Cyclic Scheduling for Supply Chain Network

Grzegorz Bocewicz, Robert Wójcik, and Zbigniew Banaszak

Abstract. This paper concerns the domain of Supply Chain Network Infrastructure (SCNI) usually observed in the multimodal transportation systems such as Multi-modal Passenger Transport Systems supported by lines of buses, trains, etc., and focuses on the scheduling problems encountered in these systems. SCNI can be modeled as a network of lines providing cyclic routes for particular kinds of stream-like moving transportation means. Lines and using them passengers can be seen as a multi agent system where passengers expectations compete with lines capability. The main question regards of SCNI schedulability, e.g. the guarantee the same distances in assumed different directions will require similar amount of the travel time. The declarative model of SCNI enabling to formulate cyclic scheduling problem in terms of the constraint satisfaction is our contribution.

Keywords: Cyclic scheduling, supply chain network, declarative modeling, multimodal process, constraints programming.

1 Introduction

A cyclic schedule [2], [8] is one in which the same sequence of states is repeated over and over again. In the case of Multimodal Transportation Systems (MTS) the

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appropriate cyclic scheduling problem has to take into account the constraints implied by the considered Supply Chain Network Infrastructure (SCNI), e.g. see Fig. 1. Assuming the transportation lines considered are cyclic and connected by common shared change stations a network can be modeled in terms of Cyclic Concurrent Process System (SCCP) [2]. Assuming each line is serviced by a set of stream-like moving transportation means (vehicles) and operation times required for traveling between subsequent stations as well as semaphores ensuring vehicles mutual exclusion on shared stations are given, the main question regards of SCNI timetabling, for instance guaranteeing the shortest time of the trip for passengers following a given direction. Depending on SCNI timetabling the time of the trip of passengers following different itineraries may dramatically differ. In that context the considered cyclic scheduling directly regards of multimodal processes encompassing passengers' itineraries, and indirectly regards of modeling them SCCPs. In systems of that type local lines (including transportation means) play the role of agents [1], attempting to reach their goals while following expectations of multi-modal processes. So, the considered MTS are treated as multi agent ones. The SCNI schedules sought have to follow vehicles collision- and deadlock-free flows as well as the passengers' itinerary optimization requirements. The problem considered belongs to NP-hard ones [3].

Literature Review. So far there is no research paper on cyclic scheduling of multimodal processes modeled in terms of above defined SCNI. The existing approach to solving the SCCPs scheduling problem base upon the simulation models, e.g. the Petri nets [5], the algebraic models, e.g. upon the (max,+) algebra [4] or the artificial intelligent methods [6]. The SCCP driven models assuming a unique process execution along each cyclic route, studied in [1], [2], [4] do not allow to take in to account the stream-like flow of local cyclic processes, e.g. buses servicing a given city line. So, this work can be seen as a continuation of the investigations conducted in [1], [2], [4], [7].

New Contributions. The declarative models employing the constraints programming techniques implemented in modern platforms such as OzMozart, ILOG, [1], [2] seems to be well suited to coup with SCNI scheduling problems. In that context, our contribution is a formulation of SCNI cyclic scheduling problem in terms of the constraint satisfaction one [2].

Organization. The paper is organized as follows. In Section 2, an illustrative example of SCNI and its cyclic scheduling problem statement are provided. In Section 3, a cyclic processes network is modeled. In Section 4, the selected case of multimodal processes is discussed In Section 5, we draw the conclusion.

2 Problem Formulation

The SCNI with distinguished vehicles and stations, shown in Fig. 1, is modeled in terms of the SCCP shown in Fig. 2. Four local **cyclic processes** (agents) are considered: P_1, P_2, P_3, P_4. The processes follow the **routes** (composed of transportation sectors and separating them stations) and while providing connections in two directions i.e., the north-south and the east-west, for **the two multimodal processes** (agents) mP_1, mP_2 and mP_3, mP_4, respectively. P_1, P_2 contain two sub-processes $P_1 = \{P_1^1, P_1^2\}, P_2 = \{P_2^1, P_2^2\}$ representing trains moving along the same route. The following constraints determine the processes cooperation:

- The new local process operation (the train's operation such as: passengers' transportation, boarding etc.) may begin only if the current operation has been completed and the resource designed to this operation is not occupied.
- The local processes share the common resources (the stations) in the mutual exclusion mode. The new local process operation can be suspended only if designed resource is occupied. The local processes suspended cannot be released. Local processes are non-preempted.
- The multimodal processes follow the local transportation routes. Different multimodal processes can be executed simultaneously along a local process.
- The local and multimodal processes are executed cyclically, resources occurring in each transportation route cannot repeat.

