Chapter 2
General Framework

Abstract. High complexity imposes systems' structuration in multiple levels.

Associating n-categories to levels offers a new perspective for modeling multi-level systems.

The comprehensive framework of polystochastic models, PSM, serving as flexible guideline for self-evolvable systems modeling is introduced.

Polytopic architectures for self-integrative closure and polycategories are presented.

2.1 Categories and Closure

Fundamental studies of closure concern the multi-level structure of reality and its relation to philosophical categories and mathematical categories (Peirce 1956, Hartmann 1952, Poli 2001, Brier 2008, 2009).

Complex systems exhibit hierarchical self-organization in levels under selective constraints. Self-organization will occur when individual independent parts in a complex system interact in a jointly cooperative manner that is also individually appropriate, such as to generate a higher level organization.

Complex systems have a multi-level architecture and this can be observed at different levels of investigation. For example, we can observe an industrial installation at the level of molecules, at the level of devices interactions or as embedded in its environment. The number of observation levels as that of reality levels is finite.

The concept of closure plays a relevant role in biological explanations since it is taken as a naturalized grounding for distinctive biological dimensions, as purposefulness, normativity and functionality. The contemporary application of closure to the biological domain comes from a philosophical tradition tracing back at least to Kant who claimed that living systems should be understood as natural ends, that is as self-organized structures driven by circular and reciprocal causation. The essence of living systems is a form of internal and circular causality between the whole and the parts, distinct from both efficient causality of the physical world and the final causality of artifacts or mechanisms (Kant 1987).

In general systems theory the concept of closure is used to identify or define the system relation with its environment and to explain the autonomy of the systems. Closure and circularity are critical for self-evolvability understanding and managing.

Closure does not mean that the system is not in contact with its environment. Rather the term closure refers to the loop that connects the structures and the functions of individual living-like entities.

It should be emphasized the complementary roles of the closure and opening or disclosure for self-evolvability. The swinging between closure and openness is an important tool for designing creative systems that can autonomously find solutions to highly complex construction problems.

Different closure concepts as organizational closure (Maturana and Varela 1980), closure to efficient cause (Rosen 1991), operational closure (Luhmann 1995), and semantic closure (Pattee 1995, 2001) are important for self-evolvability studies.

According to Pattee, biological organization consists of the integration of two intertwined dimensions, which cannot be understood separately. On the one side, the organization realizes a dynamic and autopoietic network of mechanisms and processes, which defines itself as a topological unit, structurally coupled with the environment. On the other side, it is shaped by the material unfolding of a set of symbolic instructions, stored and transmitted as genetic information.

The dynamic, that is, mechanistic and the informational dimensions realize a distinct form of closure between them, which Pattee labels semantic or semiotic closure. This concept refers to the fact that while symbolic information must be interpreted by the dynamics and mechanisms that it constrains, the mechanisms in charge of the interpretation and the material translation require that information for their own production. Semantic closure, as an interweaving between dynamics and information, constitutes an additional dimension of organizational closure of biological systems, complementary to the operational or efficient one.

In *On a New List of Categories*, Peirce formulates a theory of categories that can demonstrate what the universal conceptions of reality and of thought are.

Peirce's categories are meant to provide a basis for an exploration of a large variety of phenomena, including natural, biological, reasoning and technological.

Peirce proposed an initial list of five philosophical categories: *substance*, *quality*, *relation*, *representation* and *being*.

Later, Peirce discarded substance and being from his initial list of five categories and focused mainly on quality, relation and representation which he called in his technical terms *firstness*, *secondness* and *thirdness*, respectively.

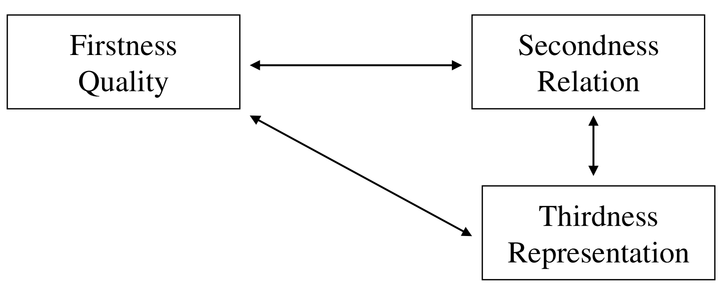

Fig. 2.1 Triadic approach

The triadic architecture of Peirce is illustrated in Fig. 2.1.

