3 Impact of Water on Offshore Structures and Infrastructure

A characteristic phenomenon in offshore engineering is the impact of water on individual devices and elements of dynamic systems. This impact is a very complex phenomenon composed, among other things, of wave action, sea currents, hydrostatic and hydrodynamic forces. These processes are difficult to describe, and there exists a handful of approaches to modelling them, which differ in the level of idealization [Newman J. N., 1977], [Mei C. C., 1989], [Faltinsen O. M., 1990]. A possibly simple description of the motion of an offshore structure's base is often desirable, e.g. in the problems of control [Fossen T. I., 1994].

3.1 Basics of Water Wave Motion Mechanics

Hydrodynamic dependencies are at the foundation of any attempt at modelling objects situated in maritime environment. Formulas presented in this chapter allow us to determine the values which direct the motion of systems immersed in water, i.e. velocity and hydrodynamic pressure of the fluid.

Let us consider a point with coordinates given by the vector (Fig. 3.1):

$$\mathbf{x} = \begin{bmatrix} x & y & z \end{bmatrix}^T, \tag{3.1}$$

in which the velocity of particles of the fluid in an inertial system equals:

$$\mathbf{v}(\mathbf{x},t) = \begin{bmatrix} v_x(\mathbf{x},t) & v_y(\mathbf{x},t) & v_z(\mathbf{x},t) \end{bmatrix}^T. \tag{3.2}$$

In many cases it is prudent to assume that the density of the fluid is constant, hence the continuity equation for incompressible flow [Bukowski J., 1968], [Newman J. N., 1977]:

$$div(\mathbf{v}) = 0, \tag{3.3}$$

where $div(\mathbf{v}) = \nabla \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$.

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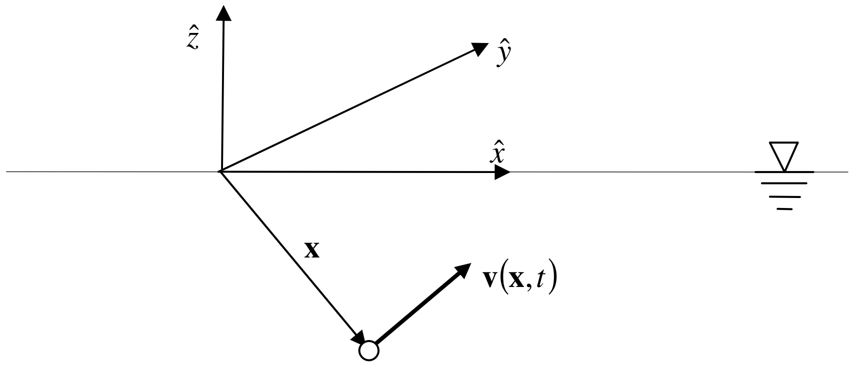

Fig. 3.1. Velocity of a particle of fluid in an inertial system

The momentum conservation principle for a Newtonian fluid is presented in the form of Navier-Stokes equations which, for an incompressible fluid, take the form [Bukowski J., 1968], [Newman J. N., 1977]:

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\nabla\mathbf{v} = \mathbf{F} - \frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{v}, \tag{3.4}$$

where $\mathbf{F} = [0 \quad 0 \quad -g]^T$ – mass forces,
$p = p(x, y, z, t)$ – pressure,
$\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$,
$\mathbf{v}$ – velocity vector,
ρ, ν – density and viscosity of the fluid.

The Navier-Stokes equations (3.4) together with the continuity equation (3.3) form a system of nonlinear partial differential equations. No general solution is known (only numerical approximations are possible). To determine approximate pressure values lack of viscosity and irrotational flow are usually assumed. The irrotationality condition [White F. M., 2006]:

$$rot(\mathbf{v}) = \nabla \times \mathbf{v} = \mathbf{0} \tag{3.5}$$

ensures the existence of a velocity field potential. A function Φ called the potential is further sought, such that:

$$\mathbf{v} = \nabla\Phi. \tag{3.6}$$

Determining the potential Φ allows for calculation of velocity as the gradient of the potential. Applying the formula (3.6), the continuity equation (3.3) may be rewritten as a Laplace equation [Newman J. N., 1977], [El-Hawary F., ed., 2001], [White F. M., 2006]:

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}=0. \tag{3.7}$$

This equation should be completed with boundary conditions.

