before the filtering. **b** BER versus bit rate for three systems under CMO. The meanings of the symbols are the same as those in **a**. The relaxation oscillation frequency of the lasers is assumed to be 12 GHz [after Liu et al. (2002b); © 2002 IEEE]

$$\mathrm{SBER} = \frac{\text{error bits caused by desynchronization bursts}}{\text{total number of bits tested}} \tag{13.19}$$

The desynchronized bit error rate (DBER) induced by desynchronization bursts is defined by

$$\mathrm{DBER} = \frac{\text{error bits caused by synchronization deviation}}{\text{total number of bits tested}} \tag{13.20}$$

Then the total BER is simply defined by the sum of the SBER and the DBER as

$$\mathrm{BER} = \mathrm{DBER} + \mathrm{SBER} \tag{13.21}$$

Figure 13.18 is the result of BERs calculated by numerical simulations under the assumption of SNR= 30 dB (Liu et al. 2002b). BER can be improved by applying a filter to the decoded messages. In this example, the testing filter used for examining the characteristics of error reduction is a digital Chbyshev Type I filter with the cutoff frequency equal to the bit rate, the sampling frequency equal to 20 times the cutoff frequency, 0.5 dB ripple on the pass-band and −30 dB attenuation on the stop-band. The best performance of BER is obtained for the optoelectronic feedback system with CMO, which is a good coincidence with the result in Fig. 13.17. Figure 13.18b is the performance of BER obtained by changing the bit rate of the data transmission in CMO schemes. The BERs in optical feedback and optical injection systems stay constant for the change of the bit rate. However, we can expect a great improvement of BER in optoelectronic feedback systems for a lower data-transmission rate.

13.8 Security of Chaotic Communications

The successful demonstrations of chaotic data transmissions in laser systems including semiconductor lasers, and solid-state lasers, and fiber lasers have proved that these schemes are robust to some degree. However, there is still much to be done in terms of evaluating the robustness and practical tradeoffs with the security based on parameter sensitivity. The security in chaotic communications is guaranteed by the coincidence of the system parameters between the transmitter and receiver including the device parameters and the operation conditions of the lasers. Namely, one cannot achieve chaos synchronization in the system without knowing parameters as a key for communications even if one can know the system configurations. Chaotic communications are essentially hardware-based techniques. However, we can make a virtual system of the chaotic communication on computer software when we know completely the system configurations and their mathematical descriptions. Even for such a case, it is still difficult to imitate the synchronous chaotic signal without knowing each system parameter value. Therefore, the tolerance for the parameter mismatch is quite important for the realization of secure chaotic communications and we require systems that have strict conditions for the parameter mismatches for communications. However, as noted, there is a tradeoff of the difficulty for the range of synchronization in real systems.

One of the aspects of the security of messages hidden in chaotic waveforms is the difficulty of separating the message from the chaotic carrier by analyzing recorded waveforms. It has been demonstrated that systems with low-dimensional chaos are not secure for data transmissions, in the sense that a low- dimensional attractor is easily reconstructed from time series data, and system parameters are easily estimated from this attractor (Short 1994, 1996). Decoding without knowing key parameters becomes more difficult with the increase of the dimension of the chaotic dynamics (Dachselt and Schwarz 2001). The chaotic signal from a semiconductor laser with optical feedback, as described by the theoretical model, is embedded in an infinite dimensional system due to the delay. However, the actual dimension of the dynamics is typically much lower, being restricted by the intrinsic response times in the laser. Quantitative analysis of the dimension of the synchronized chaos and the degree of security remains an important challenge for future study. Here, we limit our discussion to the issue of the matching of the receiver laser for message recovery by synchronization.

Another important issue is the noise problem. Not only optical and electronic circuits to generate chaotic signals but also optical channels include noises; however, there is little study on the effects of noises on the performance of chaotic communications. The result of Fig. 13.18 is such an instance. It is a well-known result that chaos is sensitive to the initial conditions of the system. When the systems include noises, it may be considered that they disturb the systems and the receiver is not able to output synchronous oscillations even if the two nonlinear systems consist of the same components and have the same parameters. Contrary to the expectations, two nonlinear systems can synchronize with each other even they include noises. This fact is verified by numerical simulations and experiments, although the basis for the phenomenon has not been theoretically proved yet. Chaos induced by the nonlinearity of a system and statistical noises are completely separated and the system seems to discriminate them. Although noises are additive to chaotic signals, a chaotic evolution of a system is essentially determined by the pure initial conditions of the system without noises. For example, a chaotic attractor has slight deviations from the original one in the presence of noises, but the chaotic route and chaotic dynamics do not change, while, for example, the maxima and minima of the output in fixed and periodic states in the bifurcation are not points but have finite widths due to the external noises.