Firstness is the conception of existing independent of anything else. Secondness is the conception of existing relative to, the conception of reaction with something else. Thirdness is the conception of mediation, whereby a first and second are brought into relation.

Firstness may be manifested by quality, feeling or multiplicity. Secondness may be manifested by relation, action, reaction, causality, or actuality. Thirdness may be manifested by representation, modality, thought, continuity, unity, or generality.

A triadic approach to biosemiotics has been discussed by Pattee who investigated the physical conditions that are necessary for codes and symbolic controls (Pattee 2001). Pattee introduced the concept of epistemic threshold, the boundary region where local matter has not only its intrinsic physical properties governed by universal laws, but was also about something else. Epistemic matter, in other words, stands for something and the standing for relation is usually considered an emergent process that leads to a triadic Peircean relationship of matter, interpreter and referent.

Studies of emergence, embedding and self-properties for systems of growing complexity suggest reconsidering the Peirce's initial list of five categories.

The categories substance and being have been described by Peirce, as the beginning and end of all conception, respectively. Taking into account the evolution during the years of Peirce's concepts of substance and being a categorical architecture with five levels of reality: substance, firstness, secondness and thirdness, centered by the so-called fourthness identified also as being, or in other context as the Self, was considered (Iordache 2011).

Fig. 2.2 shows a polytopic representation of the Peirce's initial list of five categories and links to the associated Kantian categories, quantity, quality, relation and modality.

This architecture shows being or the Self as a centering category surrounded by substance, firstness, secondness and thirdness. The being considered in its aspect of meta-representation of the four surrounding levels is the key capability for self-evolvability.

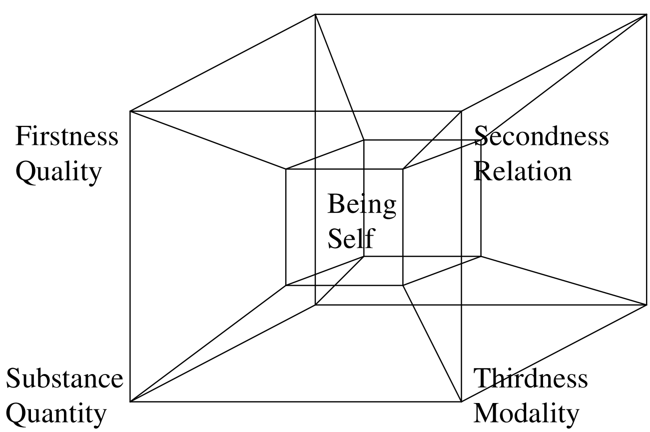

Fig. 2.2 Polytope for five categories

Peirce suggested that cognitive investigation could continue beyond the thirdness. For instance, following the study of existential graphs alpha, α, beta, β, or gamma γ, associated to firstness, secondness and thirdness, Peirce started the study of δ systems, supposed to deal with modals, that is beyond modality (Pietarinen 2003, 2006).

There exist several developments of the Peirce's triadic architectures to tetradic architectures and beyond. An interesting study is the so-called reasoning cycle of Peirce (Sowa 2006). In this case the four modules of the cognitive architecture are: World, Knowledge, Theory and Prediction. The interactions between categories as interpreted by Sowa are induction, abduction, deduction and action. Fig. 2.3 shows a polytope inspired by Peirce's cycle of cognition.

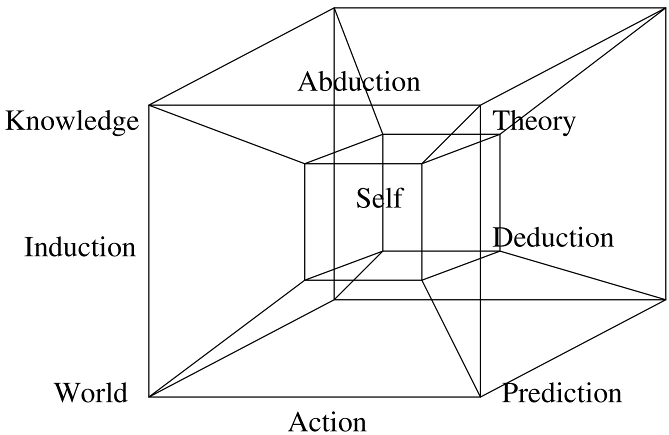

Fig. 2.3 Cognition polytope

Induction or learning starts from observations and looks for commonalities to summarize observed data. Abduction or conjecturing starts with disconnected observations and hypothesizes a theory that relates them. Deduction or inference starts with a theory, observes new data and is used to generate implications.

