A Bayesian-MAP method based on TV for CT image reconstruction from sparse and limited data

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Abstract—**Computed tomography (CT) plays an important role in the field of modern medical imaging. Reducing radiation exposure dose without significantly decreasing image's quality is always a crucial issue. Inspired by the outstanding performance of total variation (TV) technique in CT image reconstruction, a TV regularization based Bayesian-MAP (MAP-TV) is proposed to reconstruct the case of sparse view projection and limited angle range imaging. This method can suppress the streak artifacts and geometrical deformation while preserving image edges. We use ordered subset (OS) technique to accelerate the reconstruction speed. Numerical results showed that MAP-TV is able to reconstruct a phantom with better visual performance and quantitative evaluation than classical FBP, MLEM and quadrate prior MAP algorithms. The proposed algorithm can be generalized to cone-beam CT image reconstruction.**

Keywords—**Computed tomography, sparse and limited angular reconstruction, total variation, Bayesian-MAP**

I. INTRODUCTION

CT image reconstruction is one of the hottest topics in the field of medical imaging. When the projection data are not sufficient, filtered back-projection (FBP), which is frequently used in clinical environment, will produce conspicuous streak artifacts in reconstructed images [1, 2]. Instead of FBP [3], iterative reconstruction technique has the advantage of insensitivity to noise and parameter flexibility [4]. Maximum a Posterior (MAP) is one of the classical iterative reconstruction methods, which was proposed by Green in 1990[5]. However, conventional Bayesian-MAP can not get a satisfactory CT reconstructed image under the condition that projection data are sparse or collected at limited views.

In recent years, reconstruction methods based on TV are widely studied and used. In 2006, Sidky and Pan et al. proposed a total variation (TV) minimization combined with projection onto convex sets (POCS) to reconstruct accurate images from few-views and limited-angle data in divergent-beam CT [6], which achieved the state-of-the-art performance. In 2008, Pan et al. proposed a TV minimization based adaptive steepest-descent POCS algorithm (ASD-POCS) for circular cone beam CT reconstruction[7], which appears to be robust against cone-beam artifacts, and may be particularly useful when the angular range is limited or when the angular sampling rate is low. In 2011, Xun Jia et al. developed a fast GPU based algorithm to reconstruct high quality CBCT images from undersampled and noisy projection data [8]. The CBCT is reconstructed by minimizing an energy functional consisting of a data fidelity term and a TV regularization term. In their study, 40 x-ray projections can sufficiently reconstruct CBCT images with satisfactory quality for clinical purposes, while at the same time 100 times faster than similar regularized iterative reconstruction approaches.

Due to the outstanding performance of TV regularization, a TV regularization based Bayesian-MAP is proposed for CT image reconstruction from sparse view projection and limited angle range. The motivation for using TV regularization is that it is extremely effective for recovering edges of images while preserving its homogenous background. Ordered subset (OS) technique [9, 10] is adopted to accelerate the convergence of the proposed method.

The rest of this paper is organized as follows. In Section 2, we describe the x-ray CT measurement model, Bayesian-MAP and TV, then the MAP-TV algorithm in sparse and limited views is presented. In Section 3, we show the numerical results using our method, and compared with FBP, MLEM [11], conventional quadrate prior Bayesian-MAP, validating the efficiency of MAP-TV. Section 4 provides a conclusion of the paper.

II. METHOD

A. X-ray CT measurement

In X-ray computed tomography, measurements obey the underlying statistical properties of Poisson distribution. They are approximately modeled as a set of independent Poisson random variables.

$$p_i \sim poisson\left\{\left[Ax\right]_i + error_i\right\} \qquad (1)$$

$$\left[Ax\right]_i = \sum_{j=1}^{N} a_{ij} x_j, i = 1,2,...,M, j = 1,2,...,N$$

where $x = (x_1, x_2, ..., x_N)'$ is the vector of reconstructed image, $p = (p_1, p_2, ..., p_M)'$ is the measurement data, $error_i$ is the error in ith detector bin, A is the system matrix (also called projection matrix),M is the number of projections and

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N is the number of image pixels, a_{ij} represents the element in system matrix A.

B. Maximum a Posterior (MAP)

The maximum a posterior estimate is, given measurement p, to find x^*, maximizing

$$P(x\mid p)=\frac{P(p\mid x)P(x)}{P(p)} \quad (2)$$

The log form of the above formula is written as follows:

$$\ln P(x\mid p)=\ln P(p\mid x)+\ln P(x)+cons \quad (3)$$

The goal is to find $x^*=\max(\ln P(x\mid p))$.

