Impedance Method for Modeling and Locating Leak with Cylindrical Geometry

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Abstract. This paper uses transient frequency methods to locate a leak in a pipeline system when a leak is modeled as a *small cylindrical pipe element open to atmospheric pressure on one end*. The impedance of the leak as a small cylindrical pipe is included in the transfer function equation at the closed valve in a reservoir-pipe-valve system. The impedance diagram of this system with cylindrical leak is plotted and analyzed; the influence of the *pipe thickness* on leak locations is studied. The numerical study relying on impedance method shows that the cylindrical leak geometry approach allows *secondary peaks to be detected only for thick pipes*. By imposing a periodic pattern on the resonant responses, the leak with cylindrical geometry can be located and its location is based on an *empirical relation*. The computed results show satisfactory agreement with previous works when leak is modeled by a circular orifice.

Keywords: Cylindrical leak, impedance diagram, Transient flow, leak depth, wave reflection.

1 Introduction

Energy producing mechanisms are generally formed by piping systems used for transporting fluids between their different components. Leaks in piping systems pose a major problem such as energy dissipation and physical process disruption causing a decrease of the energetic system efficiency. In previous works such as (Mpesha et al. 2001), (Chaudry et al. 2002), (Ferrante and Brunone 2003-a, 2003-b), (Al-Khomairi 2005), (Covas et al. 2005), (Hadj-Taieb 2007), (Lee et al 2006, 2007) and (Gao et al 2009) different methods are developed for leak detection and location in pipeline system, such as Frequency Response Diagram, Extended Kalman Filter, Characteristic Numerical Method, Standing Wave Difference Method, Impedance Method, Impulse response method and cross correlation method etc.

Numerous works have investigated the behavior of different leak geometries such as (Brunone, and Ferrante 2001). Overall previous works use to treat the leak discharge as an orifice relation and lump all losses in the discharge coefficient.

In this paper the leak is considered as a small pipe element open to the atmospheric pressure on one end. The basic cylindrical surface of the leak is located on the pipe circumferential and with small diameter.

The leak locations are based on the harmonic analysis of the presence of additional peaks due to the leak depth.

Two location cases are studied either leak near the reservoir or near the valve.

The impedance method is also used to study the influence of the leak depth on frequency response of leaking pipeline system.

The numerical results are compared with (Covas et al. 2005).

2 Mathematical Flow Modeling

2.1 Motion Equations

The simplified one dimensional continuity and momentum equations that describe transient flow in horizontal cylindrical pipe, of linear elastic behavior according to Hooke law, are adapted from the analytical model developed by Wylie et al. (1993):

$$\frac{\partial Q}{\partial x}+\frac{gA}{C^2}\frac{\partial H}{\partial t}=0 \tag{1}$$

$$\frac{\partial H}{\partial x}+\frac{1}{gA}\frac{\partial Q}{\partial t}+\frac{\lambda Q|Q|}{2gDA}=0 \tag{2}$$

where Q is the fluid discharge, A is the pipe cross section area, C is the wave celerity, λ is the friction coefficient, D is the diameter of the main pipe, H is the pressure head, t is the time and x is the distance along the pipe.

2.2 The Impedance Method

Equations (1) and (2) can be solved in frequency domain by the impedance method, which allows explaining the harmonic analysis of the pressure wave through the pipe.

The pressure head H and the discharge Q are composed of two parts, the average values $\overline{H}$ and $\overline{Q}$ and the fluctuated oscillatory complex terms h and q.

Substituting H and Q in Eqs. (1) and (2), and considering the average terms time independent, the linearized equations are:

$$\frac{\partial q}{\partial x}+\frac{gA}{C^2}\frac{\partial h}{\partial t}=0 \tag{3}$$

$$\frac{1}{gA}\frac{\partial q}{\partial t}+\frac{\partial h}{\partial x}+Rq=0 \tag{4}$$

The solution of this set of equations can be obtained by the technique of separation of variables (Wylie and Streeter 1993).