Staat ascribed the Peirce's categories, firstness, secondness, thirdness to abduction deduction and induction (Staat 1993). Fig. 2.3 shows that abduction is rooted in Knowledge, and deduction is rooted in Theory. If we consider Prediction, and World, together as the category thirdness, the induction will appear to be rooted here.

Taking inspiration from Peirce's philosophy, Brier formulated a transdisciplinary theory of information, semiotics, consciousness and cultural social communication illustrated by the fourfold cybersemiotic star (Brier 2008). Fig. 2.4 shows a polytope based on the Brier's cybersemiotic star.

The four legs correspond to the four main areas of knowledge that is: Material, Living, Consciousness and Mentality. A comparison with the Hartmann's ontological hierarchy is of interest (Hartmann 1952).

The center of the cybersemiotic star was related by Brier to semiotic mind. It may be considered as a meta-representation of the fourfold star and it is linked to self-evolvability.

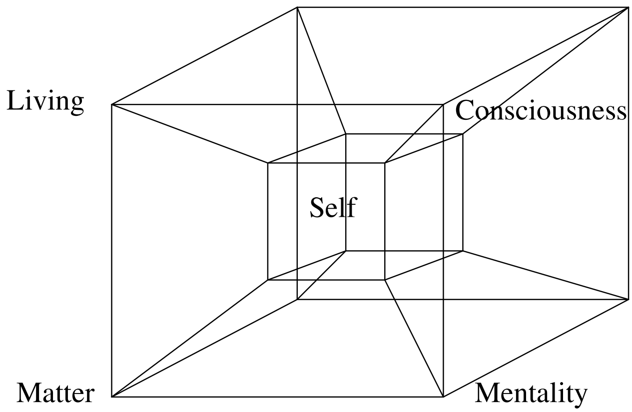

Fig. 2.4 Cybersemiotic polytope

This central category, the Self, is crucial for the transition from evolvable systems to self-evolvable ones. Briers' cybersemiotic star may be interpreted as a 2D-projection of a more general cybersemiotic polytope.

2.2 General PSM Framework

Polystochastic models, PSM, study started by considering complex systems to be compound processes organized hierarchically in levels as arrays of systems within systems (Iordache 1987).

The PSM is developed now as a modeling tool for high-level complexity, mainly for evolvable and self-evolvable systems investigation. The complexity was portrayed in PSM studies using concepts such as hierarchy and conditioning levels, real and formal or in other words non-standard time and probability algebraic frames, and by methods as categorification and integrative closure. Conventional methods, applied in specific ways, joined new ones resulting in a distinctive understanding of complexity (Iordache 2010).

The elements of basic PSM frame are quadruple of vectors [S, K, U, P] denoted also SKUP. The notations are: S-States, K-Conditions, U-Operators, and P-Possibilities.

Observe that the early SKUP framework involves only two levels or realms, S and K. The relation with random systems with complete connections theory is clearly identifiable (Iosifescu and Grigorescu 1990). But it should be emphasized that the two level architectures have limited efficiency for high complexity problems.

As for other approaches to complexity, it was assumed that the complexity can be managed through supplementary hierarchical layering. The elements of SKUP have been considered as vectors. The conditioning levels have been correlated to time and to space scales (Iordache 2011).

Each component of the vectors corresponds to a different conditioning level and a different time scale.

The basic elements of the SKUP have been considered as vectors:

$S = (s^0, s^1, \ldots, s^n, \ldots, s^M)$; $K = (k^0, k^1, \ldots, k^n, \ldots, k^M)$;
$U = (u^0, u^1, \ldots, u^n, \ldots, u^M)$; $P = (p^0, p^1, \ldots, p^n, \ldots, p^M)$.

Here s^n represents the particular state at the level n, and k^n represents the particular condition at the level $n \leq M$. Upper indices are reserved to levels, while lower indices are reserved to time steps. The components of U are operators such as: u^n: $k^n \times s^{n'} \rightarrow s^{n''}$

PSM should describe parallel evolutions. Moreover, S and K are associated to different types of algebraic fields. Despite algebraic framework differences, S and K are interconnected. This interconnection is described by operators U and possibilities P.