Omitting the last summand in the equation (3.4) standing for viscosity and taking the dependency $\mathbf{F} = -\nabla gz$ into account allows us to rewrite (3.4) in the form:

$$\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\nabla\mathbf{v} = -\nabla\left(\frac{p}{\rho}+\Gamma\right) \tag{3.8}$$

where $\Gamma = gz$,

or, when the irrotationality condition is assumed, as a Bernoulli equation [Bukowski J., 1968], [Newman J. N., 1977], [El-Hawary F., ed., 2001]:

$$\frac{p}{\rho}+\frac{\partial\Phi}{\partial t}+\frac{1}{2}\left(\nabla^2\Phi\right)+\Gamma = const. \tag{3.9}$$

It makes it possible to determine the pressure if the potential Φ is known. Integrating the pressure over the surface of the body immersed in the fluid yields forces acting on it. In most practical cases, the hydrodynamics of ideal fluids is a sufficient theory for modelling the dynamics of systems occurring in offshore engineering. Viscosity, which was omitted from the equation (3.9), is sometimes re-added by stipulating additional empirical relations [Hoerner S. F., 1958], [Sarpkaya T., Isaacson M., 1981].

Equations of ideal fluid hydrodynamics enable determination of velocity and pressure fields for a regular wave (Fig. 3.2). In this case it is assumed that the profile of the wave (the free surface) is described by the formula:

$$z = \xi(x, y, t). \tag{3.10}$$

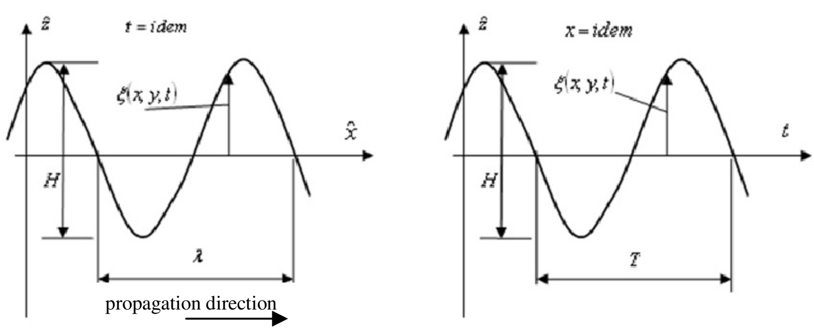

Fig. 3.2. Profile of a regular wave

Two boundary conditions for the free surface need to be also introduced. In the fist one, called the dynamic condition of free surface, the pressure of the fluid at the free surface is assumed to equal the atmospheric pressure. In the other, the so-called kinematic condition of free surface, the particles of the fluid on this surface are bound to it (they cannot break loose from it) [Newman J. N., 1977], [Faltinsen O. M., 1990].

3.1.1 Linear Model

Assuming the value of the quotient $\frac{p_0}{\rho}$ (p_0 – atmospheric pressure) in the Bernoulli equation (3.9) to be constant, taking the pressure on the free surface as p_0 and the profile of the wave to be described with the function [Newman J. N., 1977]:

$$\xi = \bar{\xi} \sin(\omega t - kx + \alpha), \tag{3.11}$$

a solution of the boundary problem is obtained in the form of a potential function (details are given, among others, in [Lighthill J., 1978], [Dean R. G., Dalrymple R. A., 1998]):

$$\Phi = \bar{\Phi} \cos(\omega t - kx), \tag{3.12}$$

where $\bar{\Phi} = \frac{\omega \bar{\xi}}{k} \frac{\cosh(k(d+z))}{\sinh(kd)}$,

$k = \frac{2\pi}{\lambda}$ – wave number,

λ – wave length.

Solving the Laplace equation with appropriate boundary conditions and using the definition of potential, the velocities of particles of the fluid may be calculated [DNV-RP-C205, 2007]:

$$v_x = \frac{\partial \Phi}{\partial x} = \omega \bar{\xi} \frac{\cosh(k(d+z))}{\sinh(kd)} \sin(\omega t - kx), \tag{3.13}$$

$$v_z = \frac{\partial \Phi}{\partial z} = \omega \bar{\xi} \frac{\sinh(k(d+z))}{\sinh(kd)} \cos(\omega t - kx). \tag{3.14}$$

Substituting the functions of potential (3.12) and profile (3.11) to the dynamic condition of free surface gives:

$$\omega^2 = gk \tanh(kd). \tag{3.15}$$

Pressure can be determined by using the potential obtained (3.12) and the Bernoulli equation (3.9):

$$p = -\rho g z + p_w \sin(\omega t - kx), \tag{3.16}$$

where $p_w = \rho g \bar{\xi} \dfrac{\cosh(k(d+z))}{\sinh(kd)}, \qquad z < 0.$

For large depths, i.e. for $kd \to \infty$, the above formulas take a simpler form:

velocity v_x, v_z:
$$v_x = v_z = \omega \bar{\xi} e^{kz}, \tag{3.17}$$

dispersion:
$$\omega = \sqrt{g k_0}\,,\ \lambda_0 = \frac{gT^2}{2\pi}, \tag{3.18}$$

pressure:
$$p = -\rho g z + \rho g \bar{\xi} e^{kz}. \tag{3.19}$$

Functions describing the trajectories of the fluid's particles are obtained by integration of velocity over time. The trajectories in local coordinate systems, when a wave profile conforming to (3.11) is assumed, change with sea depth (Fig. 3.3) forming ellipses. For $kd \to \infty$ the ellipses turn into circles whose radii may be approximated with the expression $\bar{\xi} e^{kz}$. For a shallow see the trajectories are flattened and tend to horizontal limes for $z \to -d$.

