Incidental Neural Networks as Nomograms Generators

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Abstract. In this paper we developed a new architecture of neural networks for generating nomograms based on series of data vectors. The paper was inspired by the XIII Hilbert's problem which was presented 1900 in the context of nomography, for the particular nomographic construction. The problem was solved by V. Arnold (a student of Andrey Kolomogorov) in 1957. For numeric data of unknown functional relation we developed the *incidental neural networks* as nomograms generators – the graphic calculating devices.

Keywords: nomography, feedforward neural networks, function approximation.

1 Introduction

A nomogram is a graphical calculating device developed by Belgian engineer Junius Massau and French mathematician Maurice d'Ocagne in 1884 [20]. The definition of a nomogram can be stated as follows: a nomogram is a function plotted on two-dimensionally space with n parameters, and knowing $n-1$ parameters, the unknown one can be find in easy way. Generally, nomograms are used in such applications where an approximate answer is appropriated and useful; otherwise, the nomogram may be used to check the answer obtained from an exact calculation method.

One of the best monographs devoted to nomograms was written by Polish mathematician Edward Otto, Professor of Technical University of Warsaw, entitled *Nomography* issued by Oxford Pergamon Press in 1964 [16].

Since the 1970s developments of electronic calculators as well as computers have eliminated out needs of using nomograms for approximated solutions of complex functional relations. However, in spite of the main fault of nomograms, namely limited accuracy of reading, nomograms are still in use e.g. in hydraulic calculations, electrical engineering, in enterprises, banks and so on for estimating considered values. No doubt, there is one extremely important merit of nomograms – they give capability to represent a multidimensional space on a plane.

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The simplest nomogram is represented by a plot of a function $y = f(x)$ drown on a plane. In general, it is assumed that nomograms represent the functional relation given in the analytical form ([3], [4], [6], [15], [16], [18], [20]), e.g.

$$F(u, v, w) = 0 \tag{1}$$

In order to find a value of one variable knowing values of the rest often are used nomograms.

There is a very interesting problem to generate nomograms when the analytical form of the functional relations (1) is unknown, and data of some relation are available in a table, e.g.:

No.	u	v	w
1	u_1	v_1	w_1
2	u_2	v_2	w_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$
N	u_n	v_n	w_n

In this paper a novel architecture of artificial neural networks is proposed. For complex process of calculating and drawing of nomograms a new as well as specialized architecture of neural networks was developed – the new architecture of neural networks was named as *incidental neural networks*. The term of *incidence* is known in geometry and is understood in the following way: a point is incidental to a line if and only if the point lies on the line.

The new architecture of neural networks is constructed in the following way, a number of feedforward single input and single output neural networks – called *an elementary neural network*) are joined into one. Each elementary neural network is associated with a single dimension of a considered problem. All the elementary neural networks are merged via their outputs by so called Soreau equation ([4], [15]). The Soreau equation just describes the incidence properties of elementary neural networks outputs.

Such an incidental neural network after learning is able to generate nomograms. An example was performed in order to show proper functioning of the developed incidental neural networks.

2 Nomograms

In the seventeenth century Rene Descartes introduced the coordinate system allowing algebraic equations to be expressed in geometric way and created analytical geometry ([2], [17]), the bridge between algebra and geometry. The next step was the introducing a log-log plane by Leon Lalaane in 1843 [4, 6]. However nomograms developed by Maurice d'Ocagne in the 1880s became groundbreaking in the graphical calculations, and in geometric solutions of algebraic functions. Since that time nomograms have become commonly used as calculating devices by engineers during almost a hundred years.

How important nomograms were it is worth to notice that David Hilbert set out 23 problems during the world congress of mathematicians in Paris in 1900. Hilbert's XIII problem was presented in the context of nomography, for the particular nomographic construction. The problem was solved by 19 year old Vladimir Arnold (a student of Andrey Kolomogorov) in 1957.

It is necessarily to notice that Polish mathematicians were also involved in development of the theory of nomography, e.g. Hugo Steinhaus [18, 19, 20], Edward Otto [15, 16].

There were developed several types of nomograms e.g. [4, 6, 16], but generally we can distinguish two main categories of nomograms: the first are called collinear nomograms and the second – the grid nomograms [6, 15, 16]. In this work we will focus on the collinear nomograms.