A common Bayesian prior is the Gibbs distribution of the form: $P(x)=\frac{e^{-\beta U(x)}}{z}$, β is a regularization factor, which controls the smoothness of image. $U(x)$, as a regularization term, is a prior energy function.

Green developed an Bayesian-MAP algorithm using one-step-late technique, the formulation is as follows:

$$x^{n+1}{}_j = x^n{}_j * \frac{\sum_{i=1}^{M}(a_{ij}(\frac{p_i}{(Ax^n)_i}))}{\sum_{i=1}^{M}a_{ij}+\beta\frac{\partial U(x)}{\partial x}} \quad (4)$$

Conventionally, $U(x)$ is defined as a simple quadratic prior function, $U(x)=x^2/2$.

C. Total Variation (TV)

The TV method was originally brought by Rudin, Osher and Fatemi [12] to remove image noise. The form of TV can be written as:

$$\| X \|_{TV}=\sum_{l,m=1}^{I=1}\sqrt{(x_{l,m}-x_{l-1,m})^2+(x_{l,m}-x_{l,m-1})^2+\varepsilon} \quad (5)$$

As prior information, the minimization of TV is added to recover CT image, making homogeneous region of image smoothed well while preserving image edges.

D. MAP-TV for sparse and limited CT reconstruction

Inspired by the outstanding performance of total variation (TV) technique in CT image reconstruction, we developed a Bayesian-MAP method based on TV for CT image reconstruction from sparse and limited data, namely TV acts as a regularization in Bayesian-MAP algorithm.

Define $U(x)=\| X \|_{TV}$ as the prior function in Bayesian-MAP reconstruction algorithm.

Then the Bayesian-MAP algorithm can be modified as follows:

$$x^{n+1}{}_j = x^n{}_j * \frac{\sum_{i=1}^{M}(a_{ij}(\frac{p_i}{(Ax^n)_i}))}{\sum_{i=1}^{M}a_{ij}+\beta\frac{\partial \| X \|_{TV}}{\partial x_j}} \quad (6)$$

Finally，in order to accelerate the convergence speed of the proposed method, ordered subset (OS) technique is adopted for the reconstruction step.

The process of MAP-TV is as follows:

step1. Initialization： x=0.1.

step2. Calculate $\frac{\partial\|X\|_{TV}}{\partial x_{s,t}}$, Ax.

where

$$\frac{\partial\|X\|_{TV}}{\partial x_{s,t}} \approx \frac{2(x_{s,t}-x_{s-1,t})+2(x_{s,t}-x_{s,t-1})}{\sqrt{\varepsilon+(x_{s,t}-x_{s-1,t})^2+(x_{s,t}-x_{s-1,t})^2}} - \frac{2(x_{s+1,t}-x_{s,t})}{\sqrt{\varepsilon+(x_{s+1,t}-x_{s,t})^2+(x_{s+1,t}-x_{s+1,t-1})^2}} - \frac{2(x_{s,t+1}-x_{s,t})}{\sqrt{\varepsilon+(x_{s,t+1}-x_{s,t})^2+(x_{s,t+1}-x_{s-1,t+1})^2}} \quad (7)$$

step3. MAP-TV reconstruction:

$$x^{n+1}{}_j = x^n{}_j * \frac{\sum_{i=1}^{M}(a_{ij}(\frac{p_i}{(Ax^n)_i}))}{\sum_{i=1}^{M}a_{ij}+\beta\frac{\partial \| X \|_{TV}}{\partial x_j}}. \quad (8)$$

step4. Go back to step2 and step3 until the rule stop criteria is satisfied.

III. Experimental Results

In this section, the classical shepp-logan phantom is applied to demonstrate the accuracy of MAP-TV algorithm. Pixel values range from 0 to 1.Fan-beam CT configuration is defined as follows: Reconstructed image size is 128x128; The distance from x-ray source to detector is 300 mm; The distance from x-ray source to the center of phantom is 200 mm; In each view, there are 256 measurements. The projections in our simulation experiment are obtained using Siddon's algorithm [13].

