# Chapter 10
# Simulation of Fiber Composites: An Assessment

**Klaus Rohwer**

**Abstract** Two facts are the main drivers for a steady rise of models simulating fiber composites: an increasing demand on optimal utilization of the material and a drastic improvement in computational power. Though this process is still in full swing a review is considered reasonable since it facilitates guidelines for future research. Topics which comprise major development lines are micromechanics, laminate theories, design and optimization, damage and failure, and manufacturing, though a strict separation is not in all cases useful. In reviewing these topics it will turn out that their model status is of rather different maturity. Areas are identified where there are plenty of models available which are really not needed, whereas other problems cannot be modeled adequately as yet.

## 10.1 Modeling Aspects

Numerical simulation requires a model, which on one hand should reflect the desired aspects of reality accurately enough but on the other hand be as simple as possible. Due to the complexity of fiber composites the number of models and tools for simulating the structural behavior is horrendous. It is by far not possible to value them all; rather, it is the intention to identify areas where, to the author's opinion, there are still deficits or where there is a surplus of models but a lack of knowledge regarding suitable application. Furthermore, it is of utmost importance to validate the model. To that end it is not sufficient to prove an adequate prediction in two or three test cases. Rather, extensive test series with relevant structural components must be run.

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K. Rohwer (✉)
Institute of Composite Structures and Adaptive Systems,
German Aerospace Center (DLR e.V.), Lilienthalplatz 7, 38108, Braunschweig, Germany
e-mail: Klaus.Rohwer@dlr.de

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## 10.2 Micromechanics

### 10.2.1 Material Properties

Micromechanics account fibers and the matrix for separate constituents; in some cases an interface layer is assumed in addition. There are chiefly two micromechanical tasks: Determination of stress between fibers and the matrix as well as homogenization of constituents. A major difficulty is the determination of material properties. That holds especially for fiber properties in transverse direction as well as for a possible interface layer. Since proper tests are difficult to perform these values are often determined from an inverse application of homogenization formulas.

### 10.2.2 Micromechanical Stress

Calculating micromechanical stress is generally based on a representative volume element (RVE) which contains the constituents in sufficiently large numbers. With suitable boundary conditions applied the RVE is then analyzed by means of finite elements. Information about size and boundary conditions are provided by Larsson et al. [1]. Examples of stress distributions are given by Rohwer and Xie [2], Nassehi et al. [3], Zhao et al. [4], Jin et al. [5] and Huang et al. [6]. This well-established procedure is quite useful for understanding the effects of inhomogeneity.

### 10.2.3 Stiffness Homogenization

Analyzing a complete structure from fiber composite material needs a homogenized approach. Adequate stiffness properties can be determined from an RVE analysis, as for instance proposed by Kari [7]. Garnich and Karami [8] included the effect of fiber waviness. But such a laborious procedure is normally avoided. Instead, simple homogenization formulas are used. For the Young's modulus and Poisson's ration in fiber direction it is the rule of mixture the results of which deviate only slightly from more complicated formulas by Hill [9]. In transverse direction and for shear the formulas are based on the rule of mixture for the compliances. But the results must be reduced by a certain amount due to 3D-effects, as can be visualized from Fig. 10.1. Many different proposals for such a reduction are available, e.g. Tsai and Hahn [10], Hashin [11] or Chamis [12]. With reasonable fiber and matrix properties a deviation of up to ±10% between the different formulas has been determined. Besides the stiffness a number of other properties have been homogenized. Among them are the coefficients of thermal

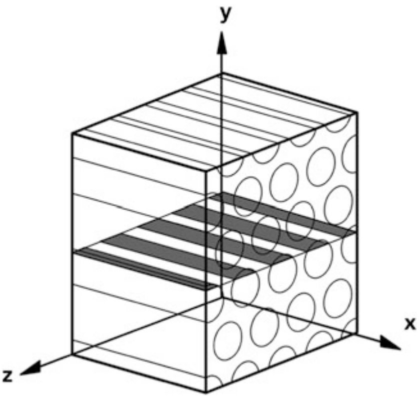

**Fig. 10.1** Rule of mixture for stiffness homogenization

expansion, e.g. Kulkarni and Ozden [13], the coefficients of moisture expansion, and the coefficients of thermal conductivity. Especially the thermal conductivity has been intensively studied; recently Al-Sulaiman et al. [14] even included the effect of voids within the matrix.

### *10.2.4 Strength Homogenization*

The status of strength homogenization is far less advanced. Some effort is put on the determination of tensile strength in fiber direction. Considering the standard composite design with an extension to failure of the matrix much higher than that of the fiber, the composite failure stress can be roughly estimated by the rule of mixture from the failure stress of the fiber and the matrix stress at fiber rupture. However, that does neither account for varying fiber strength along each single fiber nor for different strength between fibers. A number of hypotheses accounting for these variations have been proposed, e.g. Zweben [15], Rosen [16], Rosen and Zweben [17], but the application is not very convincing. More recent developments along this line are the global load sharing scheme by Curtin [18] and the simultaneous fiber-failure model by Koyanagi et al. [19]. Micromechanical simulations on the transverse failure behavior of fiber-epoxy systems are presented by Cid Alfaro et al. [20]. They pointed at a strong influence of the relative strength of the fiber-epoxy interface and the matrix.

Compressive strength models were first set up by studying the buckling of fibers with an elastic support. Depending on the fiber volume fraction Rosen and Hashin [21] found different failure modes. Dharan and Lin [22] extended this approach by accounting for an interface layer around the fibers. Further models were proposed by Xu and Reifsnider [23] and by Budiansky and Fleck [24]. A micromechanical analysis of the kink band formation after fiber buckling including the effect of fiber misalignment was performed by Jensen and Christoffersen [25]. Lately, Gutkin et al. [26, 27] determined two different failure mechanisms, (1) shear-driven fiber compressive failure and (2) kinking/splitting. Mishra and Naik [28] used the inverse micromechanical method to calculate fiber properties and applied them to determine the compressive strength for a composite with a different fiber volume fraction. A formulation capable of obtaining the maximum compression stress, and the post-critical performance of the material once fiber buckling has taken place was proposed by Martinez and Oller [29]. Only recently, micromechanical shear strength analyses were performed by Ng et al. [30] and Totry et al. [31].

In summary, micromechanics of composites have some benefits with respect to understanding the effects of inhomogeneity. All these methods, however, suffer from the difficulty to measure the micromechanical properties, in particular those of the fiber and the interface. Homogenization models in general are well advanced, but determination of transverse and shear strength needs further attention.

## 10.3 Laminate Theories

### *10.3.1 Laminate-Wise Approximations*

After homogenization the behavior of a single layer of composite material can be described by an anisotropic material law. Structures predominantly consist of several layers with different fiber orientation. Attempts to homogenize these in thickness direction did not prove reasonable. Instead, certain assumptions are made for the laminate behavior in thickness direction. The straightforward one is the Bernoulli hypothesis which, together with a plane state of stress, leads to the classical lamination theory (CLT). The structural behavior is then described by the so-called ABD-matrix. CLT is still first choice for thin-walled composite laminates.

Since the relation between shear modulus and Young's modulus in fiber direction is relatively small Whitney and Pagano [32] suggested using the First-Order Shear Deformation Theory (FSDT) instead. In addition to the ABD-matrix that needs a $2 \times 2$-matrix of transverse shear stiffness which cannot be determined without further assumptions. Rohwer [33] proposed an explicit computation, Altenbach [34] has presented a method, which works for sandwiches as well, and Schürg et al. [35] have published an energy minimization approach to reach that aim. FSDT is rather popular also because many plate and shell finite elements are based on this theory, so that only the anisotropic material law must be taken care of.

From the early 1980s onwards an increasing number of higher order theories have been developed. It started with the cubic distribution in thickness direction for membrane displacements by Murthy [36] and Reddy [37]. At the expense of additional functions that allows the determination of more reasonable transverse shear stresses from the material law. Following an idea of Reissner [38], Lo et al. [39] added a quadratic distribution of the transverse displacement which yields even better results but needs to specify eight functions. Using the method of multiple scales, Wu et al. [40] presented an asymptotic expansion of the 3D elasticity equations with material properties piecewise continuous in thickness direction. Shell thickness over radius is used as the perturbation parameter. Successive integration leads to a process embracing the CLT, the FSDT, and the cubic model by Reddy, respectively, as first-order approximations to the three-dimensional theory. The procedure allows improving the solutions obtained by the respective theories in an adaptive and hierarchical manner without increasing the number of functions.