U characterizes the K to S transition and P characterizes the S to K transitions, that is:

$U: K \rightarrow S$ and $P: S \rightarrow K$.

Operators U should be able to describe change of conditioning level and splitting of levels. Possibilities P replacing and generalizing probabilities have been studied in game theory (Hammond 1994), in fuzzy logic (Dubois and Prade 2001), and in other domains.

The possibilities P may be defined by vectors such as:

$P (K) = (p (k^0), p (k^1), \ldots, p (k^m), \ldots, p (k^M))$.

The component $p (k^n)$ is an evaluation of the condition k^n.

An innovative aspect for PSM concerns the differential model for K defined process. The elements of K are resulting as solutions of differential equations (Iordache 2009, 2010).

These models have been used as generic models producing other models.

The last development stage for PSM, the categorical approach, appears as a categorification of stochastic transition systems for growing n-dimensional problems.

Categories are linked to the different levels of reality. The notion of level or reality which was firstly studied from an intuitive point of view may be approached from a more formal point of view based on category theory. In this case, the levels and sub-levels of reality are characterized and distinguished by their categories and sub-categories.

A category is specified by objects and arrows called also morphisms.

In numerous situations the physical systems are objects while the morphisms correspond to processes. For PSM frameworks the conditions K may represent the category describing the types of component processes. The process types are the objects of category. Interactions among types can be modeled as morphisms.

The arrows, that is the morphisms, describe the transition relations between the states of the component processes. Different algebraic frameworks for states-S (dynamical, analogical, and physical) and conditions-K (symbolic, digital, and formal) have been considered.

One can define a special category which has categories as objects. The morphism between two categories is then defined as functor. Functors as denoted by U are accounting for interactions in K, and between categories K and S. Other functors as the possibilities P supplement the probabilities to express potentiality, fuzziness, uncertainty, and emergence.

Advancements in modeling higher complexity, the evolvability request, required to take into account multiple levels and multiple SKUPs interaction.

Replacing K by several levels is mandatory for the study of higher complexity.

Instead of the category K, the categories K1, K2 and K3 have been considered as a preliminary development of SKUP (Iordache 2010).

Centered, four realms PSM frameworks, resulting by integrative closure, have been presented as the architecture shared by numerous autonomous systems (Iordache 2011).

The pursuit of self-evolvability for systems imposes the polytopic architecture associated to the general PSM framework as shown in Fig. 2.5.

The basic levels and categories are S, K1, K2 and K3 represented on the front face of the outer cube and S′, K1′, K2′ and K3′ represented on the back face of the outer cube.

The swinging between the two faces of the outer cube is mediated by the inner cube identified as the Self.

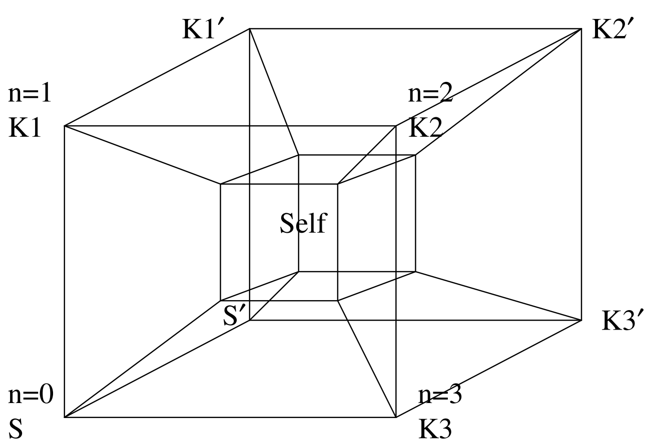

Fig. 2.5 Polytope for general PSM framework

Notable theoretical perspectives resembling the PSM approach are the memory evolutive systems (Ehresmann and Vanbremeersch 2007) and the hierarchical cycles (Louie and Poli 2011). These research directions rely on the power of category theory, too. They develop, in different ways, the idea of iterative constructions of systems over systems in which the system of different layers presents specific properties.

2.3 Self-Integrative Closure

Integrative closure appeared as the direct consequence of mutual restrictedness or exclusiveness of the new levels relative to the previous ones, and of the finite number of levels to be considered. Integrative closure approach is not looking for an identity between the philosophical and mathematical categorical viewpoints but for a structural analogy and a common methodology shared by different domains as knowledge organization, problem solving or technological developments (Iordache 2010).