There are two types of chaos synchronization; one is complete chaos synchronization and the other is generalized chaos synchronization. In the following, we discuss the differences of the security in chaotic communications between the two types of synchronization. Take as an example optical feedback systems. As shown in Fig. 12.7, complete chaos synchronization is only realized at zero frequency detuning and a lower optical injection in the map of the unstable injection-locking region. On the other hand, generalized chaos synchronization is attained in a wide range of frequency detuning and optical injection in the stable injection-locking region. From the standpoint of security, the conditions for generalized chaos synchronization are loose compared with those for the complete case. We have investigated the effects of mismatches of the laser device parameters for chaos synchronization in Fig. 12.8. In that case, good synchronization is attained for a very small range of the parameter mismatches in complete chaos synchronization and the accuracy of synchronization rapidly becomes worse for the increase of the parameter mismatches. On the other hand, the tolerance for the parameter mismatches is rather large for the case of the generalized chaos synchronization. Strict conditions are imposed for the case of complete chaos synchronization and, as a result, the security is better than that of generalized chaos synchronization. As a whole, the scheme of complete chaos synchronization is suited for chaotic communications with respect to a high degree of security.

Another interest concerning the security of chaotic communications is the dependence of the system structures, namely the system is either open-loop or closed-loop. Each system has advantages and disadvantages as discussed in the preceding sections. Since the receiver is not subject to feedback in the open-loop system, this configuration is mechanically more stable and easier to implement compared with the closed-loop system. The open-loop system is also very robust against frequency detuning and small parameter mismatch, and has a shorter resynchronization time. On the contrary, the closed-loop is less stable in spite of high degree of synchronization. Therefore, not only the device parameters but also the external conditions have to be matched within high precision, otherwise the synchronization quality is greatly deteriorated. In general, the closed-loop system is less robust than the open-loop system and has a longer resynchronization time. However, the degree of synchronization in the open-loop system is worse than that in the closed-loop, especially when the open-loop is working in the strong injection regime. The use of large amplitude modulations makes the message less encrypted. Soriano et al. (2009) investigated the degree of security in a system of semiconductor lasers with optical feedback. They used the average mutual information as a measure for correlations between two signals and tested for a CMO system both for the open and closed configurations. As results, they numerically found that higher privacy and security can be achieved when the closed-loop scheme is used in the receiver architecture, instead of the open-loop scheme.

The methods of analog chaotic communications are based on the technique of embedding or hiding a message into a chaotic carrier as a secure code. The study of the security issue is still under way. Finally, we here briefly address other alternative techniques proposed at present. Among them, the method of code scrambling based on chaotic signal generations as discrete-sequence optical CDMA is used for chaos communications as digital techniques as discussed before (Kennedy et al. 2000). In the meanwhile, the study has been carried out on developing chaotic algorithms, algorithms using the iteration of nonlinear functions, to efficiently generate random sequences with improved randomness and correlation properties for use in spread-spectrum, code-division multiplexing and error correction (Chen and Wornell 1998; Chen et al. 2001; Uchida et al. 2003a). Since a chaotic time series is truly an irregular oscillation, which is not predictable for the future. Using this property, one can generate fast physical random numbers from such as chaos from semiconductor lasers. Such example is treated in the next Chapter. In any event, it is noted that the methods of analog message encoding and decoding discussed here may not be the best ones for secure optical communications for chaotic data transmissions or for ultimate secure communication systems.