Laminate-wise models have been evaluated by Noor and Burton [41], Rohwer [42], Yang et al. [43], Timarci and Aydogdu [44] and Carrera et al. [45]. As a result it can be stated that for standard applications the FSDT with suitable shear correction delivers sufficiently accurate results at acceptable expenses. But in special cases it may be necessary to increase the order of polynomials in thickness direction. Rohwer et al. [46] have presented such a case, where local thermal loads require polynomials of up to 5th order for modeling in-plane as well as transverse displacements in order to determine reliable stresses. Because of the considerably large effort involved with an application of such high-order models they should be restricted to cases where they are really needed.

### *10.3.2 Layer-Wise Approximations*

In a technical note Pagano and Hatfield [47] have shown a pronounced layer-wise zigzagging distribution of the in-plane displacements at slenderness ratios $a/h < 10$. This observation gave rise to develop a substantial number of analysis models, which consider each layer separately. For several of these models the number of functions depends on the number of layers. Since in real structures the layer number sometimes is in excess of 100 the large number of functions is an evident handicap. Computation time increases drastically since not only the number of degrees of freedom but also the bandwidth of the stiffness matrix is proportionally expanded. Therefore, it can be stated that in general layer-wise models with the number of functions depending on the number of layers are not suitable for practical application.

By means of compatibility and equilibrium conditions applied at layer interfaces one can achieve layer-wise models for which the number of functions does not depend on the number of layers. A considerable research effort in this field has led to many models of that type of which only a limited selection can be cited here. Sun and Whitney [48] assumed the in-plane displacements to vary linearly over each layer, whereas the transverse displacements remain constant. By enforcing not only the compatibility but also the transverse shear equilibrium at layer interfaces they could reduce the number of functions to five. Di Sciuva [49] later

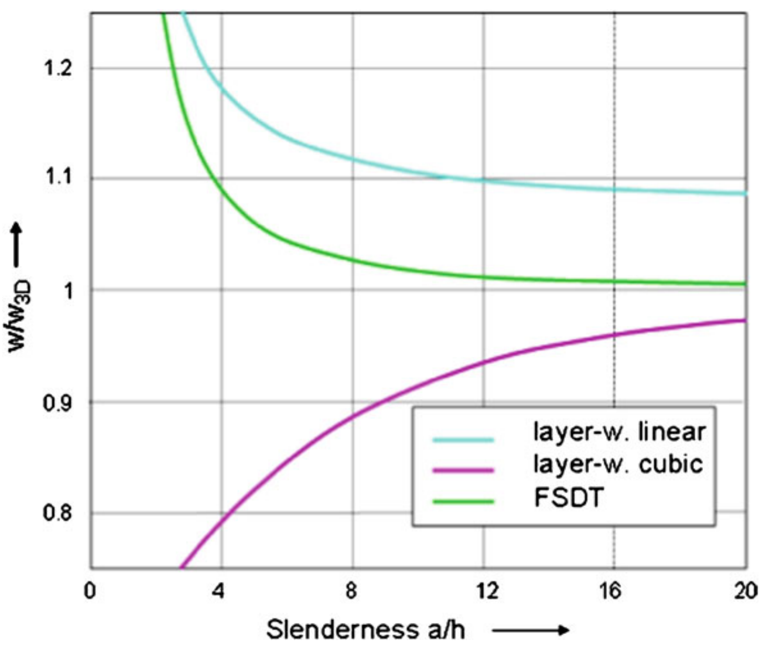

**Fig. 10.2** Center deflection of a rectangular [0, 90]s plate with layer thicknesses [0.4, 0.1]s compared to 3D elasticity solution

used a different formulation but ended up in an identical model. Lee et al. [50] assumed the in-plane displacements layer-wise cubically distributed and the transverse displacement remaining constant. Displacement compatibility and equilibrium against transverse shear at layer interfaces are utilized for eliminating unknowns. The model is thus left with five functions. The same approach, somewhat more elegantly presented using the Heaviside unit step functions, was provided by Savithri and Varadan [51]. A study by Rohwer et al. [52] has shown that especially the layer-wise linear approach can deliver rather accurate results, but for certain configurations they are worse than those obtained by the FSDT. As an example Fig. 10.2 shows the center deflection determined by means of different models with five functional degrees of freedom each. Lo et al. [53] have superimposed cubic laminate-wise in-plane displacements with layer-wise cubic ones; the transverse displacements are assumed linear. Using compatibility and equilibrium conditions for elimination the number of functions could be reduced to 16; the results obtained with this approach are remarkably good, but the expenses are high as compared to FSDT.

Another possibility to account for the effect of layer differences is superimposing specific zigzag functions onto a continuous displacement approximation in thickness direction. Examples for such an approach are proposed by Murakami [54], Di and Ramm [55], Brank and Carrera [56] as well as Icardi and Ferrero [57].

Unfortunately there is little information available about the efficiency as compared to more standard models. A unified formulation accounting for higher-order, zig-zag, layer-wise and mixed theories has been provided by Carrera [58].

### 10.3.3 Transverse Stresses

Multidirectional laminates are susceptible to delaminations. Information about the transverse shear and normal stresses are needed to cover this threat. With simple laminate models like CLT or FSDT these stresses cannot be determined from the material law. But if the membrane stresses are determined accurately enough equilibrium conditions can be applied locally for the transverse shear stresses, and their derivatives in turn can be integrated to determine the transverse normal stress. Corresponding procedures as proposed by Pryor and Barker [59], Lo et al. [60] and Engblom and Ochoa [61] suffer from the need of higher order shape functions, a constraint which has been reduced by the Extended 2D method of Rolfes and Rohwer [62]. An alternative approach is lately proposed by Schürg et al. [35]. Stresses determined by means of the equilibrium conditions can be iteratively improved. For this purpose predictor-corrector processes are invented by Noor et al. [63] and Noor and Malik [64]. Further, the re-analysis method described by Park and Kim [65] and Park et al. [66] can be applied for that purpose. A totally different approach by Guiamatsia [67] bases the homogenization in thickness direction not on a priori displacement assumptions but on the superposition of certain fundamental states. Published results look promising, but limitations due to the superposition process are not clear.

An up-to-date and elaborate overview over the different approaches is provided by Kreja [68]. In conclusion it can be stated that for structures with common slenderness rates and smooth loading the FSDT with improved transverse shear stiffness can be regarded as a good choice. Transverse stresses should be determined by application of the equilibrium conditions. So far, only limited knowledge is available with respect to the accuracy of layer-wise models. Engineering judgment is still needed when deciding upon the model for stress analysis of thicker layered structures. Depending on the conditions at hand it may rather be suitable to use 3D finite elements right away.

## 10.4 Design and Optimization

### 10.4.1 Initial Design

Design is usually an ill-posed problem rather than a problem of optimization as Kolpakov and Kolpakov [69] have pointed out. Anyway, structural design aims at a solution well suited to the requirements and thus needs some type of

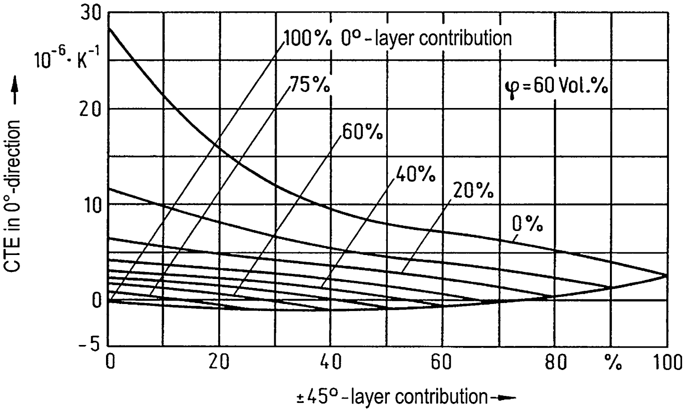

**Fig. 10.3** Design diagram for a symmetric laminate with predetermined CTE in 0°-direction (layer properties: $\mathrm{CTE}_{\mathrm{parallel}} = -0, 8 \times 10^{-6}\ \mathrm{K}^{-1}$ $\mathrm{CTE}_{\mathrm{transverse}} = 29 \times 10^{-6}\ \mathrm{K}^{-1}$)

optimization. With fiber composites this includes not only the choice of material and thickness but also fiber directions and the stacking sequence. In the early stage of the design process that requires simple and fast procedures which not necessarily yield the optimal solution but lead into the right direction. One such procedure places fibers in the direction of principle stress; another one assumes three fiber directions and applies the netting theory, which ignores the effect of the matrix material, to determine the corresponding layer thickness. Besides other deficiencies as for instance outlined by Evans and Gibson [70] both are applicable for membrane problems only. For special cases like setting up a laminate with predetermined thermal extension or certain stiffness in one direction there are diagrams available. Figure 10.3 provides such a diagram. Moreover, ranking can be applied as a design procedure, where the objective function is calculated for all suitable stacking sequences. Such an approach is used by the laminate design programs RANKHO by Tan [71], LamRank by Tsai [72] or ESAComp [73].