Accordingly, the complex head and discharge are given by transfer equations

$$h(x)=h_U\cosh(\gamma x)-Z_C q_U\sinh(\gamma x) \tag{5}$$

$$q(x)=-\frac{h_U}{Z_C}\sinh(\gamma x)+q_U\cosh(\gamma x) \tag{6}$$

Where h_U, q_U =head and discharge at the upstream end; γ= propagation constant, $\gamma=\alpha+i\beta$ where:

$$\alpha=\left(\frac{Ag\omega}{C^2}\right)^{1/2}\left[\left(\frac{\omega}{gA}\right)^2+R^2\right]^{1/4}\sin\left(\frac{1}{2}\tan^{-1}\left(\frac{RgA}{\omega}\right)\right) \tag{7}$$

$$\beta=\left(\frac{Ag\omega}{C^2}\right)^{1/2}\left[\left(\frac{\omega}{gA}\right)^2+R^2\right]^{1/4}\cos\left(\frac{1}{2}\tan^{-1}\left(\frac{RgA}{\omega}\right)\right) \tag{8}$$

The hydraulic impedance at the distance x along the pipe is the ratio of the head fluctuation h to the discharge fluctuation q Thus,

$$Z(x)=\frac{h}{q}=\frac{Z_U-Z_C\tanh(\gamma x)}{1-\frac{Z_U}{Z_C}\tanh(\gamma x)} \tag{9}$$

where:

Z_C is the characteristic impedance defined by the following equation

$$Z_C=\frac{\gamma C^2}{i\omega gA}=\frac{C^2}{\omega gA}(\beta-i\alpha) \tag{10}$$

and

Z_U is the upstream impedance at $x=0$ and Z_D is the downstream impedance at $x=L$ (fig. 1)

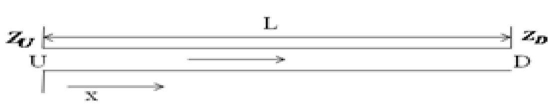


Fig. 1 Simple pipeline

2.3 Boundary Conditions

In order to apply the impedance solution to the hydraulic installation considered in this work and composed by constant head reservoir in the upstream pipe section and a valve in the downstream pipe section (fig. 2), one must represent the boundary conditions as terminal impedances.

At the upstream end, constant head reservoir

$$H_U = 0 \text{ then } Z_U = \frac{H_U}{Q_U} = 0 \tag{11}$$

At the downstream end $x = 0$, the hydraulic impedance is deduced by substituting the eq.11 in the eq.9, then:

$$Z_D = Z_{out} = \frac{H_D}{Q_D} = -Z_C \tanh(\gamma L) \tag{12}$$

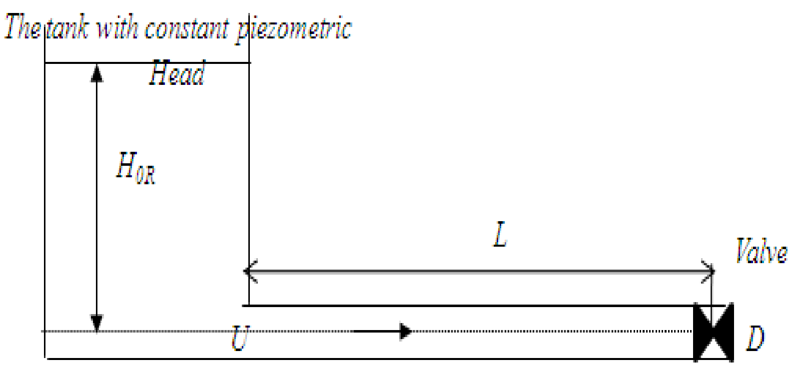


Fig. 2 Reservoir-pipe-valve systems without leak

2.4 Hydraulic Impedance for a Simple Pipeline System Having a Leak with Cylindrical Geometry

The development of the flow does not depend only on the distance but depend also on the Reynolds number, the diameter and discharge of the leak, the triggering of fully developed flow that_figures in (Drust, F et al works 2005) allows to consider that for a leak depth less than 2 cm and a low leak discharge, the *flow can be approximate to be one dimensional*. The one dimensional motion equations are used to study the flow throw the leak.

