Analytical Foundation for Energy Efficiency Optimisation in Cellular Networks with Elastic Traffic

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Abstract. Lately, energy efficiency (EE) in mobile communications is receiving growing attention as its increasing energy consumption raises concern over climate effects. As a stepping stone to improve the situation, we provide some analytical tools for optimising the EE of a base station through power control, focusing on elastic traffic in the downlink scenario. Under certain assumptions, the problem is formulated such that any rate maximisation algorithm can be incorporated to achieve the optimum EE. Furthermore, when formulated as a function of one sum power variable, optimality is illustrated graphically and Pareto optimality is discussed. Finally, an EE function for a base station in a cellular network is proposed that captures essential factors of power consumption.

Keywords: energy efficiency, optimisation, cellular network, mobile communications.

1 Introduction

Following the increasing popularity of mobile cellular communications worldwide, its rising energy consumption has started to draw the attention of both regulatory bodies and network operators. It is not only a concern due to operational costs, but also due to future energy availability and environmental preservation. Currently, about 0.2% of the global CO_2 emissions are contributed by the mobile telecommunication industry [1]. Energy efficiency (EE) will become an inevitable concern in network design and architecture. Due to restrictions on power supply on mobile units, their EE is highly optimised to provide customer satisfaction. However, the power consumption of base stations (BS), which constitutes the major portion of a cellular network [2,3], has been neglected until lately [4]. Without considering users' phones, the power consumption of a typical mobile communications network is estimated to be about 40 MW [5].

One of the first steps for enhancing EE is to adequately model the communication system and measure its EE. The accurate evaluation of the EE requires the operational power of the entire access network at different levels to be taken into account [6]. However, its prohibitive complexity makes idealisation unavoidable. There is much research done in EE optimisation for specific system models.

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Algorithms to achieve EE in OFDM systems and frequency-selective channels were presented in [7] and [8], respectively. Game theory was applied in [9] to analyse the EE of CDMA systems. In [10], the technique of switching between SIMO and MIMO to achieve EE in the uplink was investigated.

Our contribution is a framework that provides solutions to a general system model. In this work, we provide some general analytical tools and insights into optimising the EE at the BS in a cell of a network through power control, especially with downlink data transfer at the link and network level in mind. Nevertheless, the general results presented here can be adapted to include the uplink as well. Furthermore, we explore the case of elastic traffic, where the delay constraints are relaxed. This applies especially to data transfer through the Internet, which is increasingly being accessed through mobile devices [11].

In Section 2 we discuss the EE metric in a general sense. We also show that if the data rate function is a concave function of the power, we may incorporate any rate maximisation algorithm into the programme to find the optimum EE. This is formulated as a problem of a single total power variable, from which we obtain some useful insights concerning Pareto optimality w.r.t. EE and throughput. In Section 3 we present an EE function, which the results in the previous section can be applied to. This function captures essential factors for elastic data transmission while remaining general. Section 4 concludes the paper.

2 Energy Efficiency

2.1 Energy Efficiency Metric

In a general sense, efficiency can be seen as the ratio of goods produced to the resources consumed. Here, we focus on the EE in the physical and medium access control layers where goods are effective data transmitted measured in the information unit (bits or nats[1]) and the resources are the total energy (Joule) consumed for transmitting data. For static power configuration, EE can be expressed as the ratio of the sum rate r to the total expended power P_s that achieves this rate[2]:

$$EE = \frac{r(\boldsymbol{p})}{P_s\left(\boldsymbol{p}\right)} \quad \left[\frac{\text{nat}}{\text{Joule}}\right]. \tag{1}$$

We characterise a *rate function* as being non-negative and $r\left(\mathbf{0}\right) = 0$. The power function P_s can be modelled as an affine function consisting of two terms, namely the constant power scalar p_C and the total input transmission power scalar $\sum_i^n p_i$, where p_i are the components of the non-negative vector $\boldsymbol{p} \in \mathbb{R}_+^n$ that represent input transmission powers e.g. in different subcarriers (as in [7]) or to different users. This constant or *base power* can be understood as the requirement for enabling transmission and is spent independently of the transmission power, which is justified considering the power consumption model of BSs [2]. It includes baseband processing, transceiver circuits, and other auxiliary components like

[1] A more convenient unit for analytical calculations.