Vannucci et al. [74] lately proposed another procedure, which works for so-called quasi-homogeneous laminates showing identical behavior in bending and in extension. That allows reducing the optimization to the in-plane properties. For these special cases stacking sequences can be determined which provide maximum plate bending stiffness, buckling load or natural frequency. Lopes et al. [75, 76] optimized the plate stacking sequence with respect to their impact resistance and damage tolerance characteristics.

### 10.4.2 Structural Optimization

According to Eschenauer [77] structural optimization in general is based on three columns: modeling the structural behavior, defining the objective function, and selecting a suitable optimization strategy. The structural model should be as simple as possible since it must be analyzed rather often, but the important features must certainly be well described. The objective function specifies which quantity should be taken to the optimum, e.g. the structural weight or the manufacturing cost; in addition the variables as well as the constraints are defined. There are many optimization strategies on the market. As Zimmermann [78] has pointed out, there is no general optimal strategy, but it depends on the type of problem. Thus choosing the best strategy needs experience. Ghiasi et al. [79] have reviewed a large number of methods suitable for optimizing the stacking sequence of laminated composites. They differentiate between gradient-based methods, direct search methods including the popular simulated annealing and genetic algorithms, as well as specialized algorithms like layer-wise optimization. A modified simulated annealing method was proposed by Javidrad and Nouri [80] where more freedom during the cooling process leads to an improved convergence. With a similar method Akbulut and Sonmez [81] have found out that the design domain is greatly enlarged by increasing the number of distinct lamina angles and the range of values they may take. Gou et al. [82] analyzed the problem of aeroelastic tailoring for aircraft wings using the bending-twisting coupling effect of laminates. They found out that the genetic algorithm is preferable in case the flutter speed is the objective function, whereas for optimal torsion rigidity a gradient method is much faster. Lopez et al. [83] have optimized laminated plates under in-plane load using maximum stress, Tsai-Wu, and Puck failure criteria. They have demonstrated that an optimal design strongly depends on the chosen failure criterion, and that none of the failure criteria is always the most or the least conservative when different load conditions are applied. Azarafza et al. [84] have treated laminated composite circular cylindrical shells subjected to compressive axial and transverse transient dynamic loads. They performed multi-objective optimization of weight and dynamic response using a genetic algorithm. It turned out that the dominant constraint that affects the optimization process most is buckling. A cylindrical shell under pure bending was optimized by Blom et al. [85] considering the restrictions in manufacturing possibilities by means of tailored fiber placement. Increasing the stiffness at the tension side and correspondingly reducing it at the compression side increased the maximum buckling load. With the aid of a genetic algorithm Almeida and Awruch [86] presented a technique for design optimization of composite laminated structures in the geometric nonlinear range. A multiobjective optimization is performed, and a pareto-optimal set is obtained by shifting the optimization emphasis using a weighting factor. Johansen and Lund [87] applied a sensitivity analysis for maximizing the safety against failure of a fully three dimensional laminated composite structure. Gillet et al. [88] used a genetic algorithm to determine the relative importance of several design parameters. They found out that material parameters are of great influence whereas the fiber orientation is of minor importance. Using particle swarm optimization Peng et al. [89] designed fiber–metal laminates for maximum strength. After a thorough investigation Ghiasi et al. [90] rated the optimality criterion methods to be first choice if available. If not, multi-level optimization methods would be the best candidates. If neither of these can be used they recommended a genetic algorithm or a gradient-based method or a combination of these two.

Obviously there are sufficient optimization strategies and computational tools available. Lack of knowledge, however, can be stated for the situation in the early design phase. One of the few exceptions is the dissertation of Zimmermann [78], which provides general information about the optimum layer sequences of axially compressed cylindrical shells depending on the number of layers. Further, a word of caution with respect to optimization seems to be adequate. Ottino [91] has pointed to the fact that the optimal state can be a high-risk state and one should rather strive for a robust solution. That also is one of the aspects treated by Marzcyk [92]. Other than damage tolerant design, robust design adds probabilistic uncertainties to the optimization process as has been discussed by Lee et al. [93] with respect to composite panels.

## 10.5 Damage and Failure

### *10.5.1 Failure Criteria*

Not even fiber composites can withstand unlimited loads. Because of the anisotropy and inhomogeneity of composites the failure is a rather complex process. Thus the difficulties to decide even for a suitable failure criterion under quasi-static load are understandable to a certain extent. Most of the proposals are formulated in stresses, but there are still some scientists who prefer a formulation in strains. Further there is an alternative between interactive and non-interactive criteria. Though the tendency goes to interactive criteria a few non-interactive ones are still in use, sometimes with astonishingly good results. Regarding the interactive criteria some simply use a tensor polynomial as an extension of the von Mieses criterion for ductile materials, and others apply more physically based considerations. Still under discussion is the question whether or not the failure envelop should be open or closed. Already in the mid 1980s Nahas [94] has reviewed a large number of failure criteria. More recent is the excellent overview provided in the compendium by Hinton et al. [95]. Further failure criteria are developed for instance by Luccioni [96] and by Pinho et al. [97]. Cuntze [98] has improved his own criterion through a single but effective modification of one inter-fiber-failure condition. Stamblewski et al. [99] have used micromechanical analysis results to feed into a quadratic criterion for which the enclosed volume is maximized. Lee and Soutis [100] found out that the compressive strength decreases with increasing size of the test specimen, an effect which is not covered by the available failure criteria.

In general one gets the impression that there are more than enough criteria; urgently needed is a validation against reliable test data. In this respect the world-wide failure exercise, the first phase of which is published by Hinton et al. [95] is of great value.

### 10.5.2 Damage Progression

When the material strength limit is reached at a certain point in one single ply of the laminated structure, as specified by a failure criterion, it usually does not mean total failure. Damage progression under a further increasing load must be modeled in order to determine the actual behavior. In principle the gradual failure process can be modeled by fracture mechanical, damage mechanical and phenomenological approaches. To that end Knops and Bögle [101] have set up a computer program using degradation models which account for the gradual loss of stiffness. Maimi et al. [102] proposed a thermodynamically consistent damage model based on the LaRC04 failure criteria. A 3D progressive damage theory was used by Cui et al. [103] to analyze the whole process of damage initiation and development for composite laminates under impact loading as well as tensile load after impact. Basu et al. [104] have developed a progressive damage growth model for composite laminates under compression, and implemented it into the finite element package ABAQUS. Based on a micromechanical approach Ha et al. [105] proposed a progressive damage model to predict failure of composite laminates under multi-axial mechanical as well as under thermal loads. Similarly, Zhang and Zhang [106] accounted for the nonlinear material behavior as well as damage on the micro-scale in the development of a macro-scale constitutive model which describes the progressive failure process of composite laminates. A review of degradation models for progressive failure of composites is provided by Garnich and Akula [107].

### 10.5.3 Delamination

Delamination is a damage mode which needs special attention. It may be initiated from manufacturing flaws, low velocity impact or the free edge effect. The usual stress-based failure criteria are not suitable for calculating the delamination progression; instead fracture mechanics methods are preferred. Rather popular is the Vitual Crack Closure Technique as described by Rybicki and Kanninen [108], which is implemented into several finite element programs. A modified version was proposed by Riccio and Gigliotti [109]. As an alternative, Alfano and Crisfield

[110] have proposed an analysis method using a cohesive-zone model combined with interface elements. A theory presented by Xiao and Gillespie [111] improves the prediction of the shear strength enhancement in the presence of friction and compression. Davidson and Zhao [112] have set up a 'limited input bilinear criterion' for mixed-mode delamination failure which needs data only from DCB, SLB, and ENF tests for its characterization. Thus the models for simulating the delamination progression seem to be well developed.

