On the Asymptotic Stabilization of an Uncertain Bioprocess Model*

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Abstract. We study a nonlinear model of a biological digestion process, involving two microbial populations and two substrates and producing biogas (methane). A feedback control law for asymptotic stabilization of the closed-loop system is proposed. An extremum seeking algorithm is applied to stabilize the system towards the maximum methane flow rate.

1 Introduction

We consider a model of a continuously stirred tank bioreactor presented by the following nonlinear system of ordinary differential equations [4], [7], [8], [10], [11]:

$$\frac{ds_1}{dt} = u(s_1^i - s_1) - k_1\mu_1(s_1)x_1 \tag{1}$$

$$\frac{dx_1}{dt} = (\mu_1(s_1) - \alpha u)x_1 \tag{2}$$

$$\frac{ds_2}{dt} = u(s_2^i - s_2) + k_2\mu_1(s_1)x_1 - k_3\mu_2(s_2)x_2 \tag{3}$$

$$\frac{dx_2}{dt} = (\mu_2(s_2) - \alpha u)x_2 \tag{4}$$

with output

$$Q = k_4\mu_2(s_2)x_2. \tag{5}$$

The state variables s_1, s_2 and x_1, x_2 denote substrate and biomass concentrations, respectively: s_1 represents the organic substrate, characterized by its chemical oxygen demand (COD), s_2 denotes the volatile fatty acids (VFA), x_1 and x_2 are the acidogenic and methanogenic bacteria respectively. The parameter $\alpha \in (0, 1]$ represents the proportion of bacteria that are affected by the dilution.

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The constants k_1, k_2 and k_3 are yield coefficients related to COD degradation, VFA production and VFA consumption respectively; k_4 is a coefficient.

It is assumed that the input substrate concentrations s_1^i and s_2^i are constant and the methane flow rate Q is a measurable output. The dilution rate u is considered as a control input.

The functions $\mu_1(s_1)$ and $\mu_2(s_2)$ model the specific growth rates of the microorganisms. Following [11] we impose the following assumption on μ_1 and μ_2:

Assumption A1: $\mu_j(s_j)$ is defined for $s_j \in [0, +\infty)$, $\mu_1(s_1^i) \geq \mu_2(s_2^i)$, $\mu_j(0) = 0$, $\mu_j(s_j) > 0$ for $s_j > 0$; $\mu_j(s_j)$ is continuously differentiable and bounded for all $s_j \in [0, +\infty)$, $j = 1, 2$.

This model has been investigated in [10], [11], where a controller for regulating the effluent COD is proposed and its robustness is illustrated by a simulation study.

The main goal of the paper is to construct a feedback control law based on online measurements for asymptotic stabilization of the system (1)–(4). Then, by means of a numerical extremum seeking algorithm, the closed loop system is steered to that equilibrium point where the maximum methane output is achieved among all other equilibrium points.

2 Asymptotic Stabilization

Define $s^i := \dfrac{k_2}{k_1} s_1^i + s_2^i$ and let the following assumption be satisfied:

Assumption A2: Lower bounds s^{i-} and k_4^- for the values of s^i and k_4, as well as an upper bound k_3^+ for the value of k_3 are known.

Consider the control system (1)–(4) in the state space $p = (s_1, x_1, s_2, x_2)$ and define the following feedback control law:

$$k(p) := \beta\, k_4\, \mu_2(s_2)\, x_2 \quad \text{with} \quad \beta \in \left(\frac{k_3^+}{s^{i-} \cdot k_4^-},\ +\infty \right). \tag{6}$$

The feedback k depends only on β and Q, i. e. $k = k(\beta, Q) = \beta \cdot Q$.

Obviously, the number $\bar{s} := s^i - \dfrac{k_3}{\beta k_4}$ belongs to the interval $(0, s^i)$.

Denote by Σ the closed-loop system obtained from (1)–(4) by substituting the control variable u by the feedback $k(p)$.