To validate the accuracy of the reconstructed images, we adopted four evaluating indicators, defined as follows:

$$d = \left(\frac{\sum_{i=1}^{N} (t_i - r_i)^2}{\sum_{i=1}^{N} (t_i - \bar{t})^2} \right)^{1/2} \tag{9}$$

$$r = \frac{\sum_{i=1}^{N} | t_i - r_i |}{\sum_{i=1}^{N} | t_i |} \tag{10}$$

$$MSE = \frac{1}{N} \sum_{i=1}^{N} (t_i - r_i)^2 \tag{11}$$

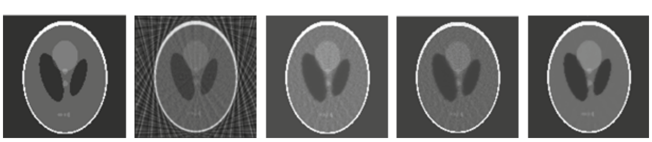
$$SNR = 10 \log\left(\frac{\sum_{i=1}^{N} (r_i - \bar{r})^2}{\sum_{i=1}^{N} (t_i - r_i)^2} \right) \tag{12}$$

where t is the reconstructed image, r is the true image, $\bar{t}$ is the mean of reconstructed image, $\bar{r}$ is the mean of true image.

A. Sparse data CT reconstruction

36 views were taken uniformly covering 360°. The number of ordered subsets is 6.

Figure 1 shows the reconstructed images from FBP, MLEM, quadrate prior Bayesian-MAP and our proposed method. Fig.1 (b) was obtained from FBP reconstruction using Hanning kernel. Serious streak artifacts are seen in the FBP reconstructed image. Fig.1 (c) and (d) were obtained from MLEM and quadrate prior Bayesian-MAP. It is obvious that images reconstructed by these two algorithms have blurring effect. The proposed MAP-TV reconstruction is presented in Fig.1 (e) .We can notice that the proposed MAP-TV algorithm outperforms the above ones in artifacts suppression and image edge preservation.



(a) (b) (c) (d) (e)

Fig.1 Shepp-Logan phantom images reconstructed by four different algorithms with 36 views covering 360° :(a) original image (b) result from FBP reconstruction using Hanning kernel. (c) result from MLEM reconstruction (d) result from quadrate prior Bayesian-MAP reconstruction (e) result from our proposed method reconstruction.

Table 1 shows d, r, MSE, SNR from different reconstruction methods. It is also clear that our proposed method behaves better than other methods.

Table 1. Error analysis

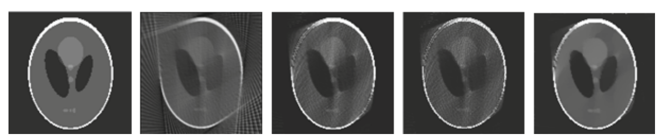
method	d	r	MSE	SNR	iteration
FBP	2.0005	0.5867	0.1824	10.0469	--
MLEM	0.1644	0.0853	0.0012	28.8170	200
Bayesian-MAP	0.1613	0.0850	0.0012	19.8855	200
Our method	0.0742	0.0288	1.4878e-4	35.7325	50

B. Limited-view CT reconstruction

30 views were taken uniformly covering 120°. The number of ordered subsets is 6.

Figure 2 shows the reconstructed images from FBP, MLEM, quadrate prior Bayesian-MAP and our method.

Due to severely sufficient projection data, FBP reconstruction using Hanning kernel leads to conspicuous and serious streak artifacts in Fig.2 (b). After 500 iterations, MLEM and quadrate prior Bayesian-MAP reconstruction still can not get satisfactory results, and the contrast of the images is low, seen from Fig.2 (c) and Fig.2 (d). Using our method of 200 iterations, we notice that the method simultaneously reduces noise and streaks artifacts effectively from Fig.2 (e).



(a) (b) (c) (d) (e)

Fig.2 Shepp-Logan phantom images reconstructed by four different algorithms with 30 views covering 120° :(a) original image (b) result from FBP reconstruction using Hanning kernel(c) result from MLEM reconstruction (d) result from quadrate prior Bayesian-MAP reconstruction (e) result from our proposed method reconstruction.

IV. CONCLUSIONS

In this paper, we have proposed a TV based Bayesian-MAP algorithm (MAP-TV) for CT image reconstruction. We have evaluated the performance of the proposed method. As numerical results shown, this method can produce accurate CT image with fewer artifacts and noise, by comparison with those obtained by FBP, MLEM and Quadratic-MAP, in condition that projection data are taken in sparse or limit views. Although the presented MAP-TV algorithm in this paper was used in the context of fan beam CT setting, it could also be easily extended to cone-beam CT(CBCT).Future work will focus on the refinements to the our method, acceleration using GPU to improve execution time, and extend 2D CT reconstruction to 3D CBCT image reconstruction.

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