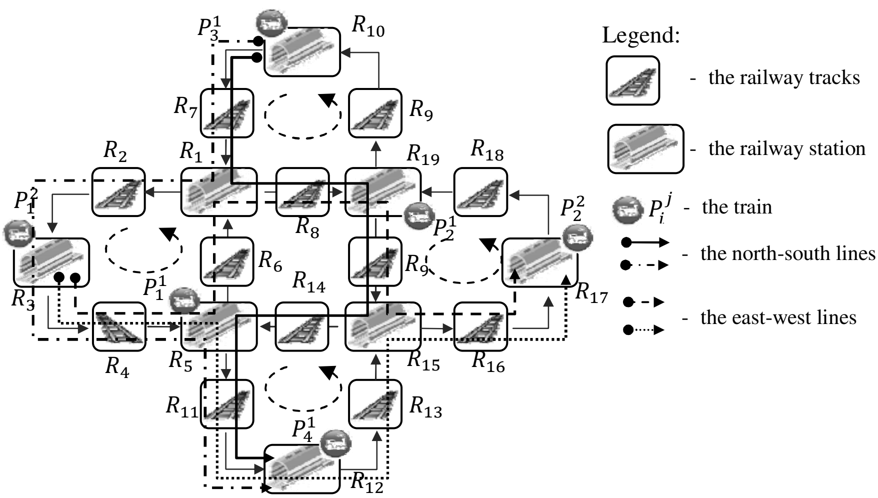

Fig. 1 An example of the SCNI

The main question concerns of SCCP cyclic steady state behavior and a way this state depends on direction of local process routes as well as on priority rules, and an initial process allocation to the system resources. Assuming the steady state there exists the next question regards of travel time along assumed multimodal process route linking distinguished destination points. Of course, the periodicity of multimodal processes depends on SCCP periodicity, i.e. characteristics of a given SCNI. That means an initial state and a set of dispatching rules can be seen as control variables allowing one to "adjust" multimodal processes schedule.

Consider a SCCP model of SCNI specified by the given dispatching rules, operation times (see Table 1), and initial processes allocation. The main question concerns: Does there exist a cyclic steady state of local and multimodal processes?

Table 1 Operation times of SCCP's o (from Fig. 2)

Streams	i	k	$t_{i,1}^k$	$t_{i,2}^k$	$t_{i,3}^k$	$t_{i,4}^k$	$t_{i,5}^k$	$t_{i,6}^k$
P_1^1	1	1	1	1	1	2	1	3
P_1^2	1	2	1	3	1	1	1	1
P_2^1	2	1	1	1	1	3	1	1
P_2^2	2	2	1	3	1	1	1	1
P_3^1	3	1	1	3	1	1	1	3
P_4^1	4	1	1	2	1	1	1	4