More involved is the simulation of buckling in the presence of delaminations. The effects of rectangular and triangular local delaminations as well as asymmetric sub-layers were treated by Wang and Lu [113]. The influence of stacking sequence on the buckling load of plates with strip delaminations was analysed by Pekbey and Sayman [114]. Lee and Park [115] applied an enhanced assumed strain solid element for better simulate the local buckling mode at the delamination zone. Two enveloped delaminations, one fully covering the other, were treated by Parlapalli et al. [116]. They determined upper and lower bounds for the buckling load which strongly depend on the locations and sizes of the two delaminations. Correspondingly, Tafreshi [117] studied the effect of the size and location of a delamination. He found out that for delaminations closer to the free surface of the laminate failure will be due to delamination growth rather than buckling. On the other hand, Aslan and Sahin [118] found out that for multiple delaminations the largest and near-surface delamination affects the buckling load most, the size of a beneath delamination has no significant effect. Kremer and Schürmann [119] treated the problem of tension-loaded plates with cut-outs, which can buckle because of local compression or shear. They showed that a shape optimized cut-out runs risk to initiate buckling before reaching the fracture load.

If the composite structure undergoes impact or crash load the dynamic aspects must be considered. Explicit finite element codes like PAM CRASH or LS-DYNA contain suitable tools. Special procedures for treating delamination under impact are proposed by Fleming [120] and Iannucci [121]. Williams et al. [122] used the strain equivalence approach for setting up a model to predict impact damage growth and its effects on the impact force histories in carbon fiber reinforced plastic laminates. A damage model proposed by Iannucci and Ankersen [123] uses damage variables assigned to tensile, compressive and shear damage at a lamina level, thus allowing the total energy dissipated for each damage mode to be controlled during a dynamic or impact event.

### 10.5.4 Fatigue

For structures from fiber composite material strength reduction due to fatigue loading is well known to be comparatively small, but not negligible in the first place. In his PhD thesis Talreja [124] has comprehensively described the failure mechanisms which happen during fatigue loading. Since then the subject has been intensively worked on leading to a large number of relevant publications.

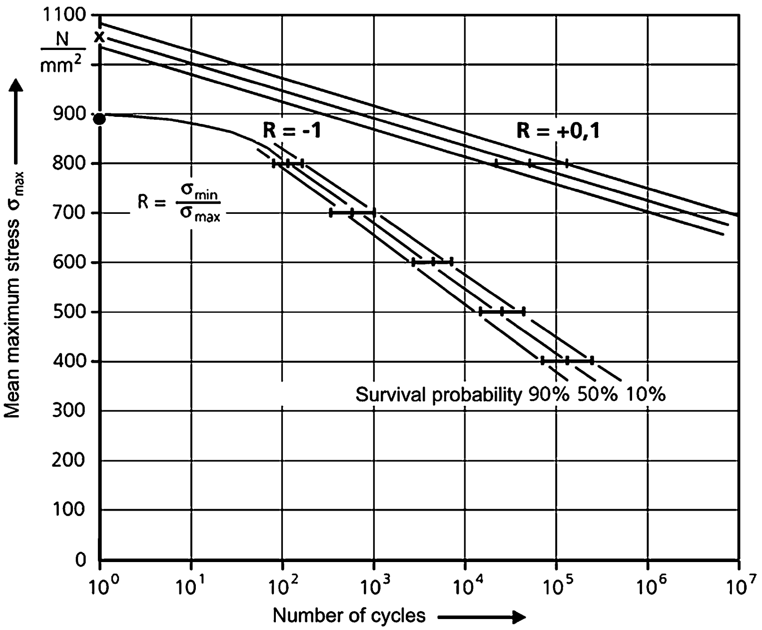

**Fig. 10.4** Effect of stress ratio on the fatigue life for a $[0_2,-45,0_2,+45,0,90]s$ laminate from T300/914C

The book recently edited by Harris [125] gives an insight into the research state of the art. Tests have shown that besides other parameters fatigue life heavily depends on the stress ratio as depicted in Fig. 10.4. Simulation models which accurately describe the damage progression during fatigue, however, are rare.

A linear damage accumulation hypothesis after Palmgren and Miner does not account for the load sequence which is important for composites. The Marco-Starkey model [126] is a nonlinear Miner rule the exponents of which must be determined by tests. The Strength-Degradation model by Yang and Liu [127] needs to measure the cycles to failure and the residual strength for every load level, and in addition it is not applicable for notched structures. The Percent-Failure rule requires fatigue tests for every load level. It can account for specific load sequences by switching between corresponding S-N curves. Moisture reduces the residual strength of GFRP equivalently to the reduction in static strength as has been observed by Cerny and Mayer [128], an effect which is not accounted for in the available models. For cross-ply laminates under constant amplitude tension-tension or compression-compression loading a progressive fatigue damage model was proposed by Shokrie and Taheri-Behrooz [129] and Taheri-Behrooz et al. [130]. As input, however, static and fatigue properties of unidirectional composites in longitudinal and transverse direction are needed. They must be measured in corresponding tests. Damage initiation and damage propagation was studied by May and Hallett [131] and May et al. [132] using cohesive interface elements.

The model requires anticipating a potential matrix crack where the interface elements are to be placed. After extensive discussions on the pros and cons of existing models Quaresimin et al. [133] conclude that they are not fully satisfying, some models even lead to non-conservative results. Especially for very high cycle fatigue and the effect of temperature increase in epoxy resin reliable models do not yet exist.

## 10.6 Manufacturing

### *10.6.1 Draping*

In case of structures with a plane or developable surface placing the fibers in the required direction is not a problem which needs simulation. However, if the structures have a non-developable surface accurate draping is not simple and simulation software can help finding an optimal solution. Accuracy and reliability of simulation models, however, are still under discussion. Hancock and Potter [134], for instance, investigated the simple pin jointed net model applied in reverse to generate formable geometries. Later, the same authors [135] stated that the conventional outputs of kinematic modeling are of limited applicability in informing the hand lay-up process for complex surfaces. They proposed a novel strategy for generating detailed unambiguous manufacturing instructions as a method for enhancing the practicality of kinematic simulation tools. Vanclooster et al. [136] compared the kinematic mapping against the explicit finite element method and found out that the kinematic mapping approach severely fails in predicting the fiber reorientation that occurs during stamp forming for non-symmetrical forming configurations. The FEM-simulation gave a reasonably good prediction of the fiber reorientation and seemed the most promising technique for draping simulations. In spite of the ongoing discussion several software packages provide tools for draping. In this context the program systems Catia V5 CPD [137], Simulayt Advanced Fiber Modeler [138], Vistagy FiberSIM [139] and Anaglyph Laminate Tools [140] are to be mentioned.

### *10.6.2 Resin Flow and Curing*

Resin transfer molding has proven to be an efficient manufacturing technology. Simulating the resin flow and the curing can help avoiding pitfalls. To prevent dry spots Minaie and Chen [141] simplified the multiport resin flow as many individual one-dimensional flows each of which extend from the gate to the vent and are controlled independently by adjusting the pressure of the related inlet. In a vacuum assisted resin transfer molding process Johnson and Pitchumani [142]

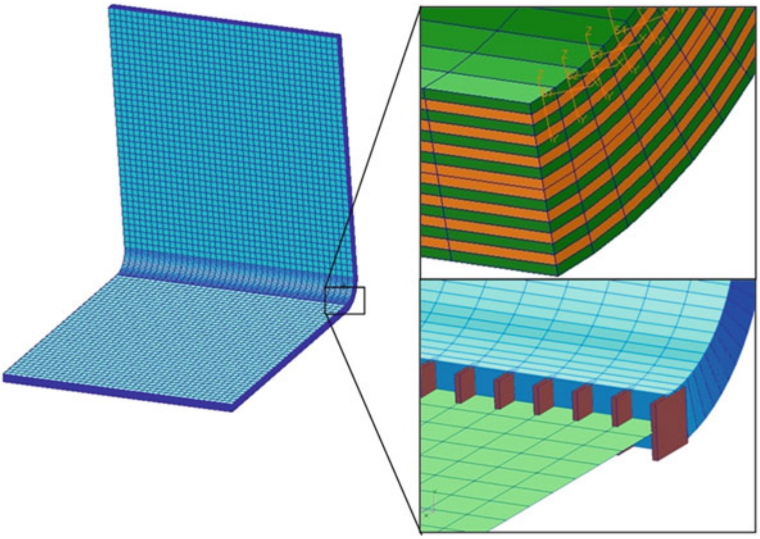

**Fig. 10.5** Modeling L-profile for spring-in simulation

used induction heating as a means of locally reducing viscosity to counteract the effects of nonhomogeneity in the permeability of preform lay-ups. García et al. [143] simulated the mould filling process within an updated Lagrangian framework. The use of a meshless technique allowed avoiding the numerical difficulties associated to the fluid properties transport through the whole domain in fixed mesh simulations. For saving computer time Dong [144] proposed a two-step method, where first, a design of experiment (DOE) approach is coupled with a 2-D control volume finite element method simulation to calculate the equivalent permeability and porosity for various process variable combinations. Second, the algorithm for the through-thickness flow front construction is developed. A commercially available software packages for flow simulation is RTM-Worx [145], which is based on the finite element and control volume methods to solve the physical equations that govern flow of a resin through a porous medium. Further, there is PAM-RTM [146], which simulates preforming, considers different types of resin, preheating of the mold, filling and curing using the Kamal Sourour model.