13.9 Chaotic Carrier and Bandwidth of Communications

The relaxation oscillation frequency is an important indicator of the maximum possible rate of data transmissions in chaotic systems of semiconductor lasers. The relaxation oscillation frequency ν_{R} of the solitary laser is given by $\nu_{\mathrm{R}} = \sqrt{gS/\tau_{\mathrm{ph}}}/2\pi$. The relaxation oscillation frequency of currently available semiconductor lasers is of the order of several GHz to 10 GHz. Chaotic variations in semiconductor lasers has a broad spectrum and the attainable maximum frequency is usually larger than the relaxation oscillation frequency of the solitary lasers. Therefore, we can transmit a signal which contains higher frequency components than the relaxation oscillation. For example, a message that contains more than the frequency components over 10 GHz was successfully transmitted in a chaotic communication system (Liu et al. 2001b). Also, over 100 Mbps messages were transmitted through a communication channel composed of systems of solid-state lasers that had a relaxation oscillation frequency of less than several MHz (VanWiggeren and Roy 1998a,b).

The semiconductor laser with a high frequency of chaotic carrier is desirable as a light source of chaotic communications to perform high data-rate transmission. Semiconductor laser with fast response is also essential for other applications such as common optical communications and mass-data storage systems. By carefully choosing parameters of semiconductor materials and device structures, the effort for fabricating faster response semiconductor lasers is still ongoing. However, the attainable frequency for the relaxation oscillation is limited only by improving the materials and the device structures. On the other hand, the relaxation oscillation frequency can be greatly enhanced by strong optical injection from different lasers as discussed in Sect. 6.3.2. Wang et al. (1996) theoretically investigated the enhancement of the relaxation oscillation frequency of a semiconductor laser by using a small signal stability analysis of the laser rate equations at the stable injection-locking steady-state and gave an example where the relaxation oscillation frequency of the injected laser at the solitary oscillation of 3 GHz was increased up to 12 GHz by strong optical injection. Thus, a semiconductor laser with enhanced modulation bandwidth by strong optical injection is effective as a light source for chaotic communications with the capability of faster data transmissions.

The receiver laser does not respond immediately after the transmitter signal is injected to the receiver laser and a finite transition time is required for synchronization. The transition time depends on the device parameters and the system configurations. We again consider a particular example. The model is an optical feedback system and the scheme of CSK. The synchronization recovery time affects the quality of synchronization especially in ON/OFF CSK systems. The transition time is directly governed by the synchronization recovery time. So far, there have been few systematic studies of the recovery time for chaos synchronization. Vicente et al. (2002) investigated the synchronization recovery time for open- and closed-loop systems of semiconductor lasers with optical feedback. Figure 13.19 shows the numerical results of the synchronization time as a function of the external cavity round trip time τ of the transmitter system. In Fig. 13.19a, the recovery time is independent of the delay time in the open-loop system and the average time required for the synchronization is very small, about 0.2 ns. The reason for the constant delay time for the variation of the external cavity length is that the receiver laser does not need to adjust the time of synchronization for the parameters of the external cavity length, since the external cavity does not exist in the receiver of the open-loop system. On the other hand, the recovery time of the closed-loop system increases for the increase of the external cavity length (delay time). The time required in this case is much longer than that of the open-loop system. It is of the order of several tens of nanoseconds. In actual fact, there exist sudden bursts of synchronization even after chaos synchronization

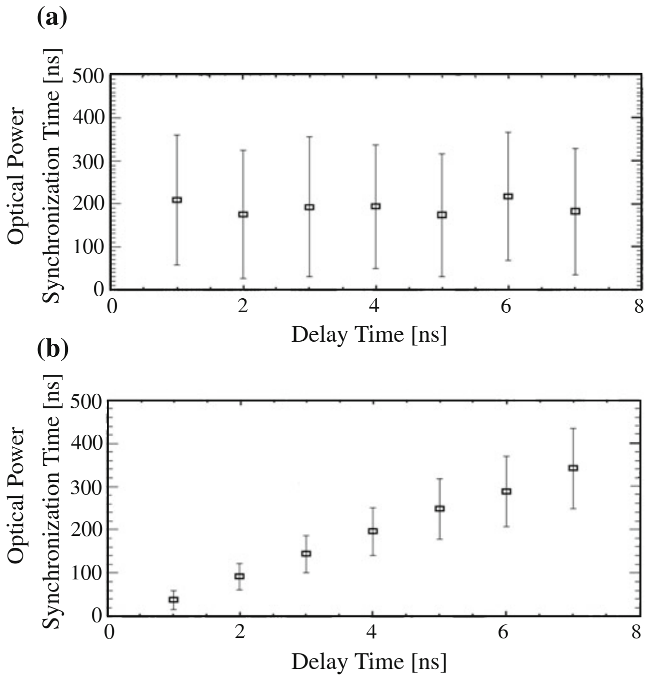

Fig. 13.19 Synchronization recovery time as a function of external-cavity roundtrip time in semiconductor lasers with optical feedback. The message encoding scheme is CSK. **a** Open-loop and **b** closed-loop systems [after Vicente et al. (2002); © 2002 IEEE]

is achieved. This burst behavior also degrades the quality of chaos communications and the property of synchronization bursts remains an important problem.