The linear model of waves described above (also called the Airy model) is adequate for waves with low amplitudes relatively to their length and sea depth. Its applicability is ruled by the following conditions:

$$\bar{\xi} k << 2\pi \text{ and } \bar{\xi} << d. \tag{3.20}$$

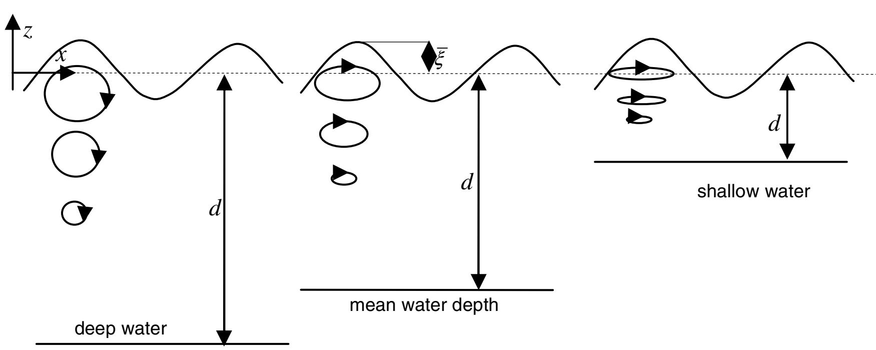

Fig. 3.3. Trajectories of particles for different sea depths

The limiting criteria allowing for the use of a given variant of the formulas produced, i.e. (3.13) – (3.16) or (3.17) – (3.19), depending on the depth d, are summarized in Table 3.1, following [Chakrabarti S. K., 2005].

Table 3.1. Criteria for depth and applicability of formulas in the linear model

Sea depth	Criterion	Wave length
large	$\frac{d}{\lambda} \geq \frac{1}{2}$	$\lambda = \lambda_0 = \frac{gT^2}{2\pi}$
medium	$\frac{1}{20} < \frac{d}{L} < \frac{1}{2}$	$\lambda = \lambda_0 \left[\tanh\left(\frac{2\pi d}{\lambda_0} \right) \right]^{\frac{1}{2}}$
small	$\frac{d}{\lambda} \leq \frac{1}{20}$	$\lambda = T\sqrt{gd}$

3.1.2 Stokes Model

The Stokes theory proposes models of waves which are nonlinear due to the nonlinearity of the dynamic condition of free surface in the variables $\mathbf{v}$ and p:

$$\frac{\partial p}{\partial t} + v_x \frac{\partial p}{\partial x} + v_z \frac{\partial p}{\partial z} = 0. \tag{3.21}$$

Depending on the number of components in the formula for velocity v_x, the Stokes theory is differentiated as first order, second order, third order, etc., see for example [Schwartz L., 1974], [Longuet-Higgins M. S., 1984]. Relations for the second order Stokes theory are shown in Table 3.2. In the description of waves this theory gives a fairly accurate approximation if the order of the method is increased along with the amplitude of waves. The results are especially satisfactory for deep seas.

There are numerous different models of waves deserving a mention [El-Hawary F., ed., 2001], [Webb D. J., 1978], [Komen G. J., et al., 1994], [Grue J., et al., 2003], [Tucker M. J., Pitt E. G., 2001]. Their applicability depends on depth d, wave period T and wave amplitude $\bar{\xi}$. Fig. 3.4, following [Dean R. G., Dalrymple R. A., 1998], shows a graph helpful in the choice of appropriate models (d and H are given in feet, T in seconds).

Trochoidal waves [Łomniewski K., 1969] distinguished in Fig. 3.4 are characterized by different shapes of crests (tall and narrow) and troughs (long and flat). They are an intermediate form between solitons and periodic waves (linear and nonlinear). Solitons lack troughs and their lengths tend to infinity. In shallow waters trochoidal waves transforms into solitons.

Table 3.2. Some dependencies in second order Stokes theory

potential Φ	$\Phi_l + \frac{3}{8}\frac{\pi H}{kT}\left(\frac{\pi H}{\lambda}\right)\frac{\cosh(2k(z+d))\sin(2\gamma)}{\sinh^4(kd)}$
velocity v_x	$v_{x,l} + \frac{3}{4}\frac{\pi H}{T}\left(\frac{\pi H}{\lambda}\right)\frac{\cosh(2k(z+d))}{\sinh^4(kd)}\cos(2\gamma)$
velocity v_z	$v_{z,l} + \frac{3}{4}\frac{\pi H}{T}\left(\frac{\pi H}{\lambda}\right)\frac{\sinh(2k(z+d))}{\sinh^4(kd)}\sin(2\gamma)$
pressure p	$p_l + \frac{3\rho g H}{4}\frac{\pi H}{\lambda\sinh(2kd)}\left[\frac{\cosh(2k(z+d))}{\sinh^2(kd)} - \frac{1}{3}\right]\cos(2\gamma) - + \frac{\rho g H}{4}\frac{\pi H}{\lambda\sinh(2kd)}\left[\cosh(2k(z+d)) - 1\right]$
where Φ_l, $v_{l,x}$, $v_{l,z}$, p_l are determined as in the linear wave theory (formulas (3.16) – (3.20)), λ is the wave length, $\gamma = \omega t - kx$	