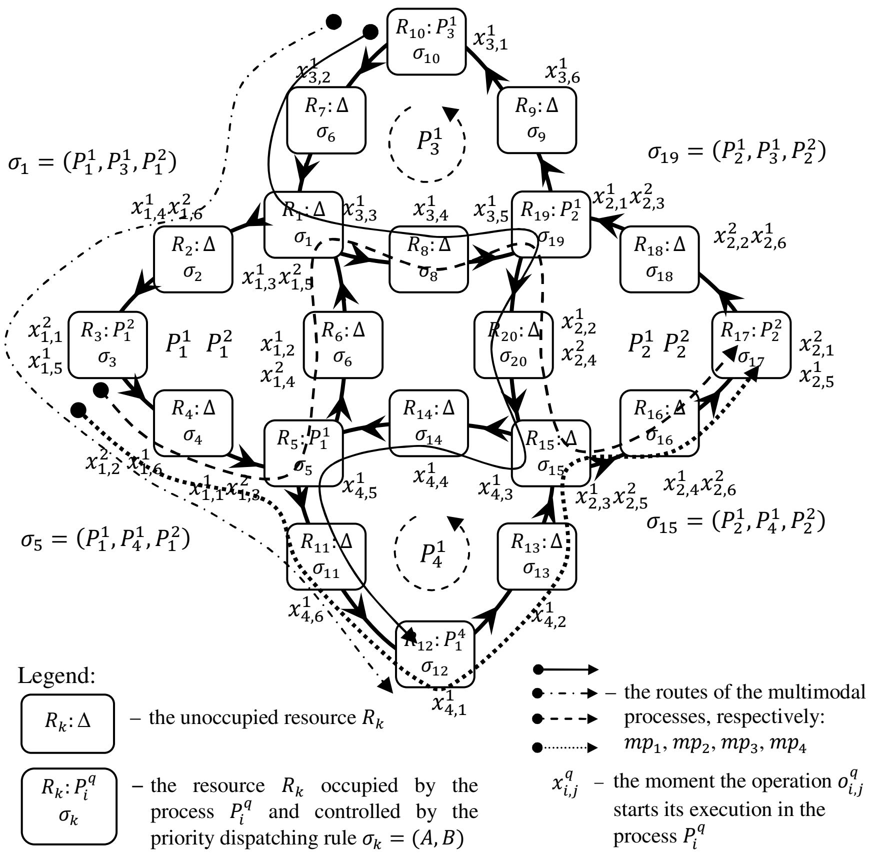

Fig. 2 SCCP of SCNI from Fig. 1

3 Modeling of Cyclic Processes Network

In the SCCP model of SCNI the following **notations** are used [1], [2]:

- A sequence $p_i^k = (p_{i,1}^k, p_{i,2}^k, \ldots, p_{i,lr(i)}^k)$ specifies **the route of the local process's stream $\boldsymbol{P_i^k}$** (k-th stream of the i-th local process P_i), and its components define the resources used in course of process operations execution, where: $p_{i,j}^k \in R$ (the set of resources: $R = \{R_1, R_2, \ldots, R_m\}$) – denotes the resource used by the k-th stream of i-th local process in the j-th operation; in the rest of the paper **the $\boldsymbol{j}$-th operation executed on resource $\boldsymbol{p_{i,j}^k}$ in the stream $\boldsymbol{P_i^k}$** will be denoted by $o_{i,j}^k$; $lr(i)$ - denotes a length of cyclic process route.
- $t_i^k = (t_{i,1}^k, t_{i,2}^k, \ldots, t_{i,lr(i)}^k)$ specifies **the process operation times**, where $t_{i,j}^k$ denotes the time of execution of operation $o_{i,j}^k$ (see Table 1).
- $mp_i = \left(mpr_j(a_j, b_j), mpr_l(a_l, b_l), \ldots, mpr_h(a_h, b_h)\right)$ specifies **the route of the multimodal process $\boldsymbol{mP_i}$**
  where: $mpr_j(a, b) = \left(crd_a p_j^k, crd_{a+1} p_j^k, \ldots, crd_b p_j^k\right), crd_i D = d_i$, for $D = (d_1, d_2, \ldots, d_i, \ldots, d_w)$, $\forall a \in \{1,2, \ldots, lr(i)\}$, $\forall j \in \{1,2, \ldots, n\}$, $crd_a p_j \in R$.
  The transportation route mp_i is a sequence of sections of local process routes. For the sake of simplicity let as assume the all operation times of multimodal processes are the same and equal to the 1 unit of time.
- $\Theta = \{\sigma_1, \sigma_2, \ldots, \sigma_m\}$ is the set of **the priority dispatching rules**, where $\sigma_i = (s_{i,1}, \ldots, s_{i,lp(i)})$ is the sequence components of which determine an order in which the processes can be executed on the resource R_i, $s_{i,j} \in P$ (the set of process streams: $P = \{P_1^1, \ldots, P_1^a, P_2^1 \ldots, P_2^b, \ldots, P_n^z\}$, each process executes periodically in infinity). Dispatching rules which determine an order on the shared train stations (resources R_1, R_5, R_{15}, R_{19}) are following: $\sigma_1 = (P_1^1, P_3^1, P_1^2)$, $\sigma_5 = (P_1^1, P_4^1, P_1^2)$, $\sigma_{15} = (P_2^1, P_4^1, P_2^2)$, $\sigma_{19} = (P_2^1, P_3^1, P_2^2)$.