2.1 Graphic Interpretation of Multiplication/Division Operation

Let us consider a simple nomogram which can realize the following relations $x_3 = x_1\ x_2$ shown in Fig. 1

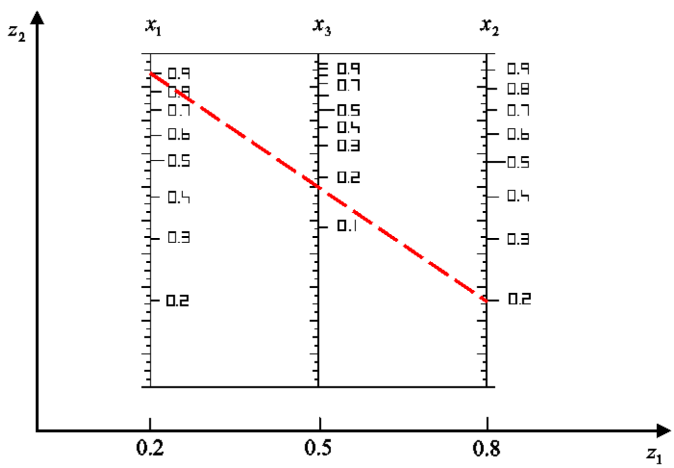

Fig. 1. Nomogram realising multiplication operation

Reading values of variables from the nomogram which consists of three axes is obvious. Connection by a straight line of a chosen point of a functional axis x_1 with a chosen point of a functional axis x_2 one obtains the solution laying on a functional axis x_3. The procedure allows finding a value of any variable under the assumption that two other variable values are known.

3 Collinear Nomograms

Let us consider a three dimensional Euclidean space. Necessary and sufficient condition in order to three points A, B, C lie on one straight line is zeroing the following matrix determinant

$$\begin{vmatrix} z_{11} & z_{12} & 1 \\ z_{21} & z_{22} & 1 \\ z_{31} & z_{32} & 1 \end{vmatrix} = 0 \tag{2}$$

where: (z_{11}, z_{12}) – the coordinates of point A, (z_{21}, z_{22}) – the coordinates of point B, (z_{31}, z_{32}) – the coordinates of point C.

The matrix determinant in (2) describes also the area of the triangle ABC. This area is equal to zero for collinear points. The exemplary matrix in (2) consists of nine entries, where the rows are related to the variables appearing in the functional relation (1); and the first column corresponds to the nomographic coordinate z_1, while the second column corresponds to the nomographic coordinate z_2, the third column consists of 1s. Such determinants are called Soreau ones, and equation (2) – Soreau equation, e.g. [16].

Equation (2) can be written as follows

$$\begin{vmatrix} z_{11}(x_1) & z_{12}(x_1) & 1 \\ z_{21}(x_2) & z_{22}(x_2) & 1 \\ z_{31}(x_3) & z_{32}(x_3) & 1 \end{vmatrix} = 0 \tag{3}$$

where:

$z_{11}(x_1)$, $z_{12}(x_1)$ are parametric functions with x_1 as a parameter,
$z_{21}(x_2)$, $z_{22}(x_2)$ are parametric functions with x_2 as a parameter,
$z_{31}(x_3)$, $z_{32}(x_3)$ are parametric functions with x_3 as a parameter.

Equation (3) can be rewritten in a form describing relations between values x_1, x_2 and x_3:

$$\begin{aligned} z_{11}(x_1)\left[z_{22}(x_2) - z_{32}(x_3)\right] - z_{21}(x_2)\left[z_{12}(x_1) - z_{32}(x_3)\right] \\ + z_{31}(x_3)\left[z_{12}(x_1) - z_{22}(x_2)\right] = 0 \end{aligned} \tag{4}$$

For instance, for the case, depictured in Fig. 1, realizing multiplication which has the following general form

$$f_3(x_3) = f_1(x_1) f_2(x_2) \tag{5}$$

and Soreau equation has the form

$$\begin{vmatrix} Z_{11} & z_{12}(x_1) & 1 \\ z_{21}(x_2) & Z_{22} & 1 \\ Z_{31} & z_{32}(x_3) & 1 \end{vmatrix} = 0 \tag{6}$$

where Z_{11}, Z_{22} and Z_{31} are constant numbers.

Up to now in nomographic practice it has been assumed that the functional relation of type (1) was given in an analytic form.

However, nowadays it happens very often in practice, data are available as series of numbers of unknown relations. In such a case following the theory of nomograms we face a problem to construct functional relation of numeric data. Let us assume that data are given as k series, each of N elements, and nothing is assumed about reciprocal relation between data within each series. This way a difficult problem of constructing nomograms for non-monotonic mappings arises. In this paper this problem is solved via introducing additional dimensions. Additionally k-element series of numbers can be represented by k parametric mappings, and these mappings can be always represented just by collinear nomograms.