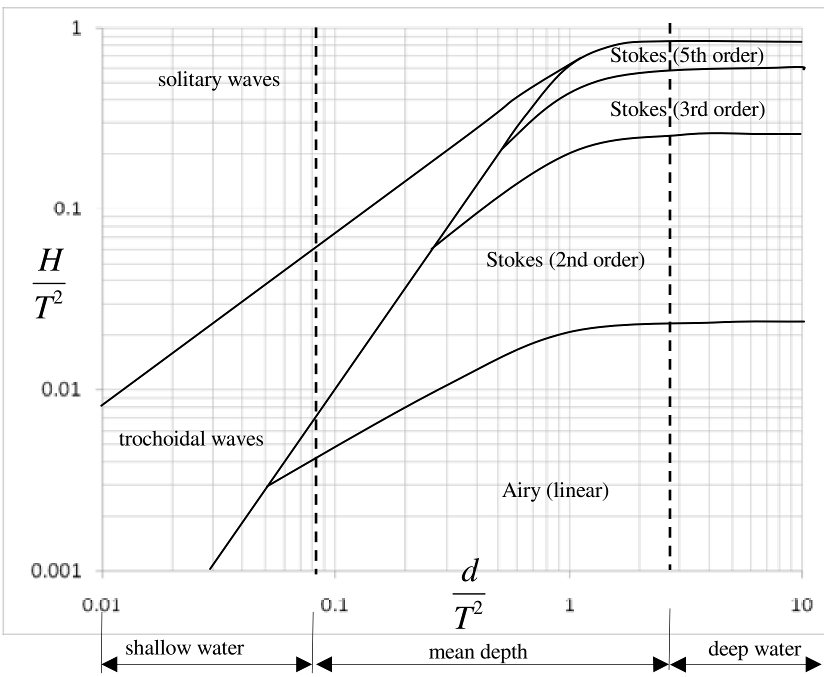

Fig. 3.4. Applicability ranges of different theories of sea waves

3.1.3 Statistical Description of Waves

In analyses and calculations, sometimes a spectral description of waves is used, which treats the phenomenon as a stationary problem within a finite time period (usually between 20 min and a few hours) [Massel S. R., 1996]. The spectrum $S(\omega)$ (spectral density of power) is defined as a function of some basic parameters of a wave, such as: significant wave height H_S, wave period T_p determined by the frequency of occurrence of function's $S(\omega)$ maxima and time T_z, calculated as mean wave period within the assumed time interval.

There are numerous methods to describe waves using a function of waves density distribution. Among the most often used are: the function devised by Pierson-Moskowitz [Pierson W. J., Moskowitz L., 1964] and its modified form, developed in the JONSWAP project [Hasselmann K., et al., 1973]. Examples of other, more complex functions are Ochi-Hubble [Ochi M. K., Hubble E. N., 1976] and Torsethaugen [Torsethaugen K., Haver S., 2004] distributions. Below, the forms proposed by Pierson-Moskowitz and JONSWAP are briefly presented, as they will be used in some mathematical models considered further herein.

The Pierson-Moskowitz spectrum is described thus:

$$S_{PM}(\omega) = \frac{5}{16} H_s^2 \omega_p^4 \omega^{-5} e^{\left(-\frac{5}{4}\left(\frac{\omega}{\omega_p}\right)^{-4}\right)} \quad (3.22)$$

where $\omega_p = \dfrac{2\pi}{T_p}$.

The JONSWAP spectrum takes additionally into account the intermediate states occurring at the onset of waves (contrary to the Pierson-Moskowitz spectrum which treats only waves already formed). The JONSWAP spectral density function is given by:

$$S_J(\omega) = A_\eta S_{PM}(\omega) \cdot \eta^{e^{\left(-0.5\left(\frac{\omega-\omega_p}{\sigma\omega_p}\right)^2\right)}} \quad (3.23)$$

where η – dimensionless shape parameter,

$$\sigma = \begin{cases} \sigma_a & \text{for } \omega \le \omega_p \\ \sigma_b & \text{for } \omega > \omega_p, \end{cases}$$

$A_\eta = 1 - 0.287 \ln(\eta)$,

σ_a, σ_b – parameters of the distribution.

For η = 1, the relation (3.23) reduces to (3.22). The method of selecting the parameters η, σ_a, σ_b is specified in norms, e.g. [DNV-RP-C205, 2007], and in literature [Clauss G. F., et al., 1992], [Holthuijsen L. H., 2007].