[2] Unless otherwise indicated, *rate* refers to the *sum rate* hereafter.

climate control (see [12,13] for more details). This constant may vary depending on factors like the number of antennas, the traffic load and the computational efficiency. In Section 3 we show that these variations can be taken into account by considering them as separate constants in different traffic conditions.

If the powers are functions of time, e.g. adapted according to time-varying parameters (e.g. the channel coefficients), EE can be expressed as

$$EE = \frac{\int_0^T r(\boldsymbol{p}(t))\,dt}{\int_0^T P_s(\boldsymbol{p}(t))\,dt} = \frac{\mathbb{E}_T\left[r(\boldsymbol{p}(t))\right]}{\mathbb{E}_T\left[P_s(\boldsymbol{p}(t))\right]} \tag{2}$$

which is the ratio of the total amount of information units transmitted during a period T, $T\mathbb{E}_T\left[r(\boldsymbol{p}(t))\right]$, to the total energy expended $T\mathbb{E}_T\left[P_s(\boldsymbol{p}(t))\right]$, where $\mathbb{E}_T\left[X(t)\right] = \int_0^T X(t)\,dt$ is the mean value of X averaged over the period T, and $\boldsymbol{p}(t)$ is a vector function of time t. Under the assumption of ergodicity and stationarity (of the channel distribution), $\mathbb{E}_\infty\left[x\right] = \lim_{T\to\infty} \frac{1}{T}\int_0^T x(t)\,dt$ is the expected value of x averaged over time.

2.2 Optimal Energy Efficiency

Our aim is to maximise (1) over $\boldsymbol{p}$ or (2) over $\boldsymbol{p}(t)$. Due to the structure of (1) and (2), this belongs to a class of nonconvex problems called fractional programming [14,15]. Depending on the objective function EE, we can determine certain properties with regard to optimality. If it is a semistrictly quasiconcave function[3] of the transmit power vector, any local maximum is also the global maximum [16]. This reduces the problem to finding any local maximum. Additionally, if the objective function is pseudoconcave[4], any stationary point ($\nabla EE = 0$) is the local as well as the global optimum. The Karush-Kuhn-Tucker (KKT) conditions are then both necessary and sufficient for optimality, which does not necessarily apply to semistrictly quasiconcave functions.

For a function $f(x)/g(x)$, it is shown in [14] that if the numerator is concave and the denominator convex, then the function is semistrictly quasiconcave. If both the numerator and the denominator are differentiable in their domain, the function is pseudoconcave, implying that the system of equations given by the KKT conditions yields the global maximum. If in addition either the numerator or denominator is strictly concave or strictly convex, respectively, the function is strictly pseudoconcave, which implies that the global maximum, if it exists, is unique. Clearly, since the sum power is an affine function of its power components, the denominator fulfils the condition of being convex in $\boldsymbol{p}$. If the rate function $r(\boldsymbol{p})$ is concave, we have a semistrictly quasiconcave EE function. If it is differentiable, it is even pseudoconcave. Further, note that the positively

[3] A function $f : X \subset \mathbb{R}^n \to \mathbb{R}$ is *semistrictly quasiconcave* on X if $f(\lambda x_1 + (1-\lambda)x_2) > \min\{f(x_1), f(x_2)\}$ for all $x_1, x_2 \in X$, $f(x_1) \neq f(x_2)$, $\lambda \in (0,1)$ [16].

[4] A differentiable function $f : X \to \mathbb{R}$, where $X \subset \mathbb{R}^n$ is an open set, is *pseudoconcave* on X if $f(x_1) > f(x_2) \Rightarrow (x_1 - x_2)\nabla f(x_2) > 0$ for all $x_1, x_2 \in X$ [16].

weighted sum of (strictly) concave functions is (strictly) concave. This can be extended to integrals, i.e. the integral of a positively weighted (strictly) concave function is (strictly) concave [17].