4 Incidental Neural Networks

The theory of neural networks was described in many papers and books, e.g. [5, 11, 12, 14]. In Fig. 2 there is shown a feedforward neural network with a single input and a single output consisting with an input and output layers, respectively, and two hidden layers. Such a network we will call *an elementary neural network*.

Artificial neural networks can be connected in many various ways. In literature one can find some examples of systems built with simple neural networks [5, 11, 14]. In considered here problem it is required to find some relation between elements of data series. For that it is proposed a new architecture of neural networks which consists of some number of elementary neural network, see Fig. 2.

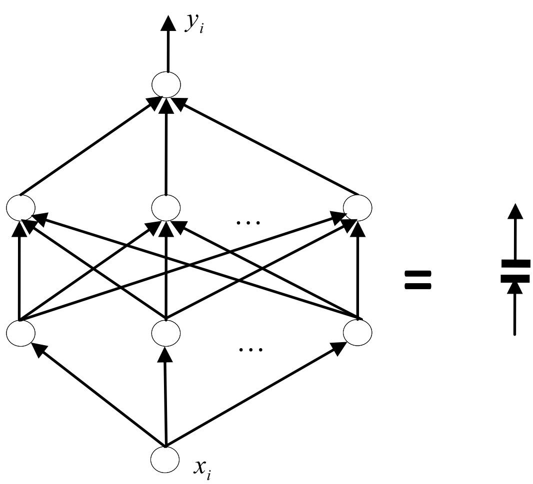

Fig. 2. Elementary neural network and it symbolic representation

It is assumed that a single elementary neural network is related or is responsible to a single dimension of the considered problem. The proposed elementary neural network consists of:

— one input neuron,
— one or two hidden layers (a number of neurons within hidden layers determines level of approximation accuracy),
— one output neuron.

Such an elementary neural network is able to model a single dimensional function, and can be used to approximate a functional axis in nomograms. From the other point of view, each element of Soreau determinant depends on one variable, and can be represented by one elementary neural network [9]. In such an elementary neural network the input is just one variable while the output constitutes a nomographic coordinate.

For instance, for the general multiplication operation (5)

$$f_3(x_3) = f_1(x_1)f_2(x_2)$$

the responsible incidental neural network is shown in Fig. 3. The exemplary new architecture consists of three elementary neural networks, each marked by two parallel thick bars, interfaced through the constraint represented by Soreau determinant depictured by a double circle.

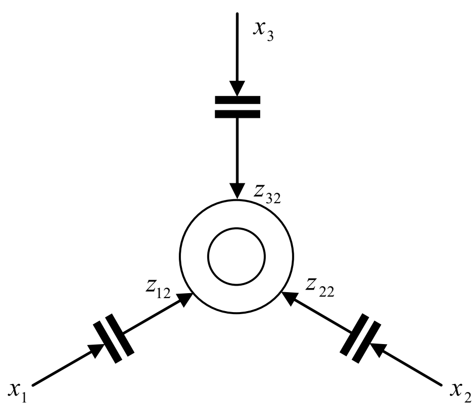

Fig. 3. The incidental neural network – a system of three elementary neural networks interfaced through Soreau determinant

The system of elementary neural networks is interfaced under some constraints as results of expressions of Soreau determinant.

It is worth to emphasise that applied special kind of elementary networks connection does not fulfil Kirchhoff low.

Adjusting Soreau determinant (6) to zero it is guaranteed that values of coordinates $z_{12}(x_1)$, $z_{22}(x_2)$ and $z_{32}(x_3)$ are coherent to relationship (3). The corresponding variables values x_1, x_2 and x_3 fulfil equation (4).

5 Illustrative Example

The nomogram presented in Fig. 1 consists of two vertical parallel functional axes and one horizontal. In this section we will solve the same problem as a collinear nomogram under the assumption that the relation $x_3 = x_1 x_2$ is given as several series of numbers data realising this multiplication operation.