In that context a SCCP can be defined as a pair [2]:

$$SC = (SC_l, SC_m), \tag{1}$$

where: $SC_l = (R, P, \Pi, T, \Theta)$ – characterizes the SCCP structure, i.e.
$R = \{R_1, R_2, \ldots, R_m\}$ – the set of resources,
$P = \{P_1^1, \ldots, P_1^a, \ldots, P_n^1, \ldots, P_n^z\}$ – the set of local processes,
$\Pi = \{p_1, p_2, \ldots, p_n\}$ – the set of local process routes,
$T = \{T_1, \ldots, T_n\}$ – the set of local process operations times,
$\Theta = \{\sigma_1, \sigma_2, \ldots, \sigma_m\}$ – the set of dispatching priority rules.
$SC_m = (MP, M\Pi)$ – characterizes the SCCP behavior, i.e.
$MP = \{mP_1, mP_2, \ldots, mP_u\}$ – the set of multimodal processes,
$M\Pi = \{mp_1, mp_2, \ldots, mp_u\}$ – the set of multimodal process routes,

The main question concerns of SCCP cyclic behavior and a way this behavior depends on direction of local transportation routes Π, the priority rules Θ, and a set of initial states, i.e., an initial processes allocations to the system resources.

CSP-driven cyclic scheduling: Since parameters describing the SCCP model (1) are usually discrete, and linking them relations can be seen as constraints, hence related to them cyclic scheduling problems can be presented in the form of the Constraint Satisfaction Problem (CSP) [1], [2]. More formally, CSP is a framework for solving combinatorial problems specified by pairs: (a set of variables and associated domains, a set of constraints restricting the possible combinations of the variable values). The CSP relevant to the SCCP can be stated as follows [2]:

$$CS = ((\{R, P, \Pi, T, \Theta, X, Tc\}, \{D_R, D_\Pi, D_T, D_\Theta, D_X, D_{Tc}\}), C), \tag{2}$$

where:

- R, P, Π, T, Θ are the decision variables describing the structure of the SCCP, i.e., (1), and X, Tc are the decision variables describing the cyclic behavior of the SCCP. $X = \{X_1^1, \dots, X_1^a, X_2^1 \dots, X_2^b, \dots, X_n^z\}$ is the set of sequences $X_i^k = (x_{i,1}^k, x_{i,2}^k, \dots, x_{i,lr(i)}^k)$, where each variable $x_{i,j}^k$ determines **the moment of** $o_{i,j}^k$ **operation beginning** in any (the l-th) cycle: $x_{i,j}^k(l) = x_{i,j}^k + l \cdot Tc,\ l \in \mathbb{Z}$, (where $x_{i,j}^k(l) \in \mathbb{Z}$ – means the moment the $o_{i,j}^k$ operation starts its execution in the l-th cycle) and Tc is the SCCP periodicity: $Tc = x_{i,j}^k(l+1) - x_{i,j}^k(l)$.
- the domains $D_R, D_P, D_\Pi, D_T, D_\Theta, D_X, D_{Tc}$ of decision variables which describe the family of: the set of resources, set of processes, sets of admissible routings, sets of admissible operation times, sets of admissible dispatching priority rules, sets of admissible coordinate values X_i^k, $x_{i,j}^k \in \mathbb{Z}$, set of admissible values of variables Tc, respectively.
- the constraints determining the relationship between the structure (specified by the quin-tuple (R, P, Π, T, Θ)) and the behavior following from this structure (specified by (X, Tc)) can be defined by the operator max [2].The constraints following assumptions imply for instance that an operation from the process P_1^1 can begin at the moment $x_{1,3}^1$ on resource R_1 only if the previous operation executed on the resource R_6 was completed at $x_{1,2}^1 + t_{1,2}^1$ and the resource R_1 has been released, i.e. if the process P_1^2 occupying the resource R_1 begins its subsequent operation at $x_{1,6}^2 - Tc + 1$. Therefore $x_{1,3}^1 = \max(x_{1,6}^2 - Tc + 1;\ x_{1,2}^1 + t_{1,2}^1)$. The rest of operation starting moments can be determined by analogy, see Table 2
- **The system's cyclic behavior** encompasses itself through values of decision variables X, guaranteeing its periodicity Tc. The parameters determining the cyclic behavior such as X and Tc are solution to the problem (2) following the set of constraints C (Table 2.), determining the SCCP's structure (1).