The main diagnostic tool in this paper is to consider that the leak has a cylindrical geometry. That yields to consider that the leak is a small cylindrical pipe which diameter is the difference between the length of the pipe without a leak and the sum of the pipe lengths upstream and downstream the leak (fig.3).

$$D_\ell = L - (L_1 + L_2) \tag{13}$$

The length of the leak pipe is the thickness e of the main pipe. Using the impedance method one can insert the impedance of the cylindrical leak in the impedance at the end section of the leaking system Z_{D2} to obtain the head pressure variation between the upstream and the downstream of the leak.

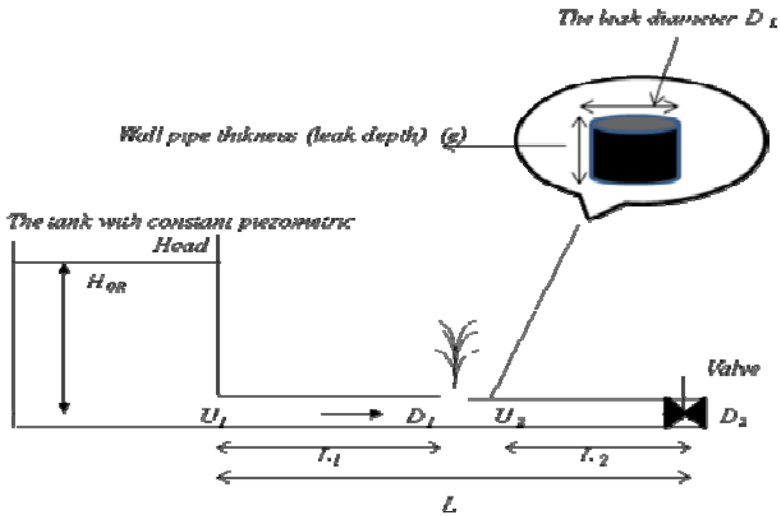


Fig. 3 Reservoir-pipe-valve systems with cylindrical leak

The leak as considered is an *opened pipe to the free atmosphere*, its impedance is written as follows:

$$Z_\ell = -Z_{C\ell} \tanh(\gamma_\ell e) \tag{14}$$

Where

$Z_{C\ell}$ is the characteristic impedance of the pipe leak and γ_ℓ is the propagation wave coefficient in the *cylindrical leak.*

To obtain the *characteristic impedance of the leak modeled by a cylindrical pipe*, one can substitute the expressions of α and β in eq.10 by α_ℓ and β_ℓ, also the pipe section A must be substituted by the leak section A_ℓ. The wave celerity is considered constant through the total pipeline system. In this case the characteristic impedance of the leak is represented by the eq.15.

$$Z_{C\ell} = \frac{C^2}{\omega g A_\ell}(\beta_\ell - i\alpha_\ell) \tag{15}$$

The impedance at the upstream and the downstream of the second pipe are respectively written:

$$Z_{U2} = \frac{Z_{D1} Z_\ell}{Z_\ell - Z_{D1}} \tag{16}$$

$$Z_{D2} = \frac{Z_{U2} - Z_{C2} \tanh(\gamma_2 L_2)}{1 - \frac{Z_{U2}}{Z_{C2}} \tanh(\gamma_2 L_2)} \tag{17}$$

By substituting the eq.16 in the eq.17, the impedance at the valve is:

$$Z_{D2} = \frac{\frac{Z_{D1} Z_\ell}{Z_\ell - Z_{D1}} - Z_{C2} \tanh(\gamma_2 L_2)}{1 - \frac{\frac{Z_{D1} Z_\ell}{Z_\ell - Z_{D1}}}{Z_{C2}} \tanh(\gamma_2 L_2)} \tag{18}$$

Substituting eq.14 in eq.18 and considering that the pipes downstream and upstream the leak have the same characteristic impedances and the same wave propagation constants, respectively $Z_{C1} = Z_{C2} = Z_C$ *and* $\gamma_1 = \gamma_2 = \gamma_\ell$.