Manufacturing fiber composites accurate to measurement needs to account for the thermal and chemical reactions at curing. They may lead to the so-called spring-in which must be determined in order to counteract by forming a suitable mould. Processing induced warping has been treated by Darrow and Smith [147], who considered thickness cure shrinkage, mould expansion, and fiber volume fraction gradients. More involved, but also more expensive is the simulation presented by Cheung et al. [148]. They obtained the thermal equilibrium and chemical kinetics during the curing phase of the resin transfer molding process subject to mould temperature history and corresponding manufacturing process plans. Sweeting et al. [149] have studied numerically the distortion of circularly curved flanged laminates. Different parameters influencing the distortion of straight L and curved C profiles have been treated by Svanberg [150]. Straight L- and Z-profiles have been analyzed by Spröwitz et al. [151]. They found out that it is sufficient to model only the curved region with volume elements whereas the flanges can do with shell elements, cf. Fig. 10.5. Based on a micromechanical analysis Brauner et al. [152] concluded that thermal and chemical shrinkage must be supplemented by matrix yielding and degradation in order to more accurately simulate the curing stresses.

A number of manufacturing aspects can be adequately modeled as yet. It seems desirable, however, to connect them to simulate the complete manufacturing chain.

## References

1. Larsson, F., Runesson, K., Saroukhani, S., Vafadari, R.: Computational homogenization based on a weak format of micro-periodicity for RVE-problems. Comput. Methods Appl. Mech. Eng. **200**, 11–26 (2011). doi:10.1016/j.cma.2010.06.023
2. Rohwer, K., Xie, M.J.: Micromechanical curing stresses in CFRP. Compos. Sci. Technol. **25**, 169–186 (1986)
3. Nassehi, V., Dhillon, J., Masciaet, L.: Finite element simulation of the micromechanics of interlayered polymer/fibre composites: a study of the interactions between the reinforcing phases. Compos. Sci. Technol. **47**, 349–358 (1993)
4. Zhao, L.G., Warrior, N.A., Long, A.C.: A micromechanical study of residual stress and its effect on transverse failure in polymer–matrix composites. Int. J. Solids Struct. **43**, 5449–5467 (2006)
5. Jin, K.-K., Huang, Y., Lee, Y.-H., Ha, S.K.: Distribution of micro stresses and interfacial tractions in unidirectional composites. J. Compos. Mater. **42**, 1825–1849 (2008). doi:10.1177/0021998308093909
6. Huang, Y., Jin, K.-K., Ha, S.K.: Effects of fiber arrangement on mechanical behavior of unidirectional composites. J. Compos. Mater. **42**, 1851–1871 (2008). doi:10.1177/0021998308093910
7. Kari, S.: Micromechanical modelling and numerical homogenization of fibre and particle reinforced composites. Dissertation, University of Magdeburg (2006)
8. Garnich, M.R., Karami, G.: Finite element micromechanics of stiffness and strength of wavy fiber composites. J. Compos. Mater. **38**, 273–292 (2004). doi:10.1177/0021998304039270
9. Hill, R.: Theory of mechanical properties of fibre-strengthened materials. J. Appl. Mech. **31,** 223–232. Errata (1965) J. Appl. Mech. **32**, 219 (1964)
10. Tsai, S.W., Hahn, H.T.: Introduction to Composite Materials. Technomic Publishing Company, Westport (1980)
11. Hashin, Z.: Analysis of composite materials—A survey. J. Appl. Mech. **50**, 481–505 (1983)
12. Chamis, C.C.: Simplified composite micromechanics equations for hygral, thermal, and mechanical properties. SAMPE Q. **15**, 14–23 (1984)
13. Kulkarni, R., Ozden, O.: Transverse and longitudinal CTE measurements of carbon fibers and their impact on interfacial residual stresses in composites. J. Compos. Mater. **40**, 733–754 (2006). doi:10.1177/0021998305055545

14. Al-Sulaiman, F.A., Al-Nassar, Y.N., Mokheimer, E.M.A.: Prediction of the thermal conductivity of the constituents of fiber-reinforced composite laminates: Voids effect. J. Compos. Mater. **40**, 797–814 (2006). doi:10.1177/0021998305055548
15. Zweben, C.: A bounding approach to the strength of composite materials. Eng. Fract. Mech. **4**, 1–8 (1972)
16. Rosen, B.W.: Tensile failure of fibrous composites. AIAA-J. **2**(11), 1985–1991 (1964)
17. Rosen, B.W., Zweben, C.H.: Tensile failure criteria for fiber composite materials. NASA CR-2057, Washington DC (1972)
18. Curtin, W.A.: Theory of mechanical properties of ceramic-matrix composites. J. Am. Ceram. Soc. **74**, 2837–2845 (1991)
19. Koyanagi, J., Hatta, H., Kotani, M., Kawadael, H.: A comprehensive model for determining tensile strengths of various unidirectional composites. J. Compos. Mater. **43**, 1901–1914 (2009). doi:10.1177/0021998309341847
20. Cid Alfaro, M.V., Suiker, A.S.J., de Borst, R.: Transverse failure behavior of fiber-epoxy systems. J. Compos. Mater. **44**, 1493–1516 (2010). doi:10.1177/0021998309360941
21. Rosen, B.W., Hashin, Z.: Analysis of material properties. In: Reinhart, T.J., et al. (eds.) Engineered Materials Handbook, Composites. Metal Park, Ohio (1989)
22. Dharan, C.K.H., Lin, Ch.-L.: Longitudinal compressive strength of continuous fiber composites. J. Compos. Mater. **41**, 1389–1405 (2007). doi:10.1177/0021998306068078
23. Xu, Y.L., Reifsnider, K.L.: Micromechanical modeling of composite compressive strength. J. Compos. Mater. **27**, 572–588 (1993). doi:10.1177/002199839302700602
24. Budiansky, B., Fleck, N.: Compressive failure of fiber composites. J. Mech. Phys. Solids **41**, 183–211 (1993)
25. Jensen, H.M., Christoffersen, J.: Kink band formation in fiber reinforced materials. J. Mech. Phys. Solids **45**, 1121–1136 (1997)
26. Gutkin, R., Pinho, S.T., Robinson, P., Curtis, P.T.: Micro-mechanical modelling of shear-driven fibre compressive failure and of fibre kinking for failure envelope generation in CFRP laminates. Compos. Sci. Technol. **70**, 1214–1222 (2010). doi:10.1016/j.compscitech.2010.03.009
27. Gutkin, R., Pinho, S.T., Robinson, P., Curtis, P.T.: On the transition from shear-driven fibre compressive failure to fibre kinking in notched CFRP laminates under longitudinal compression. Compos. Sci. Technol. **70**, 1223–1231 (2010). doi:10.1016/j.compscitech.2010.03.010
28. Mishra, A., Naik, N.K.: Inverse micromechanical models for compressive strength of unidirectional composites. J. Compos. Mater. **43**, 1199–1211 (2009). doi:10.1177/0021998308104133
29. Martinez, X., Oller, S.: Numerical simulation of matrix reinforced composite materials subjected to compression loads. Arch. Comput. Methods Eng. (2009). doi:10.1007/s11831-009-9036-3
30. Ng, W.H., Salvi, A.G., Waas, A.M.: Characterization of the in-situ non-linear shear response of laminated fiber-reinforced composites. Compos. Sci. Technol. **70**, 1126–1134 (2010). doi:10.1016/j.compscitech.2010.02.024
31. Totry, E., Molina-Aldareguía, J.M., González, C., LLorca, J.: Effect of fiber, matrix and interface properties on the in-plane shear deformation of carbon-fiber reinforced composites. Compos. Sci. Technol. **70**, 970–980 (2010). doi:10.1016/j.compscitech.2010.02.014
32. Whitney, J.M., Pagano, N.J.: Shear deformation in heterogeneous anisotropic plates. J. Appl. Mech. **37**, 1031–1036 (1970). doi:10.1115/1.3408654
33. Rohwer, K.: Improved transverse shear stiffness for layered finite elements. DFVLR-Forschungsbericht 88-32, Braunschweig, (1988)
34. Altenbach, H.: An alternative determination of transverse shear stiffnesses for sandwich and laminated plates. Int. J. Solids Struct. **37**, 3503–3520 (2000)