Assumption A3. There exists a point $\bar{s}_1$ such that

$$\mu_1(\bar{s}_1) = \mu_2\left(\bar{s} - \frac{k_2}{k_1}\bar{s}_1\right), \quad \bar{s}_1 \in \left(0, s_1^i\right).$$

The above Assumption A3 is called in [7] regulability of the system.

Define

$$\bar{s}_2 = \bar{s} - \frac{k_2}{k_1}\bar{s}_1, \quad \bar{x}_1 = \frac{s_1^i - \bar{s}_1}{\alpha k_1}, \quad \bar{x}_2 = \frac{1}{\alpha\beta k_4}. \tag{7}$$

It is straightforward to see that the point

$$\bar{p} := (\bar{s}_1, \bar{s}_2, \bar{x}_1, \bar{x}_2)$$

is an equilibrium point for the system (1)–(4). We shall prove below that the feedback law (6) asymptotically stabilizes the closed-loop system Σ to $\bar{p}$.

Denote

$$s := \frac{k_2}{k_1} s_1 + s_2$$

and define the following sets

$$\begin{aligned}
\Omega_0 &= \{(s_1, x_1, s_2, x_2) |\ s_1 > 0,\ x_1 > 0,\ s_2 > 0,\ x_2 > 0\},\\
\Omega_1 &= \left\{(s_1, x_1, s_2, x_2) |\ s_1 + k_1 x_1 \le \frac{s_1^i}{\alpha},\ s + k_3 x_2 \le \frac{s^i}{\alpha}\right\},\\
\Omega_2 &= \left\{\left(s_1, x_1, \bar{s} - \frac{k_2}{k_1} s_1, \bar{x}_2\right) |\ 0 < s_1 < \frac{k_1}{k_2}\bar{s},\ x_1 > 0\right\},\\
\Omega &= \Omega_0 \cap \Omega_1.
\end{aligned}$$

Assumption A4. Let $\mu_1'(s_1) + \frac{k_2}{k_1}\mu_2'\left(\bar{s} - \frac{k_2}{k_1}s_1\right) > 0$ be satisfied on $\Omega \cap \Omega_2$.

Assumption A4 is technical and is used in the proof of the main result. It will be discussed in more details later in Section 4, where the growth rates μ_1 and μ_2 are specified as the Monod and the Haldane laws and numerical values for the model coefficient are introduced.

Theorem 1. *Let Assumptions* A1, A2, A3 *and* A4 *be satisfied. Let us fix an arbitrary number* $\beta \in \left(\frac{k_3^+}{s^{i-} \cdot k_4^-}, +\infty\right)$ *and let* $\bar{p} = (\bar{s}_1, \bar{x}_1, \bar{s}_2, \bar{x}_2)$ *be the corresponding equilibrium point. Then the feedback control law* $k(\cdot)$ *defined by (6) stabilizes asymptotically the control system (1)–(4) to the point* $\bar{p}$ *for each starting point* p_0 *from the set* Ω_0.

Proof. Let us fix an arbitrary point $p_0 \in \Omega_0$ and a positive value $u_0 > 0$ for the control. According to Lemma 1 from [7] there exists $T > 0$ such that the value of the corresponding trajectory of (1)–(4) for $t = T$ belongs to the set Ω. Hence the corresponding trajectory of (1)–(4) starting from the point p_0 enters the set Ω after a finite time. Moreover one can directly check that each trajectory staring from a point from the set Ω remains in Ω. For that reason we shall consider the control system (1)–(4) only on the set Ω.