The impedance at downstream of the pipe with leak begins (Ayed. L et al 2011)

$$Z_{D2} = \frac{Z_C Z_{C\ell} \tanh(\gamma_\ell e) \tanh(\gamma L_1) + Z_{C2} Z_{C\ell} \tanh(\gamma_\ell e) \tanh(\gamma_2 L_2) - Z_C^2 \tanh(\gamma L_1) \tanh(\gamma L_2)}{-Z_{C\ell} \tanh(\gamma_\ell e) + Z_C \tanh(\gamma L_1) - Z_{C\ell} \tanh(\gamma_\ell e) \tanh(\gamma L_1) \tanh(\gamma L_2)} \tag{19}$$

3 Cylindrical Leak Detection Using Impedance Frequency Spectrum Analysis

The hydraulic system installation in fig. 3 is characterized by the wave celerity $C = 1200\, ms^{-1}$, the friction coefficient $\lambda = 0.04$, the pipe thickness $e = 0.02m$, the pipe diameter's $D = 1m$, the resolution of the frequency is equal to $\Delta\omega = 10^{-4}$ and the pipe length is $L = 1600m$.

In this part of the paper, attention is focused on the transients occurring in a single pipe system with a known transfer function at the upstream end.

The problem is to determine the impedance at the downstream end of the system without leak, or with leak modeled by a small cylindrical pipe. The leak discharge is $Q_{\ell 0} = 10\% Q_0$, where the flow discharge at the upstream of the main pipe is $Q_0 = 0.1 m^3 s^{-1}$.

3.1 The Leak Is Near the Tank

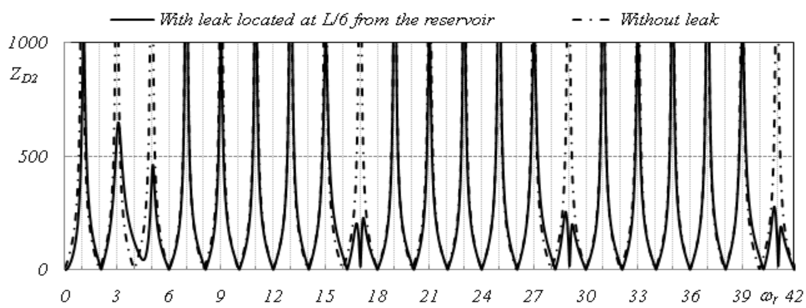
Figures 4(a) and 4(b) represent a comparison of the impedance at the downstream end of the pipe without leak to the impedances at the downstream end of the pipe with leak located respectively at $L/6$ *and* $2L/5$, from the reservoir .

In this case the leak is near the tank $(L_1 \leq L/2)$.

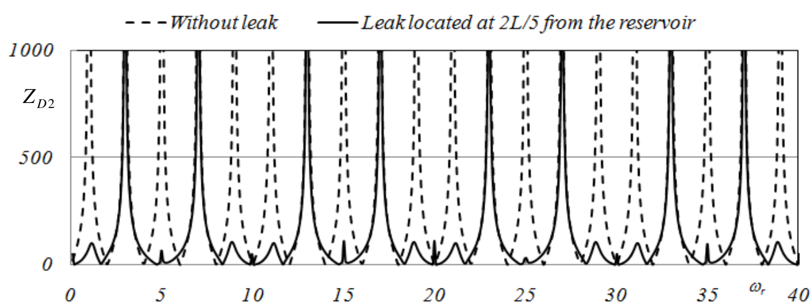
The impedance curves at the downstream end of the identical pipe with and without leak show an interesting result, which is giving rise to new (secondary) peaks, due to the leak depth consideration.

Only one new peak type is born due to the leak presence in different leak location with the same leak size represented by the leak area $A_\ell = 28.26\,mm^2$.

For different leak locations, the impedance diagram analysis shows that two peaks occurring successively at the normalized frequencies $\omega_{r\ell k-1}$ and $\omega_{r\ell k}$ are separated by the same band of frequency for each leak location.