Spectral density $S(\omega)$ may be used to generate an irregular wave. Such waves are closer to actual waves than those considered by many authors in their papers. The simplest example of an irregular wave is a sum of a given number N of harmonic components:

$$\xi_r(t) = \sum_{k=1}^{N} A_k \cos(\omega_k t + \varepsilon_k), \tag{3.24}$$

where A_k – random amplitude, calculated as $A_k = \sqrt{2 \cdot \Delta\omega \cdot S(\omega_k)}$,

$\varepsilon_k \in \langle 0,\dots,2\pi \rangle$ – random initial phase with uniform distribution,

$\Delta\omega_k = \omega_k - \omega_{k-1}$.

The dependency (3.24) allows us to determine the waves parameters according to the formulas (3.12) – (3.16) or those given in Table 3.2. Summation is performed for every calculated value. For example, the velocity from the formula (3.13) is determined thus:

$$v_x = \sum_{i=1}^{N} \omega_i \overline{\xi}_i \frac{\cosh(k_i(d+z))}{\sinh(k_i d)} \sin(\omega_i t - k_i x + \varepsilon_i). \tag{3.25}$$

The number of components N may reach a few hundred and more (guidelines are specified in appropriate norms). A pre-filled array of values needed, combined with interpolation over time, is therefore desirable when integrating the equations of a system's dynamics. Such approach ensures much shorter times taken by computations.

3.2 Determination of Forces Acting on Objects Immersed in Water

Let us consider a body immersed in a liquid. Action of the liquid on the body results from the motion of the body and of the liquid itself. Diffraction and radiation phenomena must be taken into consideration in the general case, as the body may influence the motion of the liquid. However, in performing analyses of structures with small characteristic dimensions relatively to the wave length, a simplifying assumption may be introduced that the Morison equation [Morison J. R., et al., 1950] governs the forces acting on a body immersed in water [Faltinsen O. M., 1990], [Sarpkaya T., Isaacson M., 1981]. Their determination requires the knowledge of coefficients whose values are yielded from appropriate laboratory experiments. In examples contained in the current book, interaction between water and pipelines is analyzed. The pipelines are modelled with beam elements. It can be easily proven that the Morison equation's applicability criterion, defined in [DNV-RP-C205, 2007] as:

$$\lambda > 5D, \tag{3.26}$$

is satisfied even for pipelines with large diameters (D greater than 1 m). This justifies the assumption of the Morison equation's applicability in dynamic analyses of pipelines throughout the rest of the volume. The force of a liquid's action on a body in general is defined as a function of multiple dimensionless parameters [Faltinsen O. M., 1990]:

$$f = f\left(t, T, KC, \mathrm{Re}, d_{fr}, \hat{e}, \hat{u}\right), \tag{3.27}$$

where $KC = \dfrac{u_0 T}{D}$ – Keugelan-Carpenter number,

$\mathrm{Re} = \dfrac{u_0 D}{\nu}$ – Reynolds number,

$d_{fr} = \dfrac{\pi D}{\lambda}$ – diffraction parameter,

$\hat{e} = \dfrac{K}{D}$ – reduced body surface roughness,

$\hat{u} = \dfrac{u_0}{f_s D}$ – reduced velocity of the liquid,

f_s – angular frequency of the body's oscillations,

u_0 – amplitude of the liquid's velocity,

ν – viscosity of the liquid,

K – surface roughness,

D – diameter (characteristic dimension) of the body.

The force of the liquid's action on a segment of a given body may be decomposed into tangent and normal components (Fig. 3.5b). For an asymmetric segment, turbulence and other effects, a lifting force F_L may additionally occur, which acts in the direction perpendicular to the liquid's flow (Fig. 3.5c).

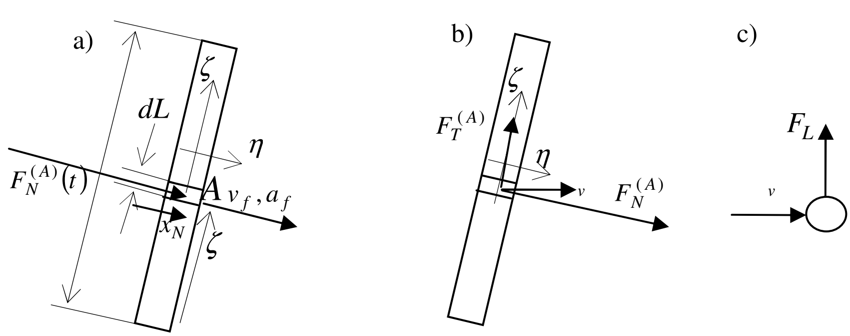

Fig. 3.5. Forces acting on a segment of a body: a) determination of the normal component, b) normal and tangential components, c) lifting force