For the case of static power configuration, we can formulate the programming problem as follows

$$\max_{\boldsymbol{p}}. \ \tfrac{r(\boldsymbol{p})}{P_s(\boldsymbol{p})}. \tag{3}$$

If $r(\boldsymbol{p})$ is concave and differentiable, finding any feasible solution that satisfies the KKT conditions will yield the global optimum. Assume a convex constraint set of sum and individual power, and the optimal point $\boldsymbol{p}^*$ is found in it. Thus, we obtain the stationarity condition by setting the first derivative of (3) to zero, which after rearrangement yields

$$\frac{\frac{\partial}{\partial p_i} r(\boldsymbol{p})\Big|_{\boldsymbol{p}=\boldsymbol{p}^*}}{\frac{\partial}{\partial p_i} P_s(\boldsymbol{p})\Big|_{\boldsymbol{p}=\boldsymbol{p}^*}} = \frac{r(\boldsymbol{p}^*)}{P_s(\boldsymbol{p}^*)} = EE^*(\boldsymbol{p}^*) \tag{4}$$

for all $i = 1, ..., n$. The denominator on the left is one, if P_s is an unweighted sum of powers, $p_C + \sum p_i$. This means that at the vector $\boldsymbol{p}^*$ where the gradient of the rate equals its corresponding EE, we have the global optimum. If the optimal point is not within the constraint set, Lagrangian multipliers are utilised to obtain the solution.

In the case of time-varying powers, we obtain a similar result. Consider a case where the powers are adapted according to the channel coefficients $\boldsymbol{\alpha}$, which vary with time. We assume that the system has perfect channel information. The programming problem is finding the functional $\boldsymbol{p}(\boldsymbol{\alpha}) = (p_1(\boldsymbol{\alpha}), p_2(\boldsymbol{\alpha}), ..., p_n(\boldsymbol{\alpha}))$ that maximises the EE as follows:

$$\max_{\{\forall i, p_i(\boldsymbol{\alpha})\geq 0\}}. \frac{\int_H r(\boldsymbol{p}(\boldsymbol{\alpha}))f(\boldsymbol{\alpha})\ d\boldsymbol{\alpha}}{\int_H P_s(\boldsymbol{p}(\boldsymbol{\alpha}))f(\boldsymbol{\alpha})\ d\boldsymbol{\alpha}}, \tag{5}$$

where $f(\boldsymbol{\alpha})$ is the probability density function (PDF) of $\boldsymbol{\alpha}$, and H is the set of all values $\boldsymbol{\alpha}$ can have. We assume that P_s is an affine function of $p_1(\boldsymbol{\alpha}), p_2(\boldsymbol{\alpha}), ...,$ $p_n(\boldsymbol{\alpha})$. Consider the case when the function r is concave and differentiable. Then by treating each $p_i(\boldsymbol{\alpha})$ as an infinitesimal vector where $p_i(\hat{\boldsymbol{\alpha}})$ for a particular value $\hat{\boldsymbol{\alpha}}$ and i is a component of it, we derive the objective function with respect to each component and obtain the following stationarity condition:

$$\frac{\frac{\partial}{\partial p_i(\hat{\boldsymbol{\alpha}})} r(\boldsymbol{p}(\boldsymbol{\alpha}))\Big|_{\boldsymbol{p}(\boldsymbol{\alpha})=\boldsymbol{p}^*(\boldsymbol{\alpha})}}{\frac{\partial}{\partial p_i(\hat{\boldsymbol{\alpha}})} P_s(\boldsymbol{p}(\boldsymbol{\alpha}))\Big|_{\boldsymbol{p}(\boldsymbol{\alpha})=\boldsymbol{p}^*(\boldsymbol{\alpha})}} = \frac{\int_H r(\boldsymbol{p}^*(\boldsymbol{\alpha}))f(\boldsymbol{\alpha})\ d\boldsymbol{\alpha}}{\int_H P_s(\boldsymbol{p}^*(\boldsymbol{\alpha}))f(\boldsymbol{\alpha})\ d\boldsymbol{\alpha}}, \tag{6}$$

for all is and all $\hat{\boldsymbol{\alpha}} \in H$, where $\boldsymbol{p}^*(\boldsymbol{\alpha})$ is the functional that maximises the problem above. Again, the denominator on the left is unity, if P_s is an unweighted sum of powers, $p_C + \sum p_i(\boldsymbol{\alpha})$. This means that when the differential increment of the rate function w.r.t. every component of $\boldsymbol{p}(\boldsymbol{\alpha})$ for all $\hat{\boldsymbol{\alpha}}$ is equal to its corresponding EE, we obtain the optimal function $\boldsymbol{p}(\boldsymbol{\alpha})$. This is treated in depth in [18].