The conventional hierarchical structures cannot serve as general models for multiple-level knowledge organization. Confronting higher complexity the task of knowledge integration remains pertinent.

The hierarchical structure should be closed and replaced by a network. Finally, it is generally acknowledged that both trees and cycles are necessary. It is the timing of activation and blend of both that matters.

Fig. 2.6 shows the polytope of categories and illustrates self-integrative closure concept.

Fig. 2.6 proposes an extended structural analogy that of the hypothetical integrative closure architecture including philosophical categories architectures as studied by Peirce (substance, firstness, secondness, thirdness and fourthness) and the mathematical n-categories (n=0, 1, 2, 3 and 4). Fig. 2.6 outlines the links to Kantian categories, quantity, quality, relation and modality.

We refer to this overarching framework as *self-integrative closure*.

Adoption of the n-categorical standpoint, suggested the initial extending the investigation to four levels or realms. Support for the four-level architectures is offered by different domains. A primary source is in data processing and neurodynamics (Cowan 2000). According to Cowan the capacity of short-term memory is limited to the number of four items to which attention can be simultaneously directed. There exists a central capacity limit of four chunks in short-term explicit memory presumably corresponding to the focus of attention. This theory assumes that attention is depending on oscillation of cortical potentials. A cortical wave of about 10 Hz is supposed to select items from a large short-term store. Other wavelets at a frequency at about 40 Hz then select one item each. Such considerations do not exclude to direct attention to more than four items or realms, but the resulting processes may be transient.

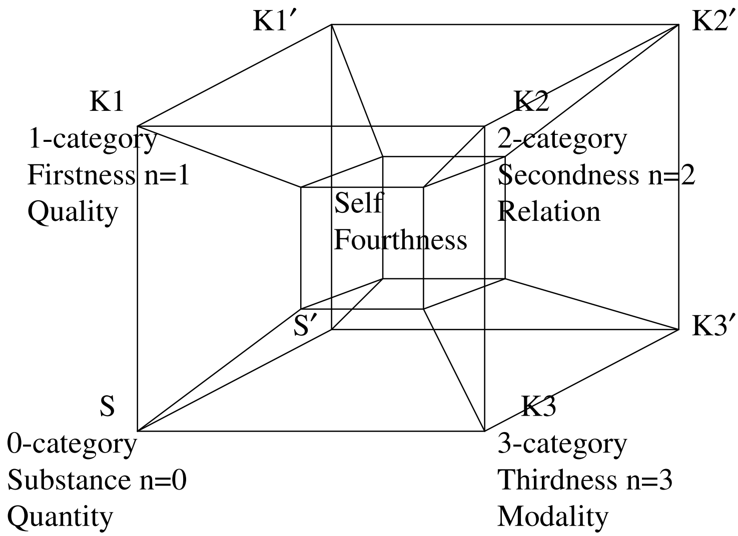

Fig. 2.6 Polytope for self-integrative closure

Significant support for the four levels or four categories architectures in data processing is given by mathematical category theory too (Leinster 2004). Apparently complexity of n-categories rises with n, dramatically. Developing the study of centered four levels means to include the categorical approach to 4-categories.

The four levels are associated in increasing order of complexity starting from 0-category that is from sets, to 1-category that is to conventional categories, to 2-categories, and then to 3-categories. The internal cube, the Self, is associated in this representation to n-categories with n≥4.

The difficulty to work with mathematical n-categories is that as the number of dimensions increases the complexity of the necessary rules to be specified increases rapidly. For one dimension the rules may be written down on one line, and those for two dimensions may be expressed in diagrams occupying a typical page. For four dimensions the detailed diagrams are so large that they will not fit in acceptable sized book. The 4-category diagram techniques just start to be developed. The difficulty of presentation was considered as a supplementary reason to restrict the majority of studies to 3-categories (Iordache 2010). However we cannot exclude 4-categories or higher ones in the long run. Clearly some other ways of approaching and presenting the theory should be envisaged.

The *self-integrative closure* is based on the hypothesis that there exists a structural correlation between philosophical and mathematical categorification architectures.

As shown in Fig. 2.6, the four levels of reality or the four philosophical categories of Kant or Peirce have been associated to the corresponding mathematical n-categories.

Philosophical categorification is the philosophical counterpart of categorification introduced in mathematics, but replacing logical concepts for categorical concepts, and also set-theoretic notions by category-theoretic notions in order to investigate concepts. The categories are attempts to distill the essence of a certain domain. This is also the goal of people working in that domain.