The normal force acting on the segment determined by a coordinate ς (Fig. 3.5a) can be described with the Morison equation [DNV-RP-C205, 2007]:

$$F_N^{(A)}(t,\varsigma) = \rho A a_f + \rho C_A A a_r + \frac{1}{2}\rho C_D D v_r |v_r|, \tag{3.28}$$

where $F_N^{(A)}(t,\varsigma)$ – normal force acting on the segment of the body at the coordinate's value ς ,

ρ – density of the liquid,

$A = A(\varsigma)$ – area of the body's segment determined by coordinat ς ,

a_f – acceleration of the liquid,

$a_r = a_f - \ddot{x}_N$ – relative acceleration,

x_N – displacement of the body in the direction normal to the longitudinal axis,

$v_r = v_f - \dot{x}_N$ – velocity of the liquid relative to the body,

C_A – dimensionless coefficient of added mass,

C_D – dimensionless drag coefficient.

The Morison equation (3.28) may be used if the following relations hold [Chakrabarti S. K., 2005]:

$$KC > 6 \text{ and } d_{fr} << 0.5. \tag{3.29}$$

E.g., by substituting data of the largest diameters of pipelines, i.e. $D = 1$ m, with wave period $T = 10$ s and its amplitude $A = 1$ m, and taking into account the linear theory of waves (formula (3.13)), we obtain $KC = 6.28$, $d_{fr} = 0.056$. The condition (3.29) ought to be checked supplementing the general condition (3.26) due to the fact that the diffraction parameter determines the magnitude of wave dispersion as the result of meeting the object. When the number KC is small, the Morison equation should be replaced with calculations using e.g. potential theory or Froude-Krylov forces [Chakrabarti S. K., 2005]. The force given by (3.28) is particularly suitable for modelling structures such as ropes, pipes, beams. It may also yield satisfactory results for small 3D objects. On the other hand, it may not be successfully applied to large objects whose influence on the motion of water particles is significant enough to cause reflections of waves.

The net normal force and its application point x_h in the local coordinate system $0\varsigma\eta$ (Fig. 3.5) with origin in the middle of the item's length can be calculated from:

$$F_N = \int_{-0.5L}^{0.5L} F_N^{(A)}(t,\varsigma)\, d\varsigma, \tag{3.30}$$

$$x_h = \frac{\int_{-0.5L}^{0.5L} F_N^{(A)}(t,\varsigma)\cdot\varsigma\cdot d\varsigma}{F_N}, \tag{3.31}$$

where $F_N^{(A)}(t,\varsigma)$ – force described by (3.28),
L – length of the object.

Coefficients C_A and C_D which appear in the Morison equation are functions of dimensionless parameters from (3.27), whereby:

$$C_A = C_A(\mathrm{Re}, KC, \hat{e}), \tag{3.32}$$

$$C_D = C_D(\mathrm{Re}, KC, \hat{e}), \tag{3.33}$$

where $\mathrm{Re}, KC, \hat{e}$ are defined in (3.27).

Further details and practical advice about how these coefficients depend on different variables are to be found, among other things, in [Sarpkaya T., Isaacson M., 1981], [API-RP-2A-LRFD, 1993], [Bai Y., Bai Q., 2005], [Chakrabarti S. K., 2005], [DNV-RP-F105, 2006], [DNV-RP-C205, 2007]. These works also include cases of calculations for items placed very close to the free surface, the bottom or another large object which changes the coefficient of the added mass.

The tangent force caused by hydrodynamic resistance should be considered mainly in analyses of long objects with rough surfaces. It may be formulated thus [DNV-RP-C205, 2007]:

$$F_T = \frac{1}{2}\rho C_{Dt} v^2, \tag{3.34}$$

where C_{Dt} – coefficient of hydrodynamic resistance in the tangent direction,
v – amplitude of the liquid's net velocity.

The resistance coefficient C_{Dt} may be described with the formula [Eames M.C., 1968]:

$$C_{Dt} = C_D(m + n\sin(\alpha))\cos(\alpha), \tag{3.35}$$

where α – angle between net velocity and the item's longitudinal axis,
m, n – coefficients from Table 3.3.

Table 3.3. Values of the coefficients *m*, *n* according to [Eames M. C., 1968]

Item type	*m*	*n*
Smooth cylindrical surfaces	0.02-0.03	0.04-0.05
Porous cables, pipes	0.25-0.5	0.25-0.50
Six-strand ropes	0.03	0.06

Also the seabed exerts forces on elements of a pipeline as it is being laid. The problem of finding a description of the seabed and its interaction with installed objects was the subject of many works. Behaviour of the seabed is highly dependent on the sea considered. It is thus difficult to formulate one universally applicable model. Empirical models are most commonly used, which reproduce approximately the character of a given type of seabed. In the present volume, a model developed by Verley and Lund [Verley R., Lund K. M., 1995] will be used. It gives the following formula for the value of seabed penetration in the normal direction:

$$\frac{\Delta u_n}{D} = 0.0071\left(YZ^{0.3}\right)^{3.2} + 0.062\left(YZ^{0.3}\right)^{0.7}, \tag{3.36}$$

where $Y = \dfrac{N_C}{Dt_u}$,

$Z = \dfrac{t_u}{D\rho'}$,

Δu_n – seabed penetration in [m],

N_C – contact force per a pipeline's length link [kN/m],

t_u – seabed material's shear strength [kPa],

ρ' – density of seabed material (wet) [kN/m^3].