35. Schürg, M., Wagner, W., Gruttmann, F.: An enhanced FSDT model for the calculation of interlaminar shear stresses in composite plate structures. Comput. Mech. **44**, 765–776 (2009). doi:10.1007/s00466-009-0410-7
36. Murthy, M.V.V.: An improved transverse shear deformation theory for laminated anisotropic plates. NASA Technical Paper 1903, Hampton VA (1981)
37. Reddy, J.N.: A simple higher-order theory for laminated composite plates. J. Appl. Mech. **51**, 745–752 (1984). doi:10.1115/1.3167719
38. Reissner, E.: On transverse bending of plates, including the effect of transverse shear deformation. Int. J. Solids Struct. **11**, 569–573 (1975). doi:10.1016/0020-7683(75)90030-X
39. Lo, K.H., Christensen, R.M., Wu, E.M.: A high-order theory of plate deformation, parts 1 and 2. J. Appl. Mech. **44**, 663676 (1977). doi:10.1115/1.3424154 and 10.1115/1.3424155
40. Wu, Ch.-P., Tarn, J.-Q., Chen, P.-Y.: Refined asymptotic theory of doubly curved laminated shells. J. Eng. Mech. **123**, 1238–1246 (1997). doi:10.1061/(ASCE)0733-9399(1997)123:12(1238)
41. Noor, A.K., Burton, W.S.: Assessment of computational models for multilayered composite shells. Appl. Mech. Rev. **43**, 67–96 (1990). doi:10.1115/1.3119162
42. Rohwer, K.: Application of higher order theories to the bending analysis of layered composite plates. Int. J. Solids Struct. **29**, 105–119 (1992). doi:10.1016/0020-7683(92)90099-F
43. Yang, H.T.Y., Saigal, S., Masud, A., Kapania, R.K.: A survey of recent shell finite elements. Int. J. Numer. Methods Eng. **47**, 101–127 (2000). doi:10.1002/(SICI)1097-0207(20000110/30)47:1/3<101::AID-NME763>3.0.CO;2-C
44. Timarci, T., Aydogdu, M.: Buckling of symmetric cross-ply square plates with various boundary conditions. Compos. Struct. **68**, 381–389 (2005). doi:10.1016/j.compstruct.2004.04.003
45. Carrera, E., Miglioretti, F., Petrolo, M.: Accuracy of refined finite elements for laminated plate analysis. Compos. Struct. **93**, 1311–1327 (2011). doi:10.1016/j.compstruct.2010.11.007
46. Rohwer, K., Rolfes, R., Sparr, H.: Higher-order theories for thermal stresses in layered plates. Int. J. Solids Struct. **28**, 3673–3687 (2001)
47. Pagano, N.J., Hatfield, S.J.: Elastic behavior of multilayered bidirectional composites. AIAA-J. **10**, 931–933 (1972)
48. Sun, C.-T., Whitney, J.M.: Theories for the dynamic response of laminated plates. AIAA-J. **11**, 178–183 (1973)
49. Di Scuiva, M.: A refinement of the transverse shear deformation theory for multilayered anisotropic plates. Atti del Dipartimento di Ingegneria Aeronautica e Spaziale del Politecnico di Torino, Publication, No. 5 (1983)
50. Lee, K.H., Senthilnathan, N.R., Lim, S.P., Chow, S.T.: An improved zig-zag model for the bending of laminated composite plates. Compos. Struct. **15**, 137–148 (1990). doi:10.1016/0263-8223(90)90003-W
51. Savithri, S., Varadan, T.K.: Accurate bending analysis of laminated orthotropic plates. AIAA-J. **28**, 1842–1844 (1990)
52. Rohwer, K., Friedrichs, S., Wehmeyer, C.: Analyzing laminated structures from fibre-reinforced composite material—An assessment. Tech. Mechanik **25**, 59–79 (2005)
53. Lo, S.H., Zhen, W., Sze, K.Y., Wanji, C.: $C^0$-type global–local theory with non-zero normal strain for the analysis of thick multilayer composite plates. Comput. Mech. **47**, 479–491 (2011). doi:10.1007/s00466-010-0554-5
54. Murakami, H.: Laminated composite plate theory with improved in-plane responses. J. Appl. Mech. **53**, 661–666 (1986). doi:10.1115/1.3171828
55. Di, S., Ramm, E.: Hybrid stress formulation for higher-order theory of laminated shell analysis. Comput. Methods Appl. Mech. Eng. **109**, 359–376 (1993). doi:10.1016/0045-7825(93)90087-E

56. Brank, B., Carrera, E.: Multilayered shell finite element with interlaminar continuous shear stresses: A refinement of the Reissner-Mindlin formulation. Int. J. Numer. Methods Eng. **48**, 843–874 (2000). doi:10.1002/(SICI)1097-0207(20000630)48:6<843::AID-NME903>3.0.CO;2-E
57. Icardi, U., Ferrero, L.: Multilayered shell model with variable representation of displacements across the thickness. Composites: Part B **42**, 18–26 (2011). doi:10.1016/j.compositesb.2010.09.022
58. Carrera, E.: Theories and finite elements for multilayered plates and shells: a unified compact formulation with numerical assessment and benchmarking. Arch. Comput. Methods Eng. **10**(3), 215–296 (2003)
59. Pryor, C.W., Barker, R.M.: A finite element analysis including transverse shear effects for application to laminated plates. AIAA-J. **9**, 912–917 (1971)
60. Lo, K.H., Christensen, R.M., Wu, E.M.: Stress solution determination for high order plate theory. Int. J. Solids Struct. **14**, 655–662 (1978). doi:10.1016/0020-7683(78)90004-5
61. Engblom, J.J., Ochoa, O.O.: Through-the-thickness stress predictions for laminated plates of advanced composite materials. Int. J. Numer. Methods Eng. **21**, 1759–1776 (1985). doi:10.1002/nme.1620211003
62. Rolfes, R., Rohwer, K.: Improved transverse shear stresses in composite finite elements based on first order shear deformation theory. Int. J. Numer. Methods Eng. **40,** 51–60 (1997). doi:10.1002/(SICI)1097-0207(19970115)40:1<51::AID-NME49>3.0.CO;2-3
63. Noor, A.K., Burton, W.S., Peters, J.M.: Predictor-corrector procedures for stress and free vibration analyses of multilayered composite plates and shells. Comput. Methods Appl. Mech. Eng. **82**, 341–363 (1990). doi:10.1016/0045-7825(90)90171-H
64. Noor, A.K., Malik, M.: An assessment of five modelling approaches for thermo-mechanical stress analysis of laminated composite panels. Comput. Mech. **25**, 43–58 (2000). doi:10.1007/s004660050014
65. Park, J.W., Kim, Y.H.: Re-analysis procedure for laminated plates using FSDT finite element model. Comput. Mech. **29**, 226–242 (2002). doi:10.1007/s00466-002-0336-9
66. Park, J.W., Lee, K.C., Kim, Y.H.: Comparative study of finite element based response evaluation methods for laminated plates. Comput. Mech. **32**, 115–133 (2003). doi:10.1007/s00466-003-0466-8
67. Guiamatsia, I.: A new approach to plate theory based on through-thickness homogenization. Int. J. Numer. Meth. Eng. **84**, 1139–1165 (2010). doi:10.1002/nme.2934
68. Kreja, I.: A literature review on computational models for laminated composite and sandwich panels. Cent. Eur. J. Eng. **1**(1), 59–80 (2011). doi:10.2478/s13531-011-0005-x
69. Kolpakov, A.A., Kolpakov, A.G.: Solution of the laminated plate design problem: new problems and algorithms. Comput. Struct. **83**, 964–975 (2005). doi:10.1016/j.compstruc.2004.08.012
70. Evans, J.T., Gibson, A.G.: Composite angle ply laminates and netting analysis. Proc. R Soc. London A **458**, 3079–3088 (2002)
71. Tan, S.C.: Stress Concentrations in Laminated Composites. Technomic Publishing Co, Lancaster (1994)
72. Tsai, S.W.: Composites design. Think Composites, Dayton, (1988)
73. Anonymous: http://www.esacomp.com (2011). Accessed 20 July 2011
74. Vannucci, P., Barsotti, R., Bennati, S.: Exact optimal flexural design of laminates. Compos. Struct. **90**, 337–345 (2009). doi:10.1016/j.compstruct.2009.03.017
75. Lopes, C.S., Seresta, O., Coquet, Y., Gürdal, Z., Camanho, P.P., Thuis, B.: Low-velocity impact damage on dispersed stacking sequence laminates. Part I: Experiments. Compos. Sci. Technol. **69**, 926–936 (2009). doi:10.1016/j.compscitech.2009.02.009
76. Lopes, C.S., Camanho, P.P., Gürdal, Z., Maimí, P., Gonzálezet, E.V.: Low-velocity impact damage on dispersed stacking sequence laminates. Part II: Numerical simulations. Compos. Sci. Technol. **69**, 937–947 (2009). doi:10.1016/j.compscitech.2009.02.015