Of course perfect channel information is hardly available in real systems. Moreover the energy needed for the information feedback also needs to be accounted for, so that the true EE may be reflected. However, the solution to this problem provides us with theoretical knowledge of the upper bound of EE.

2.3 Equivalent Problems

Although we will focus on the EE maximisation problem shown in (3), we mention the variety of problem formulations that yield the same solution. For example, the problem can be formulated as the minimisation of the inverse of EE over power, $\frac{1}{EE} = P_s(\boldsymbol{p})/R(\boldsymbol{p})$. Since EE is non-negative, the solution $\boldsymbol{p}^*$ to $\max . EE(\boldsymbol{p})$ is identical to that of $\min . \frac{1}{EE(\boldsymbol{p})}$. The optimal value of one problem is just the reciprocal of the other. This is evident from the fact that if $EE(\boldsymbol{p}^*) \geq EE(\boldsymbol{p})$ for any $\boldsymbol{p}$ in the constraint set, simply inverting it yields $\frac{1}{EE(\boldsymbol{p})} \geq \frac{1}{EE(\boldsymbol{p}^*)}$.

In [8], the cost function, energy consumption per bit, corresponding to the dissipated power divided by the throughput is formulated as a function of rates $\boldsymbol{r}$ in subcarrier channels $\frac{1}{EE(\boldsymbol{r})} = E_a = P_s(\boldsymbol{r})/R(\boldsymbol{r})$ where $R(\boldsymbol{r}) = \sum r_i$ here is the sum of all rates produced in the subcarriers. The cost function is to be minimised over $\boldsymbol{r}$. This formulation is only possible if the transmit power can be written as a function of the individual rate, $p_i(r_i)$. In this case we have a bijective power function where each power allocation corresponds to a unique rate distribution (e.g. across the subcarriers). Because of the bijective property we know that the solution to the minimisation problem is obtained when the rate distribution yields the optimal power allocation solved by $\min . \frac{1}{EE(\boldsymbol{p})}$. Therefore, this optimisation problem is equivalent to the previous one.

2.4 Energy Efficiency Optimisation Using Rate Maximisation

The EE optimisation problem can be nested into inner and outer optimisation problems, namely to maximise the rate given the sum transmit power P and then to maximise this over the single variable P in the EE function. This allows the utilisation of any rate maximising algorithm for EE optimisation. Let us formulate the programming problem as follows:

$$\max . EE(P) = \max_P . \frac{\left(\max\limits_{\sum_i p_i = P} . R(\boldsymbol{p}) \right)}{p_C + P} \tag{7}$$

$$= \max_P . \frac{R_{\max}(P)}{p_C + P}. \tag{8}$$

Based on the discussion in Section 2.2, a semistrictly quasiconcave objective function ensures that the local maximum is also the global maximum, which can then be found efficiently, e.g. using the bisection method or interior-point methods. This requires $R_{\max}$ to be concave in P. Using Corollary 1, which is attained through Lemma 5, we conclude that the objective function of (7) is semistrictly quasiconcave in P, if $R(\boldsymbol{p})$ is concave in $\boldsymbol{p}$. It is also pseudoconcave if $R_{\max}$ remains differentiable after the maximisation operation.

Corollary 1. *If $R(\boldsymbol{p})$ is concave in $\boldsymbol{p}$, $R_{\max}(P) = \max\left\{R(\boldsymbol{p}) : \mathbf{1}^{\boldsymbol{T}}\boldsymbol{p} = P\right\}$ is concave in P.*

Note that this result is applicable if we additionally impose individual power constraints, provided that the set $\left\{\boldsymbol{p} : \mathbf{1}^{\boldsymbol{T}}\boldsymbol{p} = P,\, 0 \leq p_i \leq p_{i,max}\, \forall i\right\}$, which is convex, is feasible.