Category theory could serve as a *lingua franca* that lets us translate between certain aspects in different domains and eventually build a general science for complex systems and processes (Baez and Stay 2008).

Traditionally philosophical categories were not studied in terms of mathematical n-category theory.

The significance of the hypothetical structural analogy between categorical approach in philosophy and mathematics needs more study. The fundamental problem of categorification and decategorification was discussed by Kant (Kant 1987). Kant distinguished two ways of analysis, a qualitative one and a quantitative one. The first may be linked to the relation between conditioned and the condition that is, to the way S→K1→K2→K3 while the second corresponds to the way K3′→ K2′→K1′→S′, from the whole to the parts (Fig. 2.6).

The need for both epistemological ways finds a strong support in the studies of metastable coordination dynamics of the brain (Kelso 2002, Kelso and Tognoli 2009). Metastability has been highlighted as a new principle of behavioral and brain function and may point the way to a truly complementary neuroscience. From elementary coordination dynamics it was shown explicitly that metastability is a result of a symmetry-breaking caused by the subtle interplay of two forces: the tendency of the components to couple together and the tendency of the components to express their intrinsic independent behavior. The metastable regime reconciles the well-known tendencies of specialized brain regions to express their autonomy, that is differentiation, and the tendencies for those regions to work together as a synergy, that is integration.

Nevertheless, the architectural similarities between philosophical categories and mathematical category theory cannot be interpreted as a coincidence. Peirce, inspired by Kant, is acknowledged today as a precursor of higher-dimensional algebra and in this way of n-category study. So, it would be interesting to re-evaluate Peirce's work about categories, in terms of mathematical n-categories. This will relate n-categories to pragmatism as founded by Peirce. What may be called categorical pragmatism refers to Peirce's fundamental concern to discover the basic elements or principles essential in the process of inquiry, rather than to just formulate a criterion of truth by means of which the results of inquiry are to be judged for their truth value.

Table 2.1 summarizes the categorification aspects for PSM frameworks.

The study of PSM framework for self-integrative closure and the emergence of the Self corresponding to $n \geq 4$ represent a challenge from both conceptual and mathematical points of view.

Table 2.1 Categorification for PSM framework

Level	S (K0)	K1	K2	K3	Self
-	n=0	n=1	n=2	n=3	n=4
Categories	0-category	1-category	2-category	3-category	4-category
Example	sets	Set	Cat	Fun	-

For *n*-category theory, a category such as Set is a 1-category, with 0-objects that is sets, for objects and 1-morphisms, that is functions, for arrows (Appendix 1). A functor is the morphism between categories. Actually a functor between two categories is also defining as mapping objects and morphisms of one category to objects and morphisms of the other, in such a way that the morphism between two objects is mapped to the morphism between two mapped objects. Thus a functor appears as a transformation which keeps the basic structure. The category of categories, Cat, has categories for objects and functors for arrows. Thus, a functor is a 2-morphism between 1-objects, that is 1-categories, in a 2-category.

One can define a new category with functors as objects. A natural transformation is the morphism between functors. The general idea is to transform not only the underlying categories one into another but also to parameterize that transformation by any basic constituting element, translating the idea that a global transformation between complex architectures is made by local transformation with different levels of accuracy.

The functor category, Fun, has functors as objects and natural transformations as arrows. Thus, a natural transformation is a 3-morphism between 2-objects, that is functors, in a 3-category.

Using n-category theory we propose a unified framework that allows describing in a condensed manner the transformations allowed in the complex system.

The categorical approach highlights the possible transformations, given a structure of the system, and checks formally the analogy or the similarity between architectural organizations.

2.4 PSM and Polycategories

Since PSM aim to describe the composition of several processes they may be presented in the frame of polycategories (Appendix 4).

Polycategories were introduced by Lambek and Szabo with the intention of providing a categorical framework for classical logic, with multiple formulae on both left and rightsides of the sequent (Lambek 1969, Szabo 1975). Their composition law is based on the cut rule of logic.

A category allows having morphisms which go from single objects to single objects.

A polycategory allows having morphisms from lists of objects to lists of objects.