77. Eschenauer, H.A.: The ‘three columns’ for treating problems in optimum structural design. In: Bergmann, H.W. (ed.) Optimization: Methods and Applications, Possibilities and Limitations. Lecture Notes in Engineering 47, Springer-Verlag Berlin, Heidelberg, (1989)
78. Zimmermann, R.: Optimierung axial gedrückter CFK-Zylinderschalen. Dissertation, Universität-Gesamthochschule Siegen (1991)
79. Ghiasi, H., Pasini, D., Lessard, L.: Optimum stacking sequence design of composite materials Part I: Constant stiffness design. Compos. Struct. **90**, 1–11 (2009). doi:10.1016/j.compstruct.2009.01.006
80. Javidrad, F., Nouri, R.: A simulated annealing method for design of laminates with required stiffness properties. Compos. Struct. **93**, 1127–1135 (2011). doi:10.1016/j.compstruct.2010.10.011
81. Akbulut, M., Sonmez, F.O.: Design optimization of laminated composites using a new variant of simulated annealing. Comput. Struct. **89**, 1712–1724 (2011). doi:10.1016/j.compstruc.2011.04.007
82. Gou, S., Cheng, W., Cui, D.: Aeroelastic tailoring of composite wing structures by laminate layup optimization. AIAA-J. **44**, 3146–3149 (2006)
83. Lopez, R.H., Luersen, M.A., Cursiet, E.S.: Optimization of laminated composites considering different failure criteria. Composites: Part B **40,** 731–740 (2009). doi:10.1016/j.compositesb.2009.05.007
84. Azarafza, R., Khalili, S.M.R., Jafari, A.A., Davar, A.: Analysis and optimization of laminated composite circular cylindrical shell subjected to compressive axial and transverse transient dynamic loads. Thin-Walled Struct. **47**, 970–983 (2009). doi:10.1016/j.tws.2009.01.004
85. Blom, A.W., Stickler, P.B., Gürdalet, Z.: Optimization of a composite cylinder under bending by tailoring stiffness properties in circumferential direction. Composites: Part B **41**, 157–165 (2010). doi:10.1016/j.compositesb.2009.10.004
86. Almeida, F.S., Awruch, A.M.: Design optimization of composite laminated structures using genetic algorithms and finite element analysis. Compos. Struct. **88**, 443–454 (2009). doi:10.1016/j.compstruct.2008.05.004
87. Johansen, L., Lund, E.: Optimization of laminated composite structures using delamination criteria and hierarchical models. Struct. Multi. Optim. **38**, 357–375 (2009). doi:10.1007/s00158-008-0280-1
88. Gillet, A., Francescato, P., Saffre, P.: Single- and multi-objective optimization of composite structures: the influence of design variables. J. Compos. Mater. **44**, 457–480 (2010). doi:10.1177/0021998309344931
89. Peng, W., Chen, J., Wei, J., Tu, W.: Optimal strength design for fiber-metal laminates and fiber-reinforced plastic laminates. J. Compos. Mater. **45**, 237–254 (2011). doi:10.1177/0021998310373521
90. Ghiasi, H., Fayazbakhsh, K., Pasini, D., Lessard, L.: Optimum stacking sequence design of composite materials Part II: Variable stiffness design. Compos. Struct. **93**, 1–13 (2010). doi:10.1016/j.compstruct.2010.06.001
91. Ottino, J.M.: Engineering complex systems. Nature **427**, 399 (2004)
92. Marczyk, J. Future trends in computer-aided engineering. In: Proceedings NAFEMS World Congress, Crete (2009)
93. Lee, M.C.W., Mikulik, Z., Kelly, D.W., Thomson, R.S., Degenhardt, R.: Robust design—A concept for imperfection insensitive composite structures. Compos. Struct. **92**, 1469–1477 (2010). doi:10.1016/j.compstruct.2009.09.054
94. Nahas, M.N.: Survey of failure and post-failure theories of laminated fiber-reinforced composites. J. Compos. Technol. Res. **8**, 138–153 (1986)
95. Hinton, M.J., Kaddour, A.S., Soden, P.D. (eds.): Failure Criteria in Fibre Reinforced Polymer Composites: The World-Wide Failure Exercise. Elsevier, Amsterdam (2004)
96. Luccioni, B.M.: Constitutive model for fiber-reinforced composite laminates. J. Appl. Mech. **73**, 901–910 (2006). doi:10.1115/1.2200654

97. Pinho, S.T., Davila, C.G., Camanho, P.P., Iannucci, L., Robinson, P.: Failure models and criteria for FRP under in-plane or three-dimensional stress states including shear non-linearity. NASA/TM-2005-213530, Hampton (2005)
98. Cuntze, R.: Efficient 3D and 2D failure conditions for UD laminae and their application within the verification of the laminate design. Compos. Sci. Technol. **66**, 1081–1096 (2006). doi:10.1016/j.compscitech.2004.12.046
99. Stamblewski, C., Sankar, B.V., Zenkert, D.: Analysis of three-dimensional quadratic failure criteria for thick composites using the direct micromechanics method. J. Compos. Mater. **42**, 635–654 (2008). doi:10.1177/0021998307088609
100. Lee, J., Soutis, C.: A study on the compressive strength of thick carbon fibre–epoxy laminates. Compos. Sci. Technol. **67**, 2015–2026 (2007). doi:10.1016/j.compscitech.2006.12.001
101. Knops, M., Bögle, C.: Gradual failure in fibre/polymer laminates. Compos. Sci. Technol. **66**, 616–625 (2006). doi:10.1016/j.compscitech.2005.07.044
102. Maimi, P., Camanho, P.P., Mayugo, J.-A., Davila, C.G.: A thermodynamically consistent damage model for advanced composites. NASA/TM -2006-214282, Hampton, VA (2006)
103. Cui, H.-P., Wenb, W.-D., Cui, H.-T.: An integrated method for predicting damage and residual tensile strength of composite laminates under low velocity impact. Comput. Struct. **87**, 456–466 (2009). doi:10.1016/j.compstruc.2009.01.006
104. Basu, S., Waas, A.M., Ambur, D.R.: Computational modeling of damage growth in composite laminates. AIAA-J. **41**, 1158–1166 (2003)
105. Ha, S.K., Huang, Y., Han, H.H., Jinet, K.K.: Micromechanics of failure for ultimate strength predictions of composite laminates. J. Compos. Mater. **44**(20), 2347–2361 (2010). doi:10.1177/0021998310372464
106. Zhang, Y.X., Zhang, H.S.: Multiscale finite element modeling of failure process of composite laminates. Compos. Struct. **92**, 2159–2165 (2010). doi:10.1016/j.compstruct.2009.09.031
107. Garnich, M.R., Akula, V.M.K.: Review of degradation models for progressive failure analysis for fiber reinforced polymer composites. Appl. Mech. Rev. **62**, 1–33 (2009). doi:10.1115/1.3013822
108. Rybicki, E.F., Kanninen, M.F.: A finite element calculation of stress intensity factor by a modified crack closure integral. Eng. Fract. Mech. **9**, 931–938 (1977). doi:10.1016/0013-7944(77)90013-3
109. Riccio, A., Gigliotti, M.: A novel numerical delamination growth initiation approach for the preliminary design of damage tolerant composite structures. J. Compos. Mater. **41**, 1939–1960 (2007). doi:10.1177/0021998307069908
110. Alfano, G., Crisfield, M.A.: Solution strategies for the delamination analysis based on a combination of local-control arc-length and line searches. Int. J. Numer. Methods Eng. **58**, 999–1048 (2003). doi:10.1002/nme.806
111. Xiao, J.R., Gillespie Jr, J.W.: A phenomenological Mohr–Coulomb failure criterion for composite laminates under interlaminar shear and compression. J. Compos. Mater. **41**, 1295–1309 (2007). doi:10.1177/0021998306067318
112. Davidson, B.D., Zhao, W.: An accurate mixed-mode delamination failure criterion for laminated fibrous composites requiring limited experimental input. J. Compos Mater **41**, 679–702 (2007). doi:10.1177/0021998306071031
113. Wang, X., Lu, G.: Local buckling of composite laminar plates with various delaminated shapes. Thin-Walled Struct. **41**, 493–506 (2003). doi:10.1016/S0263-8231(03)00020-X
114. Pekbey, Y., Sayman, O.: A numerical and experimental investigation of critical buckling load of rectangular laminated composite plates with strip delamination. J. Reinf. Plast. Compos. **25**, 685–697 (2006). doi:10.1177/0731684406060566
115. Lee, S.-Y., Park, D.-Y.: Buckling analysis of laminated composite plates containing delaminations using the enhanced assumed strain solid element. Int. J. Solids Struct. **44**, 8006–8027 (2007). doi:10.1016/j.ijsolstr.2007.05.023