4 Cyclic Processes Scheduling

Consider CSP stated by CS (2) and formulated for SCCP from Fig 2. The assumed set $\{\sigma_1 = (P_1^1, P_3^1, P_1^2),\ \sigma_{19} = (P_2^1, P_3^1, P_2^2),\ \sigma_5 = (P_1^1, P_4^1, P_1^2),\ \sigma_{15} = (P_2^1, P_4^1, P_2^2)\}$ of dispatching rules implies $Tc = 11$. The resultant cyclic steady state shown in Fig. 3 has be obtained in OzMozart, Dual Core 2.67, GHz, 2.0, GB RAM environment in 1 s. Obtained periodicity $(Tc = 11)$ of the SCNI behavior implies different traveling times required by different directions – the itineraries mP_4 and mP_3 following the routes mp_3, mp_4 along the east-west direction are realized in 18 and 28 time units, respectively (see the dotted and dashed lines in Fig. 1÷3). In turn, the itineraries mP_1 and mP_2 following the routes mp_1, mp_2 along the north-south direction are realized in 22 and 33 time units, respectively (see the solid and dot-dashed lines in Fig. 1÷3). So, the best line serving the east-west direction is faster than the best line serving the north-south direction.

Table 2 The constraints describing the moments $x_{i,j}^k$ of SCCP from Fig. 2

R_3:	$x_{1,1}^2 = \max\left(x_{1,6}^1 - Tc + 1; x_{1,6}^2 - Tc + t_{1,6}^2\right)$	R_4:	$x_{1,2}^2 = \max\left(x_{1,1}^1 + 1; x_{1,1}^2 + t_{1,1}^2\right)$
	$x_{1,5}^1 = \max\left(x_{1,2}^2 + 1;\ x_{1,4}^1 + t_{1,4}^1\right)$		$x_{1,6}^1 = \max\left(x_{1,3}^2 + 1; x_{1,5}^1 + t_{1,5}^1\right)$
R_5:	$x_{1,1}^1 = \max\left(x_{1,4}^2 - Tc + 1;\ x_{1,6}^1 - Tc + t_{1,6}^1\right)$	R_6:	$x_{1,2}^1 = \max\left(x_{1,5}^2 - Tc + 1; x_{1,1}^1 + t_{1,1}^1\right)$
	$x_{4,5}^1 = \max\left(x_{1,2}^1 + 1; x_{4,4}^1 + t_{4,4}^1\right)$		
	$x_{1,3}^2 = \max\left(x_{4,6}^1 + 1;\ x_{1,2}^2 + t_{1,2}^2\right)$		$x_{1,4}^2 = \max\left(x_{1,3}^1 + 1; x_{1,3}^2 + t_{1,3}^2\right)$
R_1:	$x_{1,3}^1 = \max\left(x_{1,6}^2 - Tc + 1;\ x_{1,2}^1 + t_{1,2}^1\right)$	R_2:	$x_{1,4}^1 = \max\left(x_{1,1}^2 + 1;\ x_{1,3}^1 + t_{1,3}^1\right)$
	$x_{3,3}^1 = \max\left(x_{1,4}^1 + 1;\ x_{3,2}^1 + t_{3,2}^1\right)$		$x_{1,6}^2 = \max\left(x_{1,5}^1 + 1;\ x_{1,5}^2 + t_{1,5}^2\right)$
R_{17}:	$x_{2,1}^2 = \max\left(x_{2,6}^1 - Tc + 1;\ x_{2,6}^2 - Tc + t_{2,6}^2\right)$	R_{18}:	$x_{2,6}^1 = \max\left(x_{2,3}^2 + 1;\ x_{2,5}^1 + t_{2,5}^1\right)$
	$x_{2,5}^1 = \max\left(x_{2,2}^2 + 1; x_{2,4}^1 + t_{2,4}^1\right)$		$x_{2,2}^2 = \max\left(x_{2,1}^1 + 1;\ x_{2,1}^2 + t_{2,1}^2\right)$
R_{19}:	$x_{2,1}^1 = \max\left(x_{2,4}^2 - Tc + 1;\ x_{2,6}^1 - Tc + t_{2,6}^1\right)$	R_{20}:	$x_{2,2}^1 = \max\left(x_{2,5}^2 - Tc + 1;\ x_{2,1}^1 + t_{2,1}^1\right)$
	$x_{3,5}^1 = \max\left(x_{2,2}^1 + 1; x_{3,4}^1 + t_{3,4}^1\right)$		
	$x_{2,3}^2 = \max\left(x_{2,6}^1 + 1;\ x_{2,2}^2 + t_{2,2}^2\right)$		$x_{2,4}^2 = \max\left(x_{2,3}^1 + 1;\ x_{2,3}^2 + t_{2,3}^2\right)$
R_{15}:	$x_{2,3}^1 = \max\left(x_{2,6}^2 - Tc + 1;\ x_{2,2}^1 + t_{2,2}^1\right)$	R_{16}:	$x_{2,4}^1 = \max\left(x_{2,1}^2 + 1;\ x_{2,3}^1 + t_{2,3}^1\right)$
	$x_{4,3}^1 = \max\left(x_{2,4}^1 + 1;\ x_{4,2}^1 + t_{4,2}^1\right)$		$x_{2,6}^2 = \max\left(x_{2,5}^1 + 1; x_{2,5}^2 + t_{2,5}^2\right)$
R_{12}:	$x_{4,1}^1 = x_{4,6}^1 - Tc + t_{4,6}^1$	R_{13}:	$x_{4,2}^1 = x_{4,1}^1 + t_{4,1}^1$
R_{14}:	$x_{4,4}^1 = x_{4,3}^1 + t_{4,3}^1$	R_{11}:	$x_{4,6}^1 = x_{4,5}^1 + t_{4,5}^1$
R_{10}:	$x_{3,1}^1 = x_{3,6}^1 - Tc + t_{3,6}^1$	R_7:	$x_{3,2}^1 = x_{3,1}^1 + t_{3,1}^1$
R_8:	$x_{3,4}^1 = x_{3,3}^1 + t_{3,3}^1$	R_9:	$x_{3,6}^1 = x_{3,5}^1 + t_{3,5}^1$