A typical morphism in a polycategory called also polymorphism or polymap would be denoted: f: $X_1, X_2, \ldots, X_n \to Y_1, Y_2, \ldots, Y_m$

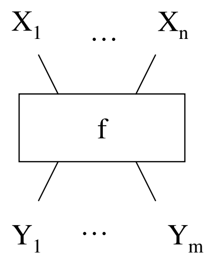

Fig. 2.7 Polymap

Fig. 2.7 shows a polymap. Here f denotes a process. The domain channels are on the top and the codomain channels on the bottom. A process acts on a number of channels by either accepting input events or producing output events in accordance with the rules or protocols associated to each channel.

Physically one could think of the channels as pipes for chemical engineering installations or wires for electrical circuits and so on.

If a codomain channel α of a process f, and a domain β of another process g, share a common protocol then f and g may be composed on α and β to form a new process.

Fig. 2.8 shows a composed process. The double lines conventionally represent strings of channels.

Such interpretations highlight the close relation between polycategories and polystochastic models, PSM, introduced as composed processes (Iordache 1987).

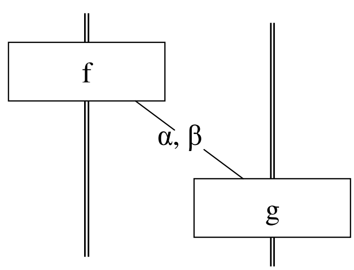

Fig. 2.8 Composed processes

There are several contexts in which the generalization related to the concept of polycategories would be useful.

As a first example consider vector spaces or any class of modules in which one can form a tensor product. A polycategory, having as objects such spaces, can be defined. The morphism of the above form may be a linear function:

$$f: X_1 \otimes X_2 \otimes \ldots \otimes X_n \rightarrow Y_1 \otimes Y_2 \otimes \ldots \otimes Y_m \tag{2.1}$$

Such polycategories have proven to be useful in the analysis of ordinary categories in which one can form tensor products of objects. Categories in which one has a suitable notion of tensor product are called monoidal.

Another significant application of polycategories is to logic. The interest is in the analysis of sequents, written: $X_1, X_2, \ldots, X_n \vdash Y_1, Y_2, \ldots, Y_m$

Here $\vdash$ denotes the implication and $X_1, X_2, \ldots, X_n, Y_1, Y_2, \ldots, Y_m$ may represent formulas in some logical system.

The above sequent holds if and only if the conjunction of $X_1, X_2, \ldots, X_n$ logically entails or implies the disjunction of $Y_1, Y_2, \ldots, Y_m$.

There is a correspondence between the sort of logical entailments considered here and categorical structures (Lambek and Scott 1986).

Notice the difference between this and the first example. When considering vector spaces, the commas on the left and rightsides were both interpreted as the tensor product. However for the logically inspired example, there are two different interpretations. Commas on the left are treated as conjunction, while commas on the right are treated as disjunction.

Thus for a categorical interpretation of polycategories one needs categories with two monoidal structures that interact in an appropriate fashion. Such categories are called linearly or weakly distributive (Cockett and Seely 1997) and they represent the appropriate framework for linear logic (Girard 1987)

Suppose we are given two polymorphisms of the following form:

$$f: X_1, X_2, \ldots, X_n \rightarrow Y_1, Y_2, \ldots, Y_m, C \tag{2.2}$$

$$g: C, Z_1, Z_2, \ldots, Z_k \rightarrow V_1, V_2, \ldots, V_j, C \tag{2.3}$$

The polymorphisms f and g are considered as processes in the PSM terminology.

Note the single object C common to the codomain of the process f and the domain of the process g. Then under the definition of polycategory, we can compose these to get a morphism of form:

$$g \circ f = X_1, X_2, \ldots, X_n, Z_1, Z_2, \ldots, Z_k \rightarrow Y_1, Y_2, \ldots, Y_m, V_1, V_2, \ldots, V_j \tag{2.4}$$

The object C which is eliminated after composition is called the cut object, a terminology derived from logic. The morphism g o f is a compound process. C controls this composition.

Fig. 2.9 illustrates the composition in polycategories.

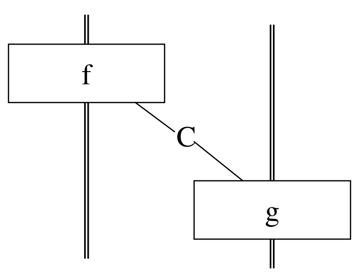

Fig. 2.9 Composition for polycategories

Composition is represented by the concatenation of the processes f and g followed by joining the incoming and outgoing edges corresponding to the cut object C.

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