Research on Method of Wavelet Function Selection to Vibration Signal Filtering

Xiangzhong Meng and Jianghong Wang

Abstract. The wavelet multi-resolution analysis is very suitable for the analysis and processing of vibration signal. During the vibration signal filtering, the different wavelet function had immediate effect on filtering; as the different vanishing moment, the support length and filter length are different which caused to the different filtering efficiency. The wavelet function selection method is put forward included the waveform of wavelet function, the vanishing moment of wavelet function and the curvilinear smoothness of filtered signal. Emulated with the vibration signal of rotor experiment platform as an example, selected Db6 as filtering wavelet function, filtering processed the vibration signal, and the effect of signal filter was very obviously.

1 Introduction

The Mallat signal decomposition and reconstruction algorithm based on wavelet multi-resolution analysis (WMA) is very suitable for the signal analysis and processing. The wavelet function is different, the curvilinear smoothness and vanishing moment are different. The different vanishing moment, the support length and filter length are different which caused to the different filtering efficiency. The wavelet function selection method is put forward including the filtered signal curve smoothness, the wavelet function waveform, and the wavelet function vanishing moment. Targeting to match the signal characteristics, Aleksandra set up the rule of selection wavelet basis so that wavelet functions to adapt to the trend of the signal. Ding ailing gave the construction algorithm of optimal match the wavelet basis function used the structured wavelet filter bank and the new waveform similarity rule.

Xiangzhong Meng
School of Optoelectronic Engineering, Xi'an Technological University, Xi'an, China
e-mail: mxiangzhong@163.com

F.L. Gaol (Ed.): Recent Progress in DEIT, Vol. 1, LNEE 156, pp. 87–92.
springerlink.com

2 Signal Decomposition and Reconstruction Based on Wavelet Multi-resolution

In 1986, Mallat and Meyer proposed the concept of multi-resolution, leading wavelet theory to produce a breakthrough. Mallat put forward the signal decomposition and reconstruction algorithm at the same time.

As to WMA, it is to the square integrable function $f(t) \in L^2(\mathbb{R})$ as the limit of gradual approach, which each level is the results to smooth the function $f(t)$ by the low-pass smoothing function $\varphi(t)$, and the function $\varphi(t)$ make step by step expansion, that is, sequential analysis the function $f(t)$ used different resolution.

Given the WMA $\{V_m\}_{m\in\mathbb{Z}}$ of $L^2(\mathbb{R})$, $\varphi(t)$ is scale function, defined the sequence $\{g_k\}_{k\in\mathbb{Z}} \in l^2(\mathbb{Z})$,and $g_k = (-1)^k \overline{h}_{(1-k)}$, $h_{(j-2k)} = \left\langle \varphi_{j+1,n}(t), \varphi_{j,k}(t) \right\rangle$, $g_{(j-2k)} = \left\langle \varphi_{j+1,n}(t), \Psi_{j,k}(t) \right\rangle$, $\frac{1}{\sqrt{2}}\Psi(\frac{t}{2}) = \sum_k g_k \varphi(t-k)$, and signal decomposition and reconstruction algorithms is as follows.

Mallat signal decomposition algorithm,

$$x_k^{(j)} = \sum_n \overline{h}_{(j-2k)} x_n^{(j+1)} \text{ , } d_k^{(j)} = \sum_n \overline{g}_{(j-2k)} x_n^{(j+1)} \tag{1}$$

In which $x_k^{(j)}$ is signal decomposed low frequency, $d_k^{(j)}$ is signal decomposed high frequency, $\overline{h}_{(j-2k)}$ is scale factor of WMA, $\overline{g}_{(j-2k)}$ is wavelet coefficients of WMA, $n = 0,1,2,\cdots,k-1$ is the number of input sequence, j is the scale of wavelet signal decomposition (there, $x_k^{(0)}$ is the original signal when $j = 0$).

Mallat signal decomposition diagram is shown in Fig. 1(a). The signal is decomposed into different frequency bands by a band pass filter. The decomposition scale factor is the low-pass filter coefficients, and the wavelet coefficients is just high-pass filter coefficients, $x_k^{(j)}$ and $d_k^{(j)}$ are the approximation and error signal of $x_n^{(j+1)}$ respectively. Filtered the input signal by h_k or g_k, sampled the filter output signal, and then the resolution of the signal obtained is only half of the input signal.

Mallat signal reconstruction algorithm

$$x_k^{(j+1)} = \sum_n h_{(k-2n)} x_n^{(j)} + \sum_n g_{(k-2n)} d_n^{(j)} \tag{2}$$

Mallat reconstruction diagram is shown in Fig. 1(b). Interpolated the approximation signal and error signal, filtered the input signal by $h_{(k-2n)}$ or $g_{(k-2n)}$, and then, added the filter output signal. The resolution obtained is reduced.