Let us remind that by Σ we have denoted the closed-loop system obtained from (1)–(4) by substituting the control variable u by the feedback $k(p)$. Then one can directly check that the following ordinary differential equations

$$\begin{aligned}
\frac{ds}{dt} &= -\beta k_4 \mu_2(s_2) x_2 (s - \bar{s})\\
\frac{dx_2}{dt} &= -\alpha\beta k_4 \mu_2(s_2) x_2 (x_2 - \bar{x}_2)
\end{aligned} \tag{8}$$

are satisfied. Let us define the function

$$V(p) = (s - \bar{s})^2 + (x_2 - \bar{x}_2)^2.$$

Clearly, the values of this function are nonnegative. If we denote by $\dot{V}(p)$ the Lie derivative of V with respect to the right-hand side of (8) then for each point p of Ω,

$$\dot{V}(p) = -\beta k_4 \mu_2(s_2) x_2 (s - \bar{s})^2 - \alpha\beta k_4 \mu_2(s_2) x_2 (x_2 - \bar{x}_2)^2 \leq 0.$$

Applying LaSalle's invariance principle (cf. [9]), it follows that every solution of Σ starting from a point of Ω is defined on the interval $[0, +\infty)$ and approaches the largest invariant set with respect to Σ, which is contained in the set Ω_∞, where Ω_∞ is the closure of the set $\Omega \cap \Omega_2$. It can be directly checked that the dynamics of Σ on Ω_∞ is described by the system

$$\frac{ds_1}{dt} = \frac{1}{\alpha}\,\chi(s_1)(s_1^i - s_1) - k_1 \mu_1(s_1) x_1$$

$$\frac{dx_1}{dt} = (\mu_1(s_1) - \chi(s_1)) x_1,$$

where $\chi(s_1) := \mu_2\left(\bar{s} - \dfrac{k_2}{k_1} s_1\right)$ (remind that $\bar{s} := s^i - \dfrac{k_3}{\beta k_4}$). According to (7) we have that $\bar{s} = \dfrac{k_2}{k_1}\bar{s}_1 + \bar{s}_2$ and $s_1^i = \bar{s}_1 + \alpha k_1 \bar{x}_1$. Then the dynamics of Σ on the set Ω_∞ can be written as follows:

$$\frac{ds_1}{dt} = -\frac{1}{\alpha}\,\chi(s_1)\cdot(s_1 - \bar{s}_1 + \alpha k_1 (x_1 - \bar{x}_1)) - k_1\left(\mu_1(s_1) - \chi(s_1)\right)\cdot x_1$$
$$\frac{dx_1}{dt} = (\mu_1(s_1) - \chi(s_1))\cdot x_1.$$

Consider the function

$$W(s_1, x_1) = (s_1 - \bar{s}_1 + \alpha k_1 (x_1 - \bar{x}_1))^2 + \alpha(1-\alpha) k_1^2 (x_1 - \bar{x}_1)^2. \tag{9}$$

This function takes nonnegative values. It can be directly checked that for each point $\left(s_1, x_1, \bar{s} - \frac{k_2}{k_1} s_1, \bar{x}_2\right)$ of the set Ω_∞,

$$\begin{aligned}\dot{W}(s_1, x_1) = &-\frac{2}{\alpha}\chi(s_1)(s_1 - \bar{s}_1 + \alpha k_1 (x_1 - \bar{x}_1))^2 \\ &- 2(1-\alpha) k_1 x_1 (s_1 - \bar{s}_1)(\mu_1(s_1) - \chi(s_1)).\end{aligned}$$

Assumptions A3 and A4 imply

$$\begin{aligned}\mu_1(s_1) - \chi(s_1) &= \mu_1(s_1) - \mu_2\left(\bar{s} - \frac{k_2}{k_1} s_1\right) = \mu_1(s_1) - \mu_2\left(\bar{s}_2 - (s_1 - \bar{s}_1)\frac{k_2}{k_1}\right) \\ &= \mu_1(\bar{s}_1) + \int_{\bar{s}_1}^{s_1} \mu_1'(\theta)\, d\theta - \mu_2(\bar{s}_2) + \frac{k_2}{k_1}\int_{\bar{s}_1}^{s_1} \mu_2'\left(\bar{s}_2 - (\theta - \bar{s}_1)\frac{k_2}{k_1}\right) d\theta \\ &= \int_{\bar{s}_1}^{s_1} \left(\mu_1'(\theta) + \frac{k_2}{k_1}\mu_2'\left(\bar{s}_2 - (\theta - \bar{s}_1)\frac{k_2}{k_1}\right)\right) d\theta,\end{aligned}$$

and therefore

$$\dot{W}(s_1, x_1) \le 0 \tag{10}$$

for each point $\left(s_1, x_1, \bar{s} - \frac{k_2}{k_1}s_1, \bar{x}_2\right)$ from the set Ω_∞.