13.10 Chaos Communications in the Real World

13.10.1 Chaos Masking Video Signal Transmissions

Many theoretical studies have been reported for secure communications using chaotic semiconductor lasers. Also, at the laboratory level, experimental chaotic communications have been demonstrated for data transmissions of sinusoidal signals and pseudorandom bit-sequence signals based on various data encryption methods. However, only a few studies have been reported on chaotic data transmission for real data. Larger et al. (2001) demonstrated data transmission of voice signals using chaotic semiconductor lasers with EO feedback through wavelength filters, which is the same system as discussed in Sect. 7.4. In that system with a CMO scheme, an AM voice signal encrypted into transmitter chaos is transmitted on a radio frequency and the

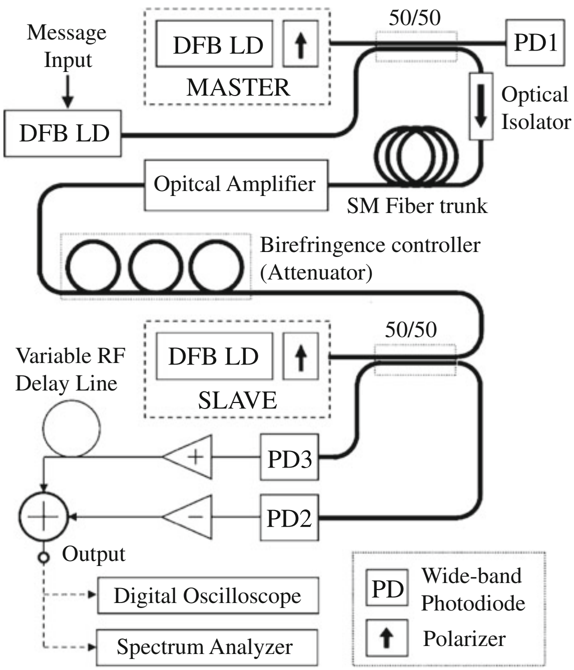

Fig. 13.20 Video signal encoding and decoding system using chaos masking method with 1.2 km fiber transmission [after Annovazzi-Lodi et al. (2005); © 2005 IEEE]

signal is successfully decrypted in the receiver system. In the following, we show another example of real world data transmissions of video signals at 2.4 GHz sideband frequency embedded into chaotic carrier in semiconductor lasers with optical feedback.

Figure 13.20 shows a CMA system for video data transmission using a chaotic generator of semiconductor laser with optical feedback (Annovazzi-Lodi et al. 2005). The light source is a DFB semiconductor laser of an oscillation wavelength at 1.55 μm. To make the laser chaotic oscillations, the emitted light is fed back from a tip of a transmission fiber, which is located at 3 cm from the laser facet. The DFB lasers used were selected between first neighbors on the same wafer. As a chaos synchronization system, the system is an open-loop configuration and there is no optical feedback in the receiver system. The injection current is biased at $1.5I_{\mathrm{th}}$ and the value of each parameter for the transmitter and receiver lasers, such as bias injection currents and temperatures, is finely tuned to coincide with each other. The levels both for the optical feedback in the transmitter and the transmission signal from the transmitter to receiver laser is set to be around 1 % in the experiment. A message is obtained as a modulation of another DFB laser and is added to chaotic transmission signal (signal from Master laser) though 50/50 fiber coupler. The fiber length of the data transmission between transmitter and receiver is of about 1.2 km. After transmission, the signal is amplified through a semiconductor optical amplifier to increase the maximum injection level from the transmitter into the receiver. Then