The original signal is decomposed into low and high frequency signal, then, the low-frequency signal is decomposed into low and high frequency signal again. During the decomposition, the signal data is half and the sampling interval is double of the former layer decomposition signal. At last, the multi-scale wavelet decomposition is completed. The various frequencies that wavelet decomposed is equivalent to the original signal time-domain analysis through the filter and reduce of sampling points, and the resolution is inevitable declined. Then, the wavelet reconstruction is done, and the resolution of the signal is reached to the original signal. The linear phase filter is adopted during the reconstruction, so that, the signal reconstruction error is reduced and reserved the advantage of the good coherence between the orthogonal wavelet bands, and there is no redundancy information between the wavelet coefficients of each scale. So, the original signal can be accurately reconstruct by using FIR.

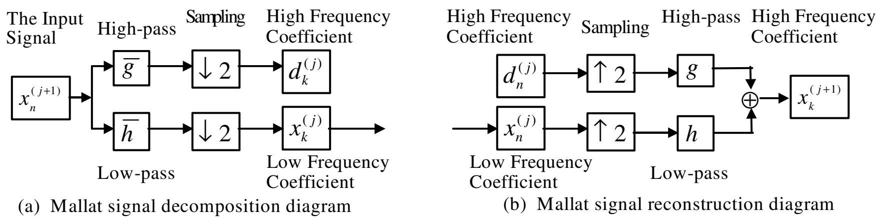

Fig. 1 Mallat signal decomposition and reconstruction diagram.

3 The Method of Wavelet Function Selection to Vibration Signal Filtering

It is very important to consider both the signal curve smoothness and the filtering efficiency. The wavelet function is different, the curvilinear smoothness, and vanishing moment and filtering effect are totally different. The wavelet family is inappropriate, the filtered signal curve is not smooth and more peaks, and caused to reduce the real-timeliness of active vibration control system. On the other hand, the vanishing moment is different, the support length and filter length are different, and obviously, the filtering efficiency is different. As thus, the selection of wavelet function is decided by the wavelet family and the vanishing moment during vibration signal filtering.

During the signal filtering process, the filter waveform should be as much as possible consistent with the signal waveform. As the vibration signal is superposed by a series of simple harmonic waves, the wavelet function waveform should be as much as possible similar to the harmonic waveform. The wavelet function waveform of Db3 is almost exactly same as Sym3, but there is very large difference between the waveform of Db6 and Sym6. It is obviously, the waveform of Db6 is more similar to the harmonic.

The Daubechies wavelet family and Symlet wavelet family are more suitable signal filtering according to the preliminary experiments and the related references. Selected the wavelet function of Daubechies wavelet family and Symlet wavelet family, the vanishing moments, the decomposition scale and the threshold method are same, filtered the vibration signal of rotor experiment platform as an example. The wavelet filtered signal curves are shown in Fig. 2.

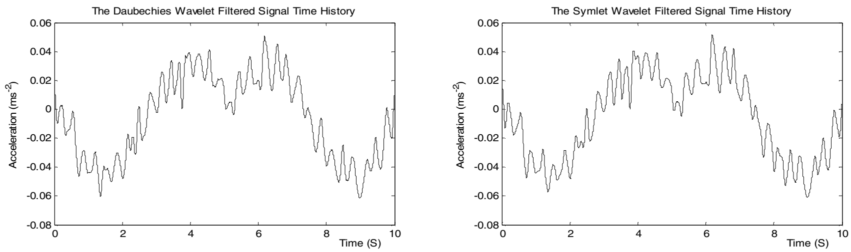

Fig. 2 The wavelet filtered signal curves

It is shown in Fig. 2 that the Daubechies wavelet filtered signal curve is smoother than the Symlet wavelet with same vanishing moment. In active vibration control system, Selected the Daubechies wavelet family, the output signal and the control effect are steadier.

During the vibration signal decomposition and reconstruction, the vanishing moment of the same wavelet family is different, the entropy values are different. The smaller entropy value means the signal decomposition and reconstruction efficiency, that is the filtering efficiency is higher, and more to meet the real-timeliness requirement of active vibration control system. The vibration signal decomposition and reconstruction entropy values of Daubechies and Symlet wavelet are shown in Tab. 1.

Table 1 Entropy Values of Daubechies and Symlet Wavelet

Vanishing Moment	Wavelet Function	Entropy Values
4	Daubechies	7.5992
	Symlet	7.5953
5	Daubechies	7.5732
	Symlet	7.5856
6	Daubechies	7.5425
	Symlet	7.6030
7	Daubechies	7.5833
	Symlet	7.6621
8	Daubechies	7.5936
	Symlet	7.7234

It is shown in Tab. 1 that, on the whole, the vibration signal decomposition and reconstruction entropy values of Daubechies wavelet are smaller than the Symlet wavelet with the same vanishing moment, and the entropy value of Db6 wavelet is the smallest.