To complete the proof we use a refinement of LaSalle's invariance principle recently obtained in [2] (cf. also [6] and [12], where similar stabilizability problems are studied). Denote by $\phi(t,p)$ the value of the trajectory of the closed-loop system Σ at time t starting from the point $p \in \Omega$. The positive limit set (or ω-limit set) of the solution $\phi(t,p)$ of the closed-loop system Σ is defined as

$$L^+(p) = \{\tilde{p} : \text{ there exists a sequence } \{t_n\} \to \infty \ \text{ with } \tilde{p} = \lim_{t_n \to +\infty} \phi(t_n, p)\}\,.$$

Let us fix an arbitrary point $p_0 \in \Omega$. The invariance of the set Ω with respect to the trajectories of Σ and the LaSalle invariance principle imply that the ω-limit set $L^+(p_0)$ is a nonempty connected invariant subset of Ω_∞.

Now consider again the function $W(\cdot,\cdot)$ defined by (9). The restriction of the Lie derivative $W(\cdot,\cdot)$ on Ω_∞ is semidefinite, meaning $\dot{W}(s_1,x_1) \le 0$ for each point $(s_1, x_1, \bar{s} - \frac{k_2}{k_1}s_1, \bar{x}_2) \in \Omega_\infty$. The proof of Theorem 6 from [2] implies that $L^+(p_0)$ is contained in one connected component of the set $L^\infty := \{(s_1, x_1, \bar{s} - \frac{k_2}{k_1}s_1, \bar{x}_2) \in \Omega_\infty : \dot{W}(s_1,x_1) = 0\}$. Taking into account (10) and Assumption A1, one can obtain that $L^\infty = \{\bar{p}\}$, and hence $L^+(p_0) = \{\bar{p}\}$. Moreover, one can verify that $\bar{p}$ is a Lyapunov stable equilibrium point for the closed-loop system Σ. This completes the proof.

3 Extremum Seeking

Let the assumptions A1, A2, A3 and A4 hold true. Denote by $\beta \in \left(\dfrac{k_3^+}{s^{i-} \cdot k_4^-}, +\infty\right)$ some constant. Consider $\bar{p}_\beta = (\bar{s}_1, \bar{x}_1, \bar{s}_2, \bar{x}_2)$ where $\bar{s}_1$, $\bar{x}_1$, $\bar{s}_2$ and $\bar{x}_2$ are computed according to (7). Assume further that the static characteristic

$$Q(\bar{p}_\beta) = k_4\, \mu_2(\bar{s}_2)\, \bar{x}_2,$$

which is defined on the set of all steady states $\bar{p}_\beta$ has a maximum at a unique steady state point

$$p^{\max}_{\beta_*} = (s_1^*,\ x_1^*,\ s_2^*,\ x_2^*),$$

that is $Q_{\max} := Q(p^{\max}_{\beta_*})$.

Our goal now is to stabilize the dynamic system towards the (unknown) maximum methane flow rate $Q_{\max}$. We apply the feedback control law

$$(Q,\beta) \longmapsto k(Q,\beta) = \beta \cdot Q. \tag{11}$$

According to Theorem 1, this feedback will asymptotically stabilize the control system (1)–(4) to the point $\bar{p}_\beta$.