The dependency (3.36) holds for $YZ^{0.3} < 2.5$. Otherwise, the following formula gives better accuracy:

$$\frac{\Delta u_n}{D} = 0.09\left(YZ^{0.3}\right). \tag{3.37}$$

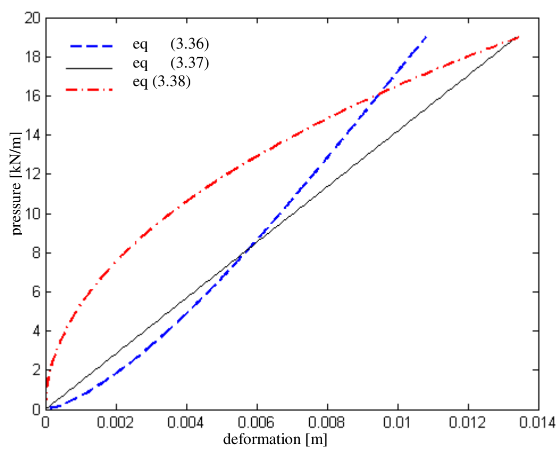

Fig. 3.6. Sample characteristics of sea beds according to Verley-Lund and bearing capacity models

Fig. 3.6 presents sample characteristics of sea beds described by (3.36) and (3.37) differentiated by bearing capacity model. In the last case, analogously to the model determining the bearing capacity, the following formulas are used [Bai Y., Bai Q., 2005]:

$$\frac{\Delta u_n}{D} = 0.5\left(1-\left(1-4\left(\frac{(b+0.5)f}{2(b+0.5f^3)}\right)^2\right)^{0.5}\right), \tag{3.38}$$

where $f = \dfrac{\alpha}{0.5(b+0.5)}$,

$\alpha = \dfrac{N_C}{2\rho' D^2}$,

$b = \dfrac{f_B t_u}{\rho' D}$,

f_b – coefficient (for large support lengths taking the value 5.14).

The graphs in Fig. 3.6 are plotted for the following parameters: $D = 0.3\,\mathrm{m}$, $t_u = 35\,\mathrm{kPa}$, $\rho' = 1.215\,\frac{\mathrm{g}}{\mathrm{cm}^3}$.

Forces caused by waves or sea currents which act on elements of pipes or other structures lying on a seabed result in their displacement in the forces' direction. The action of the seabed in the transverse (horizontal) direction needs therefore to be taken into consideration. More even so, given that the bodies in question often lie in sand or other material (e.g. clay). For objects penetrating the seabed's material only slightly, the Coulomb model of friction may be used [Bai Y., Bai Q., 2005], however, with soft materials or deeper penetration the description becomes more complex. Typical characteristics formulating the transversal forces as functions of displacement are nonlinear. Adding to this, the seabed in the contact area may become deformed and its material's consistence may vary due to displacement. Often a characteristic composed of two different linear parts is used. One of them describes the working conditions of the system up to the moment when a ground layer is shorn off. The other treats the forces in action when the object is moved along with seabed's material. This approach is presented, among other things, in the norm [DNV-RP-F105, 2006]. Guidelines concerning the selection of parameters and its applicability are also stated therein. The transversal force is calculated thus:

$$F_{lat} = \begin{cases} k_{l1}\Delta u_t & \text{for} \quad F_{lat} < \mu_t F_V \\ \mu_t F_V + k_{l2}\left(\Delta u_t - \dfrac{\mu_t F_V}{k_{l1}}\right) & \text{for} \quad \mu_t F_V \le F_{lat} < F_{lat,\max} \end{cases}, \tag{3.39}$$

where k_{l1} – equivalent transversal stiffness coefficient up to shearing off of the seabed's material,

k_{l2} – stiffness coefficient for motion developed in the transverse direction,

$\Delta\mu_t$ – transverse displacement,

F_V – vertical contact force per length of link,

μ_t – friction coefficient between the seabed and the body (transversal),

$F_{lat,\max} = F_{lat,\max}(\mu_t, F_V, D, t_u, \nu, \rho', G)$ – transverse force per length of link maximally transmissible by the seabed's material,

ν, G – Poisson number and shear modulus of the seabed's material.

There also exist other models of sea beds, including more complex ones. In [DNV-RP-F105, 2006] guidelines can be found on the use of different models and parameters of the seabed. The fact deserving a mention is that the models described above and used below omit in particular any changes of the seabed's properties due to the oscillatory impact of pipes and cables.