2.5 Visualisation of Optimal EE Involving One Power Variable

We showed how the problem can be presented as having only one power variable, making it easier to handle. In this section we illustrate some properties of optimising EE with one power variable. Let us look at a simple example to obtain some insight. Consider

$$EE(P) = \frac{R(P)}{p_C + P} = \frac{B\log\left(1 + \frac{\alpha}{B}P\right)}{p_C + P}, \tag{9}$$

where the rate $R(P)$ is only dependent on a single power variable P and is modelled here as a logarithmic function with parameter α that represents the channel gain, and B the bandwidth. Since $B\log\left(1 + \frac{\alpha}{B}P\right)$ is a strictly concave function in P, (9) is a strictly pseudoconcave function. Using (4) the maximum EE is found where

$$\left.\frac{dR(P)}{dP}\right|_{P=P^*} = \frac{B\log\left(1 + \frac{\alpha}{B}P^*\right)}{p_C + P^*} \tag{10}$$

is fulfilled, P^* being the optimal point. This can be solved with the help of the Lambert-W function[5] $W(\cdot)$ such that

$$P^* = \frac{1}{\alpha}\left[\frac{\alpha p_C - B}{W\left(\left(\alpha p_C - B\right)/B\exp(1)\right)} - B\right]. \tag{11}$$

This example is sketched in Fig. 1. Using this figure we illustrate that the solution can also be obtained graphically for any concave rate function $R(P)$. The figure shows the rate as a function of the total power $P_s = p_C + P$. The abscissa shows the magnitude of the total expended power whereas the ordinate shows the corresponding rate. p_C is the minimum power required for data transmission. The EE at any point of P_s is given as $EE = \tan\theta(P_s) = \frac{R(P)}{P_s}$. Since $R(P)$ is

[5] i.e. the inverse function of $x = W\exp(W)$ [19].

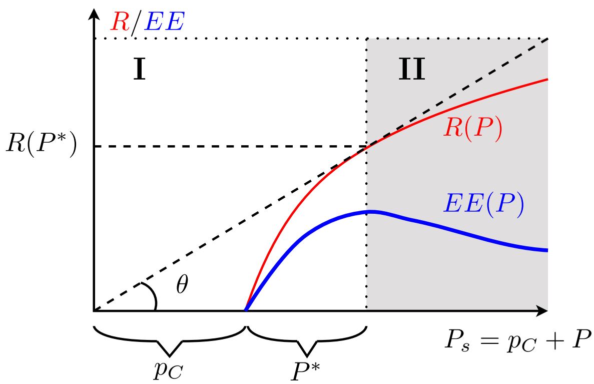

Fig. 1. Visualisation of the optimal EE. The optimal EE is obtained where the transmission power is P^*.

concave, we know that the optimal point is where (10) is fulfilled. Rearranging (10), we obtain

$$\left(p_C + P^*\right) \left.\frac{dR\left(P\right)}{dP}\right|_{P=P^*} = R\left(P^*\right), \tag{12}$$

which can be interpreted as follows. The linear function (l.h.s.) which has the slope identical to the slope of $R\left(P\right)$ at point P has to intersect the rate function (r.h.s.) in order that the optimality criterion is fulfilled. This means that the tangent of the rate function at P^* has to form a line that passes the origin of P_s. This is shown as the dashed line in Fig. 1. This is similar to the approach in [20], where the global maximiser is identified graphically for resource management problems.

Generally, maximising EE is equivalent to finding the point P along $R\left(P\right)$ that maximises θ, even when $R\left(P\right)$ is not concave, e.g. when cross-channel interference exists.

Lemma 2. *Given a concave rate function $R\left(P\right)$, the optimal point P^* of $EE = \frac{R(P)}{p_C+P}$ increases with p_C whereas the optimal value $EE^* = \frac{R(P^*)}{p_C+P^*}$ decreases with p_C.*

Proof. Intuitively, this can be observed graphically in Fig. 1. As p_C is increased, the point of contact P^* of the tangent line with the rate function increases and the angle θ decreases. To show it analytically, consider the following. It is evident that for any given P, EE decreases with an increasing p_C. This implies that the optimal value of EE also decreases with p_C. Recall the stationarity condition

$$\left.\frac{dR\left(P\right)}{dP}\right|_{P=P^*} = \frac{R\left(P^*\right)}{p_C + P^*} = EE^*.$$

Since $R\left(P\right)$ is a concave function, its derivative is a non-increasing function of P. It follows that a lower optimal EE^* would intersect with $\frac{dR}{dP}$ at a higher P^*. □

Corollary 3. *If* $p_C = 0$*, the highest possible* $EE^* = \left.\frac{dR(P)}{dP}\right|_{P=P^*}$ *is achieved, where* $P^* = 0$*.* $\left.\frac{dR(P)}{dP}\right|_{P=0}$ can be used as an upper bound for the EE^*.