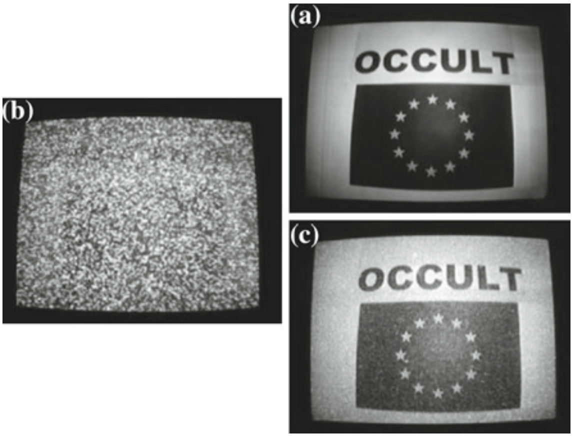

Fig. 13.21 TV frames of a still image transmitted by the setup of Fig. 13.20. **a** Original image to be send, **b** encoded pattern with chaotic signal, and **c** decoded pattern [after Annovazzi-Lodi et al. (2005); © 2005 IEEE]

the signal passed through a birefringence controller to trim and adjust the injection level to the receiver laser. Subtracting from the reproduced chaotic signal from the transmitted one, the decoded message is recovered. The type of chaos synchronization in this system is one for generalized synchronization (injection and amplification synchronization).

Figure 13.21 shows the results of data transmissions for a still TV pattern. In the system, a composite video signal with amplitude-modulated frequency at 2.4 GHz is used as a message. The quality of the received signal has been evaluated after synchronous detection and baseband filtering at the receiver output node. The signal amplitude has been adjusted as a compromise among efficient masking, low signal distortion, and good quality of the recovered message. In Fig. 13.21, three photographs of the monitor screen are shown. Figure 13.21a is an original pattern to be transmitted without added chaos. Figure 13.21b shows the picture hidden within chaos and represents the message as it would be recovered by an eavesdropper. Figure 13.21c shows the extracted message after synchronization. The signal level has been adjusted as a tradeoff between sufficient image masking by chaos and acceptable image quality after chaos cancelation. Figure 13.21c is obtained by setting the AM sideband level at about 4 dB over chaos. In these conditions, the SNR of $S/N = 16 \sim 18\,\mathrm{dB}$ is obtained for the decoded message.

13.10.2 Chaotic Signal Transmissions Through Public Data Link

Argyris et al. (2005) tested the effectiveness of chaotic data transmission in the existing public optical communication links. They employed two chaotic communication systems using semiconductor lasers as light sources; one is an EO open-loop system and the other is all optical open-loop system both based on CMA technique. Figure 13.22 shows the schematics of the systems. Figure 13.22a shows a chaotic EO

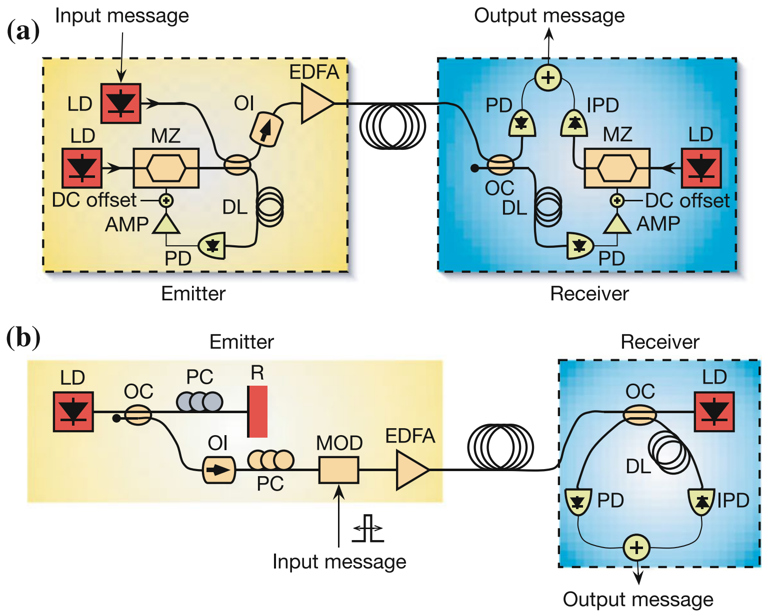

Fig. 13.22 Setups for two optical chaos communication systems. **a** Electro-optic open-loop system. **b** All optical open-loop system. LD: laser diode, MZ: electro-optic Mach–Zehnder interferometer, PD: photodiode, AMP: electronic amplifier, OI: optical isolator, DL: delay line, EDFA: erbium-doped fiber amplifier, OC: optical fiber coupler, IPD: sign-inverting amplified photodiode, R: digital variable reflector, PC: polarization controller, MOD: modulator [after Argyris et al. (2005); © 2005 Nature Pub.]