For such a case Soreau determinant has the following form

$$\begin{vmatrix} 0.2 & z_{12}(x_1) & 1 \\ 0.5 & z_{22}(x_2) & 1 \\ 0.8 & z_{32}(x_3) & 1 \end{vmatrix} = 0 \tag{7}$$

The considered problem is three dimensional therefore the incidental neural networks consists of three elementary neural networks interfaced by (7), such incidental network (see Fig. 3) represents the collinear as well as rectilinear nomogram. The collinear nomogram is built of three parallel functional axes. The first column in (7) is related to the first nomographic coordinate z_1, it means that $Z_{11} = 0.2$, $Z_{21} = 0.5$ and $Z_{31} = 0.8$; while the second column in (7) is related to the second nomographic coordinate z_2, and is responsible of changing of each nomographic coordinate, respectively. The values Z_{11}, Z_{21} and Z_{31} were chosen arbitrarily.

The equation (7) can be rewritten as follows

$$z_{12}(0.5 - 0.8) + z_{22}(0.8 - 0.2) + z_{32}(0.2 - 0.5) = 0 \tag{8}$$

Each particular elementary neural network will be taught according to the following schema

$$z_{12} = \frac{-z_{22}(0.8 - 0.2) - z_{32}(0.2 - 0.5)}{0.5 - 0.8} \tag{9}$$

$$z_{22} = \frac{-z_{12}(0.8 - 0.2) - z_{32}(0.2 - 0.5)}{0.8 - 0.2} \tag{10}$$

$$z_{32} = \frac{-z_{12}(0.5 - 0.8) - z_{22}(0.8 - 0.2)}{0.2 - 0.5} \tag{11}$$

For this numerical example each elementary neural networks consists of one neuron in the input layer, five neurons in the first and second hidden layers and one neuron in the output layer. After choosing constant point along the axis z_1 there is a task to find changeability of nomographic axes z_{12}, z_{22} and z_{32}.

For the learning process of the incidental neural network the backpropagation algorithm with momentum was applied; the parameters were adjusted as follows: the learning coefficient = 0.7, the momentum coefficient =0.3 and the number of steps for each elementary network within each learning sequence =10000.

The algorithm of learning of incidental neural networks can be shortly described as follows:

Step 1

Values of numerical series data are presented to inputs of the incidental neural network.

Step 2

The respective outputs of elementary neural networks are calculated subject to actual weights and neurons activation functions.

Step 3

The values z_{12}, z_{22}, z_{32} are obtained from (9)–(11).

Step 4

Differences between values obtained in Step 2 and in Step 3 are considered as learning errors in learning processes in each elementary neural network. The elementary networks are trained sequentially one network after another; it means the values z_{12}, z_{22}, z_{32} are used in training.

Step 5

If the assumed level of accuracy is not reached then the weights must be changed and algorithm starts from the beginning, otherwise the algorithm is stopped.

This way, the functional axes which are parallel, they are perpendicular to nomographic axis of abscissae z_1; in result the nomogram is developed. In the incidental neural network the inputs are represented by variables x_1, x_2 and x_3, while the outputs of the elementary networks $z_{12}(x_1)$, $z_{22}(x_2)$ and $z_{32}(x_3)$ represent location of x_1, x_2 and x_3 on axis of ordinates z_2 (the coordinates of the functional axes).

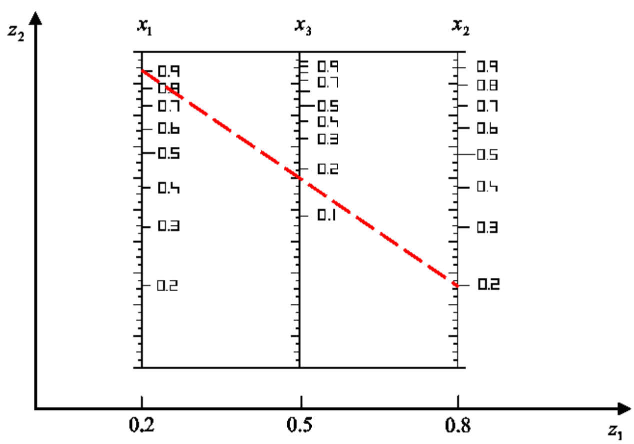

Fig. 4. Collinear nomogram realising multiplication operation

Using of nomograms is very easy, in the case of the example from Fig. 4, one needs to draw a straight line between the axes x_1 and x_2 – the result of the multiplication operation is read as the intersection of this drew line and the axis x_3.

6 Conclusions

In this paper it was shown that using collinear nomograms one can visualise and analyse causes of changeability of functional relation in multidimensional spaces.

In order to generate nomograms for numeric data of unknown relations we developed the new architecture of neural networks, here called the incidental neural networks. For such neural networks we developed the training algorithm based on the well-known backpropagation one.

Solution of many examples, also multidimensional, showed correctness of the assumptions as well as efficiency of the computer implementation.

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