(a)



(b)

Fig. 4 Frequency response functions for different leak location from the reservoir

If we note the band frequency width between two successive peaks due to the leak (frequency period of the leaking pipe impedance) $\Delta\omega_{r\ell(k-1,k)} = \omega_{r\ell k} - \omega_{r\ell k-1}$ and the frequency period of the impedance at the downstream of the pipe without leak by $\Delta\omega_{r,\text{int}\,act\;pipe}$, the leak locations from the reservoir, can be calculated by multiplying the total length of intact pipe by the ratio of the frequency period of the downstream impedance of intact pipe to the frequency period of the downstream impedance of leaking pipe. So one gets *the next empirical leak location equation:*

$$L_1 = \frac{\Delta\omega_{r,\text{int}\,act\;pipe}}{\omega_{r\ell k} - \omega_{r\ell k-1}} L \tag{20}$$

where ω_r is the normalized frequency and $\omega_{r\ell k-1}$ is the normalized frequency at which occurs the $k-1^{th}$ peak due to the leak presence (*leak depth*).

$\omega_{r\ell k}$ is the normalized frequency at which occurs the k^{th} peak due to the leak presence (leak depth).

By applying the eq.20, one can calculate the location of the leak for each figure case.

In fig.4 (a) for frequency width plotting of 40, one can detects three new peaks occurring successively at the frequency

$\omega_{r1} = 17; \omega_{r2} = 29; \omega_{r3} = 41$

In addition the normalized frequency period of the response function at the downstream end of the pipe without leak is equal to $\Delta\omega_{r,\text{int}\,actpipe} = 2$. So, and the values of corresponding series are:

$$L_1 = \frac{\Delta\omega_{r,\text{int}\,act\;pipe}}{\omega_{r\ell,(k,k-1)}} L = \frac{\Delta\omega_{r,\text{int}\,act\;pipe}}{\omega_{r\ell 2} - \omega_{r\ell 1}} L = \frac{\Delta\omega_{r,\text{int}\,act\;pipe}}{\omega_{r\ell 3} - \omega_{r\ell 2}} L = \frac{L}{6}$$

By the same procedure one can get the leak locations after calculating the normalized frequencies at which occurs the secondary peaks of responses when leak is either located at $2L/5$ from the reservoir.

3.2 The Leak Is Near the Valve

In this case, the leak is near the valve $(L_2 < L/2)$. As an example for the leak located at $L/n(n > 2)$ from the valve there are $n-1$ types of peaks. This result is clarified by the fig. 5 in which the leak is located at $L/6$ from the valve and we can observe five peak types differentiated by the monotony; the form and the amplitude. Compared to the case when the leak is near the reservoir there is a difference explained by the effect of the mother wave sent by the valve. This wave is sending back $(n-1)$ times by the leak, before being absorbed by the reflected negative wave coming from the reservoir. In the case presented and when leak is located at $L/n\,(n \geq 2)$, two successive peaks for the same peak type are separated by the same band of frequency which is the period of the response function at the downstream of pipeline system. This band of frequency is equal to $n\Delta\omega_{r,\ell}$. To deduce the location of the leak having cylindrical geometry one must calculate the frequencies at which occurs this peaks and differentiate their different types.

After the leak location is deduced by the same process like the case when leak is near the reservoir, but L_1 must be substituted by L_2 and $\Delta\omega_{r\ell,(k,k-1)}$ must be substituted by $\Delta\omega_{r\ell,(jk,jk-1)}$, so the leak location from the valve is deduced by the eq.21

$$L_2 = \frac{\Delta\omega_{r,\text{int}\,actpipe}}{\omega_{r\ell,(jk,jk-1)}} L \tag{21}$$

$\Delta\omega_{r\ell,(jk,jk-1)}$ is the *band frequency width between two successive peaks*, with the same type (j^{th} type), due to the leak depth (pipe thickness).This band frequency is also the frequency response function period.

In fig.5, the leak is located at $\frac{L}{6}$ from the valve. For the frequency width-plotting of 40, one can detect fifteen new peaks distributed on five types. As an example we consider one peak type that we note it as the first peak type. The secondary peaks of the first type are occurring successively at these frequencies:

$$\omega_{r\ell 11} = 3.36; \omega_{r\ell 12} = 15.371; \omega_{r\ell 13} = 27.36; \omega_{r\ell 14} = 39.359$$