In summary, during the vibration signal filtering process, selected the Daubechies wavelet family which the vanishing moment is 6, that is, the wavelet function Db6 as filter. The wavelet function Db6 is orthogonal and compactly supported, has extreme value phase and the maximum order of vanishing moments in the given support space. So, the wavelet function Db6 is very suitable for the real-time filtering of vibration signal.

4 The Vibration Signal Filtering Based on Wavelet Multi-resolution Analysis

The vibration signal filtering based on the wavelet multi-resolution analysis as follows: firstly, wavelet multi-scale decomposed the vibration signals: Daubechies wavelet function is adopted, and a 5-scale multi-resolution analyzed; secondly, threshold quantification processed the high-frequency signal approach to mandatory filtering. And finally, reconstructed the vibration signal using the low and high frequency coefficients according to the wavelet multi-resolution analysis, and the reconstructed signal is the filtered signal.

The wavelet transform and inverse transformation of the wavelet function Db6 can be calculated with limited discrete convolution, which is very suitable for embedded systems programming calculations online.

In order to verify the accuracy of wavelet decomposition and reconstruction, during the signal reconstruction, all of the low and high frequency component are preserved. The vibration signal and the wavelet reconstructed signal are shown in Fig. 3.

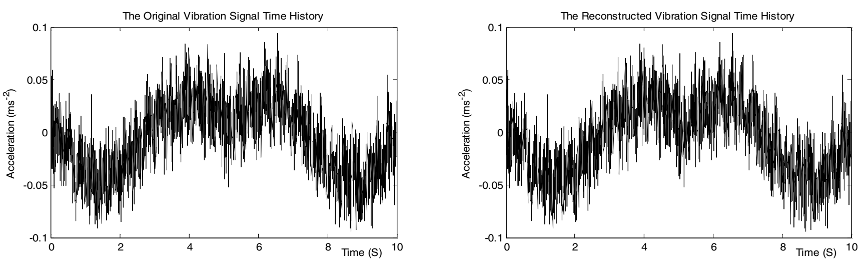

Fig. 3 The vibration signal and the wavelet reconstructed signal

Quantified the high-frequency signal coefficients by threshold, reconstructed the vibration signal, the reconstructed signal is the wavelet filtered signals. The original vibration signal and the wavelet Db6 filtered signal are shown in Fig. 4.

During the active vibration control process, the low frequency components of wavelet decomposition can be used directly as the input signal, which is very effective to the effectiveness and efficiency.

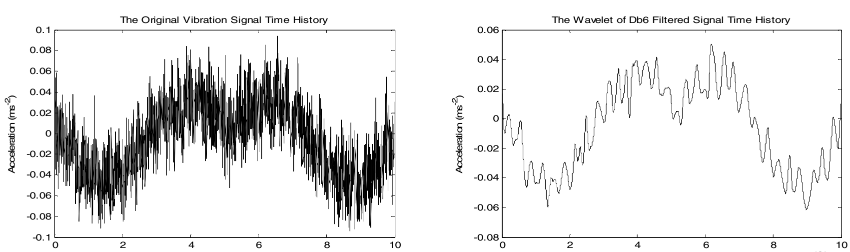

Fig. 4 The original vibration signal and the wavelet filtered signal

5 Closing

Selected an appropriate wavelet function, achieved a higher signal to noise ratio, reduced waveform distortion, enhanced the calculation efficiency, and easy to the vibration active control real-time processed. The selection method of wavelet function is researched including the filtered signal curve smoothness, the wavelet function waveform, and the wavelet function vanishing moment. The wavelet transform and inverse transformation of the wavelet function Db6 can be calculated with limited discrete convolution, which is very suitable for embedded systems programming calculations online. Selected Db6 wavelet function, emulated with the vibration signal of rotor experiment platform as an example, the vibration signal filtering realized successfully, and the effect of signal filter is very obviously.

References

[1] Cohen, A., Daubechies, I., Feauveau, J.: Biorthogonal bases of compactly supported wavelets. Commun. Pure Appl. 45(3), 485–560 (1992)

[2] Ding, A., Shi, G., Zhang, N., Jiao, L.: Signal Compression and Design of Wavelet Based on Waveform Matching. Journal of Electronics & Information Technology 29(4), 804–807 (2007)

[3] Xia, D., Zhou, B., Wang, S.: Application of wavelet's real-time filter in silicon micromachined gyroscope. Journal of Chinese Inertial Technology 15(1), 92–95 (2007)

[4] Kugarajah, T., Zhang, Q.: Multidimensional Wavelet Frames. IEEE Transactions On Neural Network 6(6), 1552–1556 (1995)

[5] Guan, S., Shi, X.: Fabric Defect Detection Based on Wavelet Lifting Scheme. Computer Engineering and Applications 44(25), 219–221 (2008)

[6] Feng, G., Li, J., Liu, P.: Noise and Vibration Source Identification in Light Bus Based on Wavelet. Journal of Vibration, Measurement & Diagnosis 26(3), 196–199 (2006)