synchronization system. In this system, chaos is not generated from the nonlinearity of the semiconductor laser itself but from the nonlinear delayed response of light due to delayed optoelectronic hybrid feedback through EO modulator (an integrated EO Mach–Zehnder interferometer: MZ). The system has very high response over 10 GHz and is frequently used as a chaos generator (Gibbs 1985; Davis 1990). The lasers used are DFB semiconductor lasers with an oscillation wavelength of 1.55 μm. A message to be sent, generated from another DFB laser, is simply added to chaotic signals through a 50/50 fiber coupler, which is an additive CMA scheme. The bandwidth of the system estimated is about 7 GHz.

Figure 13.22b shows the second case of all optical system. The transmitter is a DFB semiconductor laser subjected to optical feedback from a digital variable reflector (R) located 6 m from the laser. The system is almost the same as one for the video signal transmission system in the previous subsection. A polarization controller (PC) is used within the cavity to adjust the polarization state of the light reflected back from the variable reflector. The message is added via a $LiNbO_3$ Mach–Zehnder modulator (MOD) at the transmitter's output. The scheme is multiplicative CMA. The bandwidth of the all optical system is less than that of the EO system and it is about 5 GHz. In both schemes, an erbium-doped fiber amplifier (EDFA) is used to com-

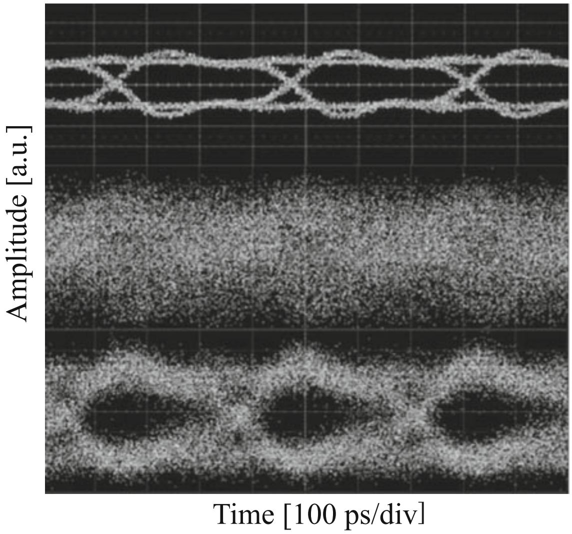

Fig. 13.23 Laboratory test of eye diagrams in the electro-optic system through 50 km fiber transmission. *Top trace* test message, *middle trace* encoded signal, *bottom trace* decoded message [after Argyris et al. (2005); © 2005 Nature Pub.]

pensate for the power lost upon transmission. Decoding is performed via subtraction of the transmitted signal from the signal filtered by the receiver. Operationally, the subtraction is performed by adding the photocurrents coming from an ordinary and a sign-inverting amplified photodiode (PD and IPD, respectively). Also the type of chaos synchronization in this system is one for generalized synchronization as is usually the case for real chaos communication systems.

Figure 13.23 shows the results for laboratory experiments of eye diagrams in the EO setup after transmission of a binary message through single mode optical fiber of 50 km and dispersion compensation fiber of 6 km. The message is a pseudo-random bit sequence of $2^7 - 1$ bits and the transmission rate is 3 Gbps. The observed bit error rates (BERs) are of the order of 10^{-7}. As already discussed, the performance of data transmission for all optical scheme is usually poor compared with that for EO system. The transmission rate to attain the similar performance of BER as that in the EO system is about 1 Gbps.