However, replacing the above assumed set of dispatching rules for the following new one $\{\sigma_1 = (P_3^1, P_1^1, P_1^2),\ \sigma_{19} = (P_2^1, P_3^1, P_2^2),\ \sigma_5 = (P_1^1, P_4^1, P_1^2),\ \sigma_{15} = (P_4^1, P_2^1, P_2^2)\}$ provides shorter cycle time $Tc = 10$, resulting in shortening of the travel time (20 time units) following the route mp_1 (north-south line), and extension of the travel time (28 time units) following the route mp_3 (east-west line).

That means, the different sets of dispatching rules implies different traveling times in assumed directions. In the case considered the difference between the shortest traveling times along two directions changes from $4 = 22 - 18$ to

$8 = 28 - 20$ time units. The open question is whether there exists such a set of dispatching rules guaranteeing the same best traveling time in both directions?

5 Concluding Remarks

In contradiction to the traditionally offered solutions the approach presented allows one to take into account such behavioral features as transient periods and deadlock occurrence. So, the novelty of the modeling framework lies in the declarative approach to reachability problems enabling an evaluation of multimodal cyclic process executed within cyclic processes environments. The approach presented leads to solutions allowing the designer to compose and synchronize elementary systems in such a way as to obtain the final SCNI system with required quantitative and qualitative behavioral features. So, we are looking for a method allowing one to replace the exhaustive search for the admissible control of the whole system by its a step-by-step structural design guaranteeing the required behavior (i.e., encompassing execution of assumed multi-modal processes).

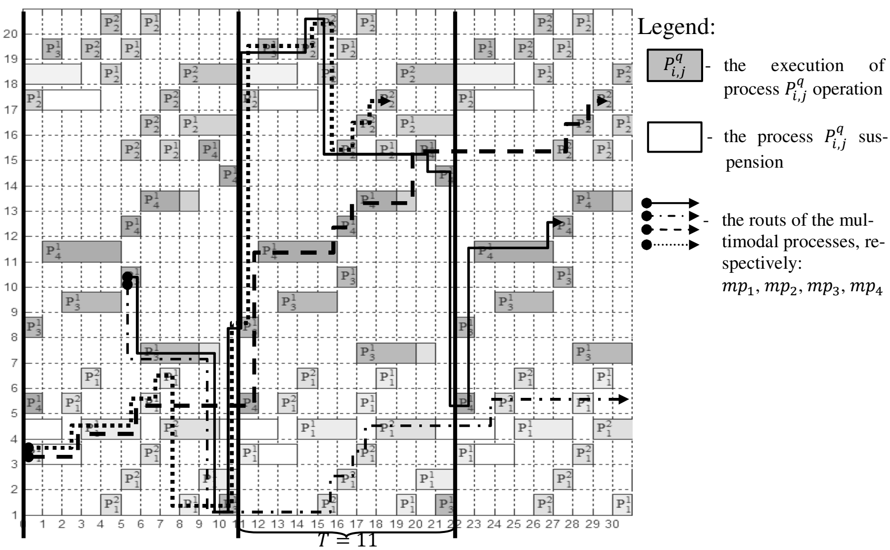

Fig. 3 Gantt diagram

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