We say that R_1 dominates R_2 if $R_1(P) > R_2(P)$ for all $P > 0$, where R_1 and R_2 are two different rate functions. It is easy to see that if R_1 dominates R_2, their corresponding EE also follow the same order, such that $EE_1 = \frac{R_1(P)}{p_C+P} > \frac{R_2(P)}{p_C+P} = EE_2$ for all P. The same applies to the optimum EE.

Corollary 4. *If* R_1 *dominates* R_2*,* $EE_1^* > EE_2^*$*.*

2.6 Pareto Optimality. Trade-Off between Rate and Energy Efficiency.

It is beneficial if we can achieve both high rates and high EE. This can be considered as a multiobjective optimisation problem. An efficient operating point is where neither the rate nor the EE can be further improved without the decline of the other. Such a point is also said to be *Pareto-optimal.*

Any point in region I (not shaded) in Fig. 1 is not Pareto-optimal since both the rate and the EE can simultaneously be increased by applying a higher power. The points in region II (shaded) are Pareto-optimal because increasing either the rate or the EE causes the other to decline. This is where there is a trade-off between the rate and the EE.

Regulatory bodies usually impose constraints on radiation power to prevent potential damage on people exposed to electromagnetic waves used in communications, and to mitigate interference among links using the same bandwidth, which is also a cause of inefficiency. As a result, we may assume a given sum transmit power constraint $P_{\max}$. If $P_{\max} < P^*$ then maximising the EE is the same as maximising the rate using $P_{\max}$. Further improvement is possible with new solutions to reduce p_C or new designs that yields a better (or more dominant) $R(P)$ function, e.g. by increasing the power amplifier efficiency, but not through power control. If $P_{\max} > P^*$ EE can be improved by reducing the sum transmit power. If a sum rate constraint $R_{\text{con}} > R_{\max}(P^*)$ exists such that $R_{\max}(P) \geq R_{\text{con}}$, it will be fulfilled with equality when maximising the EE, while the optimal EE cannot be achieved. If $R_{\text{con}} \leq R_{\max}(P^*)$ both the rate constraint and EE optimality can be simultaneously achievable.

3 Energy Efficiency Function for One Cell

In this section, we present a model for EE optimisation considering different traffic conditions. Imagine the downlink scenario of a cell in a mobile cellular network. During the day the links between the BS and the mobile stations experience higher interference from cells due to greater traffic and thus have statistically lower *signal to interference plus noise ratios* (SINR) than at night.

Assume we have the statistical information of these two traffic conditions, denoted by $i = 1$ for day-time and $i = 2$ for night-time. This stochastic information is captured as PDFs f_i of the normalised SINR α_i, which is the SINR at a unit transmit power. This can be written as a vector $\boldsymbol{\alpha}_i$ to include the normalised SINR of different components (e.g. subcarriers) of the transmission. Note that the dimension of vector $\boldsymbol{\alpha}_i$ may vary for different is. The PDF yields the probability that a certain channel realisation $\boldsymbol{\alpha}_i$ occurs during traffic condition i. Assume that conditions $i = 1, 2$ applies to 60% and 40% of a day's cycle, respectively. We describe this using weights $w_1 = 0.6$ and $w_2 = 0.4$. One may choose to have different schemes for day-time and night-time traffic (e.g. MIMO and SISO as studied in [10] for the uplink) to lower the total base power during low load period. The variations in the base power can be modelled using separate constants $p_{C,i}$.

We generalise this example and describe the EE as follows:

$$EE = \frac{\sum_i w_i \beta_i \int_0^\infty R_i\left(\boldsymbol{\alpha}_i, P_i, \gamma_i, \epsilon_i, G\right) f_i\left(\boldsymbol{\alpha}_i\right) d\boldsymbol{\alpha}_i}{\sum_i w_i\left(p_{C,i} + P_i\right)}. \tag{13}$$