In addition the normalized frequency period of response function at the downstream end of the pipe without leak is equal to $\Delta\omega_{r,\text{int}\,act\ pipe} = 2$, so the values of the corresponding series are:

$$L_2 = \frac{\Delta\omega_{r,\text{int}\,actpipe}}{\omega_{r\ell,(jk,jk-1)}} L \xrightarrow{for\ j=1\ \&\ k\in\{1;2;3;4\}} L_2 = \frac{\Delta\omega_{r,\text{int}\,actpipe}}{\omega_{r\ell,(12,11)}} = \frac{\Delta\omega_{r,\text{int}\,actpipe}}{\omega_{r\ell,(13,12)}} = \frac{\Delta\omega_{r,\text{int}\,actpipe}}{\omega_{r\ell,(14,13)}} = \frac{L}{6}$$

Also, the fig.5 shows respectively the second, the third, the fourth and the fifth types of peaks appearing for a leak located at $\frac{L}{6}$ from the valve.

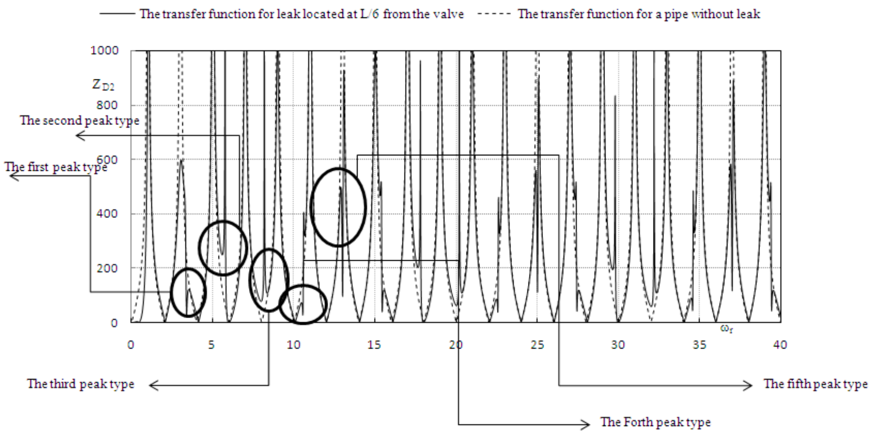


Fig. 5 Different peak types of the frequency response functions of pipe with leak located at $L/6$ from the valve

For a width frequency plotting of 40, three peaks are observed for each peak type. These peaks are occurring successively at these frequencies:

$$\omega_{r\ell 21}=5.72;\ \omega_{r\ell 22}=17.72;\ \omega_{r\ell 23}=29.7658;\ \omega_{r\ell 31}=8.152;\ \omega_{r\ell 32}=20.155;\ \omega_{r\ell 33}=32.174$$
$$\omega_{r\ell 41}=10.527;\ \omega_{r\ell 42}=22.564;\ \omega_{r\ell 43}=34.5669;\ \omega_{r\ell 51}=12.971;\ \omega_{r\ell 52}=24.981;\ \omega_{r\ell 53}=36.99$$

These frequencies are substituted in the eq. 21 to get the location of the leak.

4 Leak Depth Influence on Downstream Impedance of Leaking Pipe

The proposed method: based on the leak reflections as a tool for the leak detection, relying on cylindrical leak geometry inquiries the pipe thickness effects to demonstrate the importance of this consideration. For this reason fig. 6 is plotted to explain physical signification of the pipe thickness which is assumed in this work to be the length of the leak as a cylindrical pipe. In previous works the leak impedance *is defined as, the ratio of the double pressure head at the orifice by the leak discharge, described by linearizing the discharge law* (Wylie and Streeter). In this paper the hydraulic *leak impedance is that of a cylindrical pipe with free discharge in the atmosphere*, as figure 6 shows the downstream impedance of leaking pipes having the same conditions of the monitoring flow, but different pipe thickness. This illustrates an important result and information contained in pipe thickness influence on the leak reflections, so the necessity of considering the cylindrical leak geometry to facilitate the leak location. This figure shows for the same friction value and different thickness, the *leak reflections* are detected only in the case where $e>0$ but not detected when $e=0$.