To test performance under 'real-world' conditions of chaotic communications, the chaos-based all optical transmission system were implemented using an installed optical network infrastructure of single-mode fiber belonging to the metropolitan area network of Athens, Greece. The network has a total length of 120 km. The topology consists of three fiber rings, linked together at specific cross-connect points as shown in Fig. 13.24a. Through three cross-connect points, the transmission path follows the Ring-1 route, then the Ring-2 route, and finally the Ring-3 route. To cancel the chromatic dispersion that would be induced by the single-mode fiber transmission, a dispersion compensation fiber module is used in the link. Also, to compensate the optical losses and filtering of amplified spontaneous emission noise, EDFA and optical filters are used along the optical link. The pair of lasers of the transmitter and

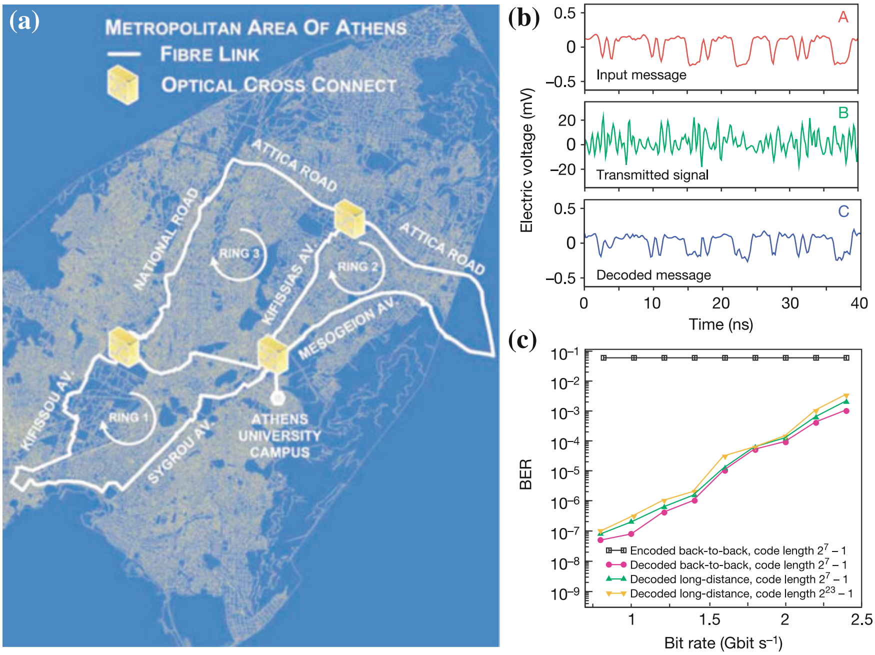

Fig. 13.24 Field experiment of fiber transmission. **a** Chaos-encoded data transmissions in the optical communication network of Athens, Greece. **b** Time trances of 1 Gbps message. *Trace A* applied message of BER$< 10^{-12}$, *trace B* carrier with the encoded message of BER$\approx 6 \times 10^{-2}$, *trace C* recovered message after 120 km transmission of BER $\approx 10^{-7}$. **c** The bit error rate (BER) performance. *Squares* encoded signal, *circles* back-to-back decoded message, *triangles* decoded message after transmission for two different code lengths [after Argyris et al. (2005); © 2005 Nature Pub.]

receiver is selected to exhibit parameter mismatches that are constrained below 3 %. The mean optical power injected into the receiver has been limited to 0.8 mW, to avoid possible damage of the anti-reflective coating of the slave laser. Test messages are NRZ pseudo-random bit sequences applied by externally modulating the chaotic carrier by means of a modulator. The message amplitude is attenuated by 14 dB with respect to the carrier to maintain the message security in the communications. As a result, the BER of the transmitted signal after filtering is always larger than 6×10^{-2}, which is the instrumentation limit. A good synchronization performance of the transmitter–receiver setup leads to an efficient cancellation of the chaotic carrier and, thus, satisfactory decoded messages are obtained as shown in Fig. 13.24b. Figure 13.24c shows the performance of the chaotic transmission system for different message bit rates up to 2.4 Gbps for two different code lengths of $2^7 - 1$ and $2^{23} - 1$. All BER values have been measured after filtering the subtracted electric signal, by using low-pass filters with bandwidth adjusted each time to the message bit rate. For transmission rates in the gigabit per second range the recovered message exhibits BER values lower than 10^{-7}. For higher transmission rates, the corresponding BERs increase due to the fact of imperfect synchronization, as shown in Fig. 13.24c. For the implementation of chaos systems in real optical communication networks, system integrations as compact chips for chaotic light generators are indispensable. We will present the development of photonic integrated circuits for chaotic light generators in next chapter.

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