- i — index for different traffic conditions or schemes at different time intervals.
- w_i — time fraction for traffic condition i such that $\sum_i w_i = 1$ and $w_i \geq 0, \forall i$.
- R_i — rate produced using scheme employed in i, a concave rate function of input power components $\boldsymbol{p}_i$ with the following parameters.
  - $\boldsymbol{\alpha}_i$ — SINR,
  - P_i — sum input power such that $\mathbf{1}^T \boldsymbol{p}_i = P_i$[6],
  - γ_i — SNR gap for scheme i,
  - G — additional imposed constraints such as individual power constraints.
  - ϵ_i — characterises the power amplifier efficiency, $\epsilon_i \in [0, 1]$, i.e. the output to input power ratio where $\epsilon_i p_i$ is the actual transmit power (more on PA efficiency in [11]).
- f_i — PDF of the SINR $\boldsymbol{\alpha}_i$ for traffic condition i.
- β_i — proportion of the rate used for data transfer (without pilot signals for channel estimation etc.), $\beta_i \in [0, 1]$.
- $p_{C,i}$ — base power required during interval i.

Note again that if R_i for all i is concave, EE is semistrictly quasiconcave (pseudoconcave if EE is differentiable), ensuring efficiency in optimisation. The programming problem can be written in the form described in (8) as:

$$\max . EE\left(\mathcal{P}\right) = \max_{\mathcal{P}} . \frac{\mathcal{R}\left(\mathcal{P}\right)}{\mathcal{P}_C + \mathcal{P}}, \quad \text{where} \tag{14}$$

[6] The vector $\mathbf{1} = (1, 1, ..., 1)^T$.

$$\mathcal{R}(\mathcal{P}) = \max_{\sum_i w_i P_i = \mathcal{P}} \sum_i w_i \beta_i \int_0^\infty R_i(\boldsymbol{\alpha}_i, P_i, \gamma_i, \epsilon_i, G) f_i(\boldsymbol{\alpha}_i) d\boldsymbol{\alpha}_i,$$
$$\text{and} \quad \mathcal{P}_C = \sum_i w_i p_{C,i}.$$

Examples of R_i are $R_i(P_i) = \left\{\sum_{n=1}^N \log(1 + \gamma_i \epsilon_i \alpha_{i,n} p_{i,n}) : \sum_n p_{i,n} = P_i\right\}$ for OFDM systems with N subcarriers and $R_i(P_i) = \left\{\log\det\left(\mathbf{I} + \gamma_i \mathbf{HQH}^H\right) : \mathrm{tr}(\mathbf{Q}) = P_i\right\}$ for MIMO systems. A rate maximising function can also be chosen such as $R_i(P_i, \boldsymbol{\alpha_i}) = \max.\left\{\sum_{n=1}^N \log(1 + \gamma_i \epsilon_i \alpha_{i,n} p_{i,n}) : \sum_n p_{i,n} = P_i\right\}$ or one w.r.t. a scheduler that selects best users, as in [21]. The integral $\int_0^\infty \{\dots\} d\boldsymbol{\alpha}_i$ implies an integration over all vector components, i.e. $\int_0^\infty ... \int_0^\infty \{\dots\} d\alpha_{i,1} ... d\alpha_{i,N}$. If the SINRs are quantised (e.g. due to channel measurements or signal feedback), the integral is to be replaced by a sum, and $d\alpha_{i,n}$ by the quantisation interval. The variable P_i can be further generalised as a function of $\boldsymbol{\alpha}_i$, such that $\sum_i w_i \int_0^\infty P_i(\boldsymbol{\alpha}_i) f_i(\boldsymbol{\alpha}_i) d\boldsymbol{\alpha}_i = \mathcal{P}$.

A great energy saving potential lies in deactivating certain components or putting them to sleep modes during inactivity [6]. This can be modelled by assigning a certain $p_{C,i}$ at which $P_i = 0$ with some time fraction w_i. In the elastic traffic scenario, it is even possible to collect enough data so that there can be continuous transmission at a later period while energy can be saved during the data collection period by shutting down inactive components. Additionally, it is more efficient to transmit during the best traffic condition (e.g. at night), that is, when the power is most effectively used due to low interference. Further improvements can be achieved by adjusting the weights w_i. This has to be done with caution since f_i may be altered by changing the time interval. Pilot and overhead signals that make channel information available at the receiver and transmitter, which takes up a fraction of the transmission, are considered in β_i.