3.3 Methods of Simplified Description of Movement of Offshore Structures

The monograph [Adamiec-Wójcik I., et al., 2008] contains a short survey of papers related to the description of a vessel's motion under wave action. Those are most often fairly complex models used e.g. to determine strains to which the hull is exposed. However, they are hardly useful in quick dynamic analyses of offshore structures. Therefore, many works in the field of dynamics of such objects, particularly cranes, take the assumption that movements of the base can be described with relatively simple functions. Often it is further assumed that the vessel moves only in the vertical plane passing through the longitudinal symmetry axis of the deck. Such propositions seem reasonable for most offshore cranes that operate predominantly on vessels which are moored and properly positioned against the waves. Instead of making assumptions about the base's motion, some papers prefer to deal with forces acting on it.

It is common practice to assume that the motion of a vessel or any given point of a crane is harmonic or pseudo-harmonic. Sinusoidal waves with angular frequencies of 0.56 and 0.74 rad/s and height of 1 m directed along a ship's longitudinal axis are considered in [Das S. N., Das S. K., 2005]. In the papers [Balachandran B., et al., 1999] and [Li Y. Y., Balachandran B., 2001] two kinds of functions are used to describe the motion of the jib's head in a crane installed on a ship. These are:

- harmonic

$$x_e = (F \sin \omega t)\cos\psi \quad ; \quad y_e = (F \sin \omega t)\sin\psi\,, \tag{3.40}$$

- periodic

$$x_e = F\left(\sin\omega t + \frac{1}{4}\sin 2\omega t + \frac{1}{9}\sin 3\omega t\right)\cos\psi\,,$$
$$y_e = F\left(\sin\omega t + \frac{1}{4}\sin 2\omega t + \frac{1}{9}\sin 3\omega t\right)\sin\psi\,, \tag{3.41}$$

where F – excitation amplitude,
ω – angular frequency of excitation,
ψ – boom inclination angle.

The papers [Osiński M., Wojciech S., 1994], [Osiński M., Wojciech S., 1998] and [Osiński M., et al., 2004] focus on the planar problem of lifting a load from a ship's deck, whereby the motion of the deck is described with a harmonic function:

$$y_d = F\sin(\omega t + \beta)\,, \tag{3.42}$$

where β is the phase angle.

General motion of the base (3 displacements and 3 rotations) defined with pseudo-harmonic functions (reducing to harmonic when a single component is taken) can be considered for the model of a crane presented in [Maczyński A., 2005], [Maczyński A., Wojciech S., 2007]. Similarly for A-frames (harmonic functions) [Fałat P., 2004], [Adamiec-Wojcik I., et al., 2009] and BOP gantries for transportation [Urbaś A., et al., 2010], [Urbaś A., 2011].

In [Ellermann K., et al., 2002] and [Ellermann K., et al., 2003] two components are distinguished among the forces exerted by the waves: a periodically changing one and a constant one (related to drifting). These forces are determined from the following formulas:

$$\mathbf{F}_w(t) = \begin{bmatrix} a\,er_x\cos(\omega t) - a\,ei_x\sin(\omega t) + a^2\,p_{drag} \\ a\,er_\theta\cos(\omega t) - a\,ei_\theta\sin(\omega t) \\ a\,er_z\cos(\omega t) - a\,ei_z\sin(\omega t) \end{bmatrix}, \tag{3.43}$$

where a – wave amplitude,
er_j, ei_j – coefficients empirically determined for a particular type of ships, whereby $j \in \{x, \theta, z\}$,
ω – wave angular frequency,
x, θ, z – generalized coordinates of the hull,
p_{drag} – drift force determined empirically.

In the article [Cha J. H., et al., 2010a], hydrodynamic forces are proposed to be present among those acting on the crane's base: one due to radiation and another excited by the wave. The forces stemming from sea waves acting on the vessel where the crane is installed are determined based on the spectrum of the wave in [Witz J. A., 1995].

Sometimes, the motion of the base is described using measurements already performed in real conditions. In [Masoud Z. N., 2000] it is assumed that a ship with a crane on board oscillates transversally and longitudinally and is subjected to the motions of heaving, swaying and rolling. Calculations are based on data obtained empirically [Fossen T. I., 1994] which describe transverse and longitudinal oscillations as well as heaving, swaying and rolling of a selected point of the ship (the reference point). Also in [Driscoll F. R., et al., 2000], measured displacements of an A-frame are used to study a model of a cage suspended in large depths (1730 m). In the paper [Pedrazzi C., Barbieri G., 1998] the ADAMS package is used to analyse the dynamics of a vessel with a crane. The sea is modelled as a massless object which moves vertically relatively to the bottom. Its motion is defined in two ways: as a spline in time based on real measurements of sea waves and as an analytic function constructed using a pseudostochastic model of a wave.

In many of the models and computer programmes discussed herein, a provision is made for defining the general motion of an offshore structure's base as a pseudo-harmonic function with arbitrary number of components.