To stabilize the dynamics (1)–(4) towards $Q_{\max}$ by means of the feedback (11), we use an iterative extremum seeking algorithm. This algorithm is presented in details in [5] and applied to a two-dimensional bioreactor model with adaptive feedback. The algorithm can easily be adapted for the model considered here. The main idea of the algorithm is based on the fact that Theorem 1 is valid for any value of $\beta > \dfrac{k_3^+}{s^{i-} \cdot k_4^-}$. Thus we can construct a sequence of points $\beta^1, \beta^2, \ldots, \beta^n, \ldots$, converging to β_*, and generate in a proper way a sequence of values $Q^1, Q^2, \ldots, Q^n, \ldots$ which converges to $Q_{\max}$. The algorithm is carried out in two stages: on Stage I, an interval $[\beta] = [\beta^-, \beta^+]$ is found such that $[\beta] > \dfrac{k_3^+}{s^{i-} \cdot k_4^-}$ and $\beta_* \in [\beta]$; on Stage II, the interval $[\beta]$ is refined using an elimination procedure based on a Fibonacci search technique. Stage II produces the final interval $[\bar\beta] = [\bar\beta^-, \bar\beta^+]$ such that $\beta_* \in [\bar\beta]$ and $\bar\beta^+ - \bar\beta^- \le \varepsilon$, where the tolerance $\varepsilon > 0$ is specified by the user.

4 Numerical Simulation

In the computer simulation, we consider for $\mu_1(s_1)$ and $\mu_2(s_2)$ the Monod and the Haldane model functions for the specific growth rates, which are used in the original model [1], [3], [4], [7], [8]:

$$\mu_1(s_1) = \frac{\mu_m s_1}{k_{s_1} + s_1}, \qquad \mu_2(s_2) = \frac{\mu_0 s_2}{k_{s_2} + s_2 + \left(\dfrac{s_2}{k_I}\right)^2}. \tag{12}$$

Here μ_m, k_{s_1}, μ_0, k_{s_2} and k_I are kinetic coefficients. Obviously, $\mu_1(s_1)$ and $\mu_2(s_2)$ satisfy Assumption A1: $\mu_1(s_1)$ is monotone increasing and bounded by μ_m; there is a point $\tilde s_2$ such that $\mu_2(s_2)$ achieves its maximum at $\tilde s_2 = k_I\sqrt{k_{s_2}}$. Simple derivative calculations imply that if $\bar s$ is chosen such that $0 < \bar s \le \tilde s_2$ then $\mu_2'\left(\bar s - \frac{k_2}{k_1} s_1\right) \ge 0$ holds true thus Assumption A4 is satisfied. Moreover, if the point $\bar s$ is sufficiently small, then Assumptions A3 and A4 are simultaneously satisfied.

Usually the formulation of the growth rates is based on experimental results, and therefore it is not possible to have an exact analytic form of these functions, but only some quantitative bounds. Assume that we know bounds for $\mu_1(s_1)$ and $\mu_2(s_2)$, i. e.

$$\mu_j(s_j) \in [\mu_j(s_j)] = [\mu_j^-(s_j), \mu_j^+(s_j)] \text{ for all } s_j \ge 0, \quad j = 1, 2.$$

This uncertainty can be simulated by assuming in (12) that instead of exact values for the coefficients μ_m, k_{s_1}, μ_0, k_{s_2} and k_I we have compact intervals for them: $\mu_m \in [\mu_m]$, $k_{s_1} \in [k_{s_1}]$, $\mu_0 \in [\mu_0]$, $k_{s_2} \in [k_{s_2}]$, $k_I \in [k_I]$. Then any $\mu_j(s_j) \in [\mu_j(s_j)]$, $j = 1, 2$, satisfies Assumption 1; it follows also that there exist intervals for the kinetic coefficients, such that Assumption 4 is satisfied for any $\mu_j(s_j) \in [\mu_j(s_j)]$, $j = 1, 2$. Such intervals are for example the following:

$$[\mu_m] = [1.2,\ 1.4], \quad [k_{s_1}] = [6.5,\ 7.2],$$
$$[\mu_0] = [0.64,\ 0.84], \quad [k_{s_2}] = [9,\ 10.28], \quad [k_I] = [15,\ 17].$$