4 Conclusion

An accurate evaluation of the EE requires adequate modelling of the whole network. Here, we present some general analytical tools for optimising EE at the BS, assuming elastic traffic. The optimisation problem can be formulated utilising rate maximisation algorithms. As a function of a single variable, insights into EE were illustrated graphically. An EE function that captures essential factors in one cell of a network is introduced and discussed.

Since this is also an idealised model, further work is needed to refine it, e.g. to describe the non-linearity of power amplifier efficiency, which has a significant influence on the EE. A model for inelastic traffic is also necessary, where more stringent quality of service demands are considered. Other figures of merit like fairness and coverage can still be incorporated into the model. The relationship between the base power and system model parameters, such as number of antennas and subcarriers, would also be a future task that enables EE optimisation over these parameters in addition to power control.

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Appendix

Lemma 5. *If f is jointly concave in $(x,y) \in \overline{C}$, $g(x) = \max_{y \in C(x)} f(x,y)$ is also concave where $C(x)$ is a convex set for any $x \in \mathbf{dom} g$ and $\overline{C} = \{(x,y) : \forall y \in C(x), \forall x \in \mathbf{dom}\, g\}$ is a convex set for all $x \in \mathbf{dom}\, g$. The domain g is defined as $\mathbf{dom}\, g = \{x : (x,y) \in \mathbf{dom}\, f \text{ for some } y \in C(x)\}$. Variables x and y can be vectors or matrices of identical dimensions.*

Proof. The function $g(x)$ is concave if

$$g(\lambda x_1 + (1-\lambda) x_2) \geq \lambda g(x_1) + (1-\lambda) g(x_2)$$

for any $\lambda \in [0,1]$ and any $x_1, x_2 \in \mathbf{dom}\, g$. We show that the following holds:

$$\begin{aligned}
&g(\lambda x_1 + (1-\lambda) x_2) \\
&= \max_{y} \{f(\lambda x_1 + (1-\lambda) x_2, y) : y \in C(\lambda x_1 + (1-\lambda) x_2)\} \\
\text{(a)} &= \max_{y_1,y_2} \{f(\lambda x_1 + (1-\lambda) x_2, \lambda y_1 + (1-\lambda) y_2) \\
&\quad : \lambda y_1 + (1-\lambda) y_2 \in C(\lambda x_1 + (1-\lambda) x_2)\} \\
\text{(b)} &\geq \max_{y_1,y_2} \{f(\lambda x_1 + (1-\lambda) x_2, \lambda y_1 + (1-\lambda) y_2) : y_1 \in C(x_1), y_2 \in C(x_2)\} \\
\text{(c)} &\geq \max_{y_1,y_2} \{\lambda f(x_1, y_1) + (1-\lambda) f(x_2, y_2) : y_1 \in C(x_1), y_2 \in C(x_2)\} \\
\text{(d)} &= \lambda \max_{y_1} \{f(x_1, y_1) : y_1 \in C(x_1)\} + (1-\lambda) \max_{y_2} \{f(x_2, y_2) : y_2 \in C(x_2)\} \\
&= \lambda g(x_1) + (1-\lambda) g(x_2).
\end{aligned}$$

The variable y is substituted by $\lambda y_1 + (1-\lambda) y_2$ in (a). We arrive at the inequality in (b) because $\{(y_1, y_2) : y_1 \in C(x_1), y_2 \in C(x_2)\}$ forms a subset of $\{(y_1, y_2) : \lambda y_1 + (1-\lambda) y_2 \in C(\lambda x_1 + (1-\lambda) x_2)\}$. The reason for this is understood by considering the following. Assume that $(x_1, y_1) \in \overline{C}$ and $(x_2, y_2) \in \overline{C}$. Because $\overline{C}$ is a convex set,$(\lambda x_1 + (1-\lambda) x_2, \lambda y_1 + (1-\lambda) y_2) \in \overline{C}$ also. By definition of $\overline{C}$, $\lambda y_1 + (1-\lambda) y_2 \in C(\lambda x_1 + (1-\lambda) x_2)$ is automatically satisfied under these assumptions. Since we now have a more restricted constraint set for y_1 and y_2 in the assumptions, a smaller maximum value may be induced. Using the property of joint concavity of f, we derive the inequality in (c). Since each summand has independent variables and the constraints can be separately considered, step (d) is derived. □

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