116. Parlapalli, M.S.R., Shu, D., Chai, G.B.: Buckling of composite beams with two enveloped delaminations: lower and upper bounds. Comput. Struct. **86**, 2155–2165 (2008). doi:10.1016/j.compstruc.2008.06.008
117. Tafreshi, A.: Instability of delaminated composite cylindrical shells under combined axial compression and bending. Compos. Struct. **82**, 422–433 (2008). doi:10.1016/j.compstruct.2007.01.021
118. Aslan, Z., Sahin, M.: Buckling behavior and compressive failure of composite laminates containing multiple large delaminations. Compos. Struct. **89**, 382–390 (2008). doi:10.1016/j.compstruct.2008.08.011
119. Kremer, T., Schürmann, H.: Buckling of tension-loaded thin-walled composite plates with cut-outs. Compos. Sci. Technol. **68**, 90–97 (2008). doi:10.1016/j.compscitech.2007.05.035
120. Fleming, D.C.: Delamination modeling of composites for improved crash analysis. J. Compos. Mater. **35**, 1777–1792 (2001). doi:10.1106/3V9W-9099-HYQ3-08GR
121. Iannucci, L.: Dynamic delamination modelling using interface elements. Comput. Struct. **84**, 1029–1048 (2006). doi:10.1016/j.compstruc.2006.02.002
122. Williams, K.V., Vaziri, R., Poursartip, A.: A physically based continuum damage mechanics model for thin laminated composite structures. Int. J. Solids Struct. **40**, 2267–2300 (2003). doi:10.1016/S0020-7683(03)00016-7
123. Iannucci, L., Ankersen, J.: An energy based damage model for thin laminated composites. Compos. Sci. Technol. **66**, 934–951 (2006). doi:10.1016/j.compscitech.2005.07.033
124. Talreja, R.: Fatigue of composite materials. Dissertation, Technical University of Denmark, Lyngby, (1985)
125. Harris, B. (ed.): Fatigue in Composite. Woodhead Publishing Ltd, Abington Cambridge (2003)
126. Marco, S., Starkey, W.L.: A concept of fatigue damage. Trans. ASME **76**, 627–632 (1954)
127. Yang, J.N., Liu, M.D.: Residual strength degradation model and theory of periodic proof tests for graphite/epoxy laminates. J. Compos. Mater. **11**, 176–204 (1977). doi:10.1177/002199837701100205
128. Cerny, I., Mayer, R.M.: Evaluation of static and fatigue strength of long fiber GRP composite material considering moisture effects. Compos. Struct. **92**, 2035–2038 (2010). doi:10.1016/j.compstruct.2009.11.024
129. Shokrieh, M.M., Taheri-Behrooz, F.: Progressive fatigue damage modeling of cross-ply laminates, I: Modeling Strategy. J. Compos. Mater. **44**, 1217–1231 (2010). doi:10.1177/0021998309351604
130. Taheri-Behrooz, F., Shokrieh, M.M., Lessard, L.B.: Progressive fatigue damage modeling of cross-ply laminates, II: Experimental evaluation. J. Compos. Mater. **44**, 1261–1277 (2010). doi:10.1177/0021998309351605
131. May, M., Hallett, S.R.: An advanced model for initiation and propagation of damage under fatigue loading—Part I: Model formulation. Compos. Struct. **93**, 2340–2349 (2011). doi:10.1016/j.compstruct.2011.03.022
132. May, M., Pullin, R., Eaton, M., Featherston, C., Hallettet, S.R.: An advanced model for initiation and propagation of damage under fatigue loading—Part II: Matrix cracking validation cases. Compos. Struct. **93**, 2350–2357 (2011). doi:10.1016/j.compstruct.2011.03.023
133. Quaresimin, M., Susmel, L., Talreja, R.: Fatigue behaviour and life assessment of composite laminates under multiaxial loadings. Int. J. Fatigue **32**, 2–16 (2010). doi:10.1016/j.ijfatigue.2009.02.012
134. Hancock, S.G., Potter, K.D.: Inverse drape modelling—an investigation of the set of shapes that can be formed from continuous aligned woven fibre reinforcements. Compos. Part A **36**, 947–953 (2005). doi:10.1016/j.compositesa.2004.12.001
135. Hancock, S.G., Potter, K.D.: The use of kinematic drape modelling to inform the hand lay-up of complex composite components using woven reinforcements. Compos. Part A **37**, 413–422 (2006). doi:10.1016/j.compositesa.2005.05.044

136. Vanclooster, K., Lomov, S.V., Verpoest, I.: Experimental validation of forming simulations of fabric reinforced polymers using an unsymmetrical mould configuration. Composites: Part A **40**, 530–539 (2009). doi:10.1016/j.compositesa.2009.02.005
137. Dassault Systems: http://www.3ds.com (2011). Accessed 7 Aug 2011
138. Simulayt: http://www.simulayt.com (2011). Accessed 7 Aug 2011
139. Vistagy http://www.vistagy.com (2011). Accessed 7 Aug 2011
140. Anaglyph: http://www.anaglyph.co.uk (2011). Accessed 7 Aug 2011
141. Minaie, B., Chen, Y.F.: Adaptive control of filling pattern in resin transfer molding process. J. Compos. Mater. **39**, 1497–1513 (2005). doi:10.1177/0021998305051082
142. Johnson, R.J., Pitchumani, R.: Flow control using localized induction heating in a VARTM process. Compos. Sci. Technol. **67**, 669–684 (2007). doi:10.1016/j.compscitech.2006.04.012
143. García, J.A., Gascón, L., Cueto, E., Ordeig, I., Chinesta, F.: Meshless methods with application to liquid composite molding simulation. Comput. Methods Appl. Mech. Eng. **198**, 2700–2709 (2009). doi:10.1016/j.cma.2009.03.010
144. Dong, C.: A modified rule of mixture for the vacuum-assisted resin transfer moulding process simulation. Compos. Sci. Technol. **68**, 2125–2133 (2008). doi:10.1016/j.compscitech.2008.03.019
145. Polyworx: http://www.polyworx.com (2011). Accessed 7 Aug 2011
146. ESI: http://www.esi-group.com (2011). Accessed 7 Aug 2011
147. Darrow Jr, A.D., Smith, L.V.: Isolating components of processing induced warpage in laminated composites. J. Compos. Mater. **36**, 2407–2419 (2002). doi:10.1177/0021998302036021784
148. Cheung, A., Yu, Y., Pochiraju, K.: Three-dimensional finite element simulation of curing of polymer composites. Finite Elem. Anal. Des. **40**, 895–912 (2004). doi:10.1016/S0168-874X(03)00119-7
149. Sweeting, R., Liu, X.L., Paton, R.: Prediction of processing-induced distortion of curved flanged composite laminates. J. Compos. Struct. **57**, 79–84 (2002)
150. Svanberg, J.M.: Predictions of manufacturing induced shape distortions—high performance thermoset composites. Dissertation, Lulea University of Technology, Sweden, (2002)
151. Spröwitz, T., Tessmer, J., Wille, T.: Process simulation in fiber-composite manufacturing – spring-in. In: NAFEMS Seminar: Simulating Composite Materials and Structures, Bad Kissingen, Germany, (2007)
152. Brauner, C., Block, T.B., Hoffmeister, C., Herrmann, A.S.: Process simulation of carbon fibre/epoxy composites on the micro level to analyse chemical and thermal induced residual stresses. In: NAFEMS Seminar: Progress in Simulating Composites, Wiesbaden, Germany 2011