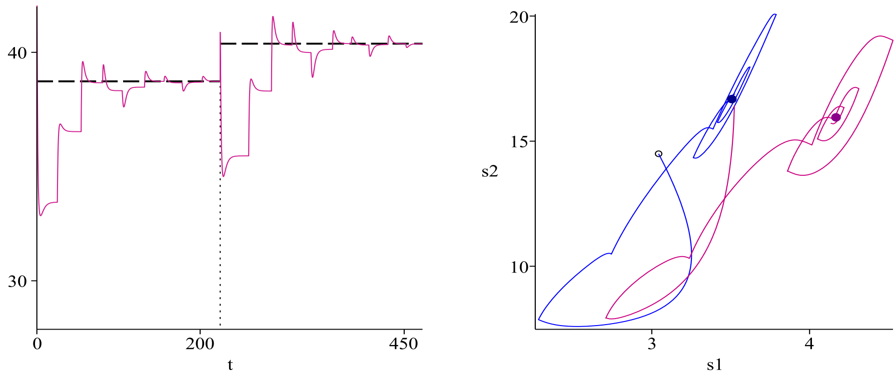

Fig. 1. Time evolution of $Q(t)$ (left) and a trajectory in the (s_1, s_2)-phase plane (right)

To simulate Assumption A2 we assume intervals for the coefficients k_j to be given, i. e. $k_j \in [k_j] = [k_i^-, k_j^+]$, $j = 1, 2, 3, 4$. Such numerical intervals are

$$[k_1] = [9.5, 11.5], \quad [k_2] = [27.6, 29.6], \quad [k_3] = [1064, 1084], \quad [k_4] = [650, 700].$$

All above intervals are chosen to enclose the numerical coefficients values derived by experimental measurements [1]; the values $\alpha = 0.5$, $s_1^i = 7.5$, $s_2^i = 75$ are also taken from [1]. In this case the inequality $\mu_1^-(s_1^i) > \mu_2^+(s_2^i)$ is fulfilled, which means that $\mu_1(s_1^i) > \mu_2(s_2^i)$ for any $\mu_j \in [\mu_j]$, $j = 1, 2$, thus Assumption A1 is satisfied.

In the simulation process we proceed in the following way. At the initial time $t_0 = 0$ we take random values for the coefficients from the corresponding intervals. We apply the extremum seeking algorithm to stabilize the system towards $Q_{\max}$. Then, at some time $t_1 > t_0$, we choose another set of random coefficient values and repeat the process; thereby the last computed values for the phase variables (s_1, x_1, s_2, x_2) are taken as initial conditions.

The left plot in Figure 1 shows the time profile of $Q(t)$; there the vertical dot-line segment marks the time moment t_1, when the new coefficients values are taken in a random way from the corresponding intervals. The horizontal dash-line segments go through $Q_{\max}$. The “jumps” in the graph correspond to the different choices of β by executing the algorithmic steps. The right plot in Figure 1 shows a projection of the trajectory in the phase plane (s_1, s_2); the empty circle denotes the initial point $(s_1(0), s_2(0))$. It is well seen how the trajectory consecutively approaches the two steady states (solid circles), corresponding to the different choice of the coefficient values.

5 Conclusion

The paper is devoted to the stabilization of a four-dimensional nonlinear dynamic system, which models anaerobic degradation of organic wastes and produces methane. A nonlinear feedback law is proposed, which stabilizes asymptotically the dynamics towards the (unknown) maximum methane rate $Q_{\max}$. The feedback depends on a parameter β, which varies in known bounds. First it is shown that for any chosen value of β, the system is asymptotically stabilized to an equilibrium point $\bar{p}_\beta = (\bar{s}_1, \bar{x}_1, \bar{s}_2, \bar{x}_2)$. Further, an iterative numerical extremum seeking algorithm is applied to deliver bounds $[\beta_m]$ and $[Q^{\max}]$ for the parameter β and for the methane flow rate Q, such that for each $\beta \in [\beta_m]$, the corresponding equilibrium point $\bar{p}_\beta$ is such that $Q(\bar{p}_\beta) \in [Q_{\max}]$. The interval $[Q_{\max}]$ can be made as tight as desired depending on a user specified tolerance $\varepsilon > 0$. The theoretical results are illustrated numerically.

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