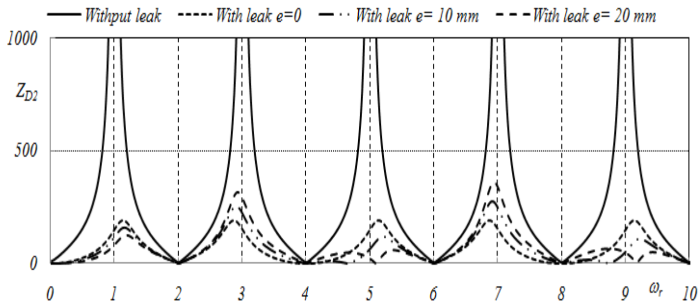


Fig. 6 frequency response functions of pipes with different pipe thickness $L_1/L = 1/2\, and\, \lambda = 0.04$

5 Validation of the Cylindrical Geometry Approach

The technique presented in this paper identifies periodicity in the frequency response at the downstream of pipeline system with cylindrical leak. The underlying mechanism is identical to that presented in numerous previous works such as (Lee, P et al 2005_2006) and (Covas et al 2005).

To validate the mathematical approach used in this paper and to focus on the importance of the cylindrical geometry of the leak figure 7 is picketed with similar conditions as shown by (Covas et al. 2005). One can observe in this figure plotted with logarithmic scale, the presence of resonant peaks occurring at the same normal frequencies calculated by Covas and related directly with the leak location $L_2 = 200m$ and the total pipe length $L = 1000\,m$. The leak with the cylindrical geometry provokes, in addition to secondary peaks due to the thickness of the main pipe as a depth of the leak modeled by a small pipe, some resonant peaks occurring at the following frequencies:

$$\omega_{r\ell 1} = 5;\ \omega_{r\ell 2} = 15;\ \omega_{r\ell 3} = 25$$

These normalized frequencies are needed to locate leak by applying the standing wave difference method for leak detection given in Covas work.

Comparing figures 7 and 8 we can conclude that for a small thickness such as $e = 0.00001\,m$, the results show a great concordance with that given by Covas in figure 8.

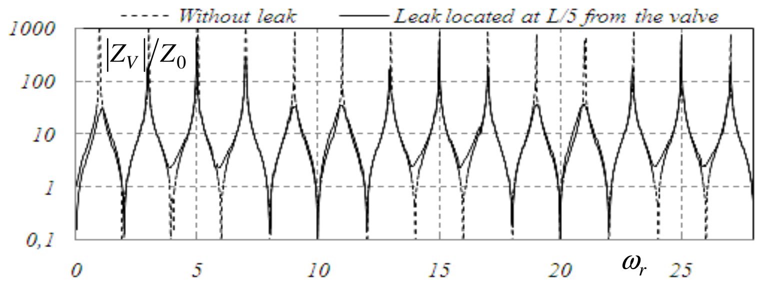


Fig. 7 Frequency responses at valve site. Undamaged pipe and leaking pipe with one leak $L_2/L=1/5 \quad e=0.00001\,m$

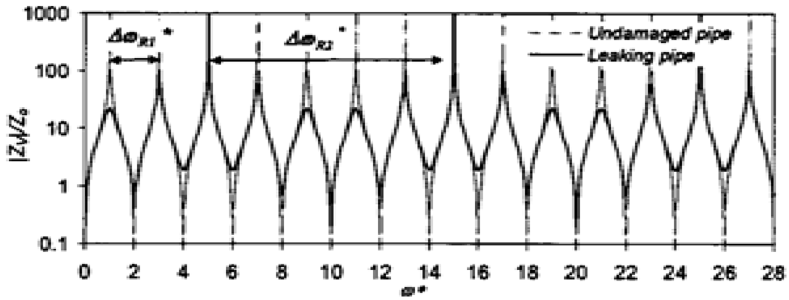


Fig. 8 Frequency responses at valve site. Undamaged pipe and leaking pipe with one leak $L_2/L=1/5$ [Covas et al. (2005)].

6 Conclusion

This paper has presented the importance of *cylindrical leak geometry* consideration to locate the leak. The impedance method indicates that it is possible to include the leak geometry in the downstream impedance by including its impedance, as a cylindrical pipe opened in the free atmosphere in the impedance equation at the downstream end of the leaking pipe (valve). This equation is needed to facilitate the analysis of the wave reflections due to the leak depth and to demonstrate that the thickness of the principal pipe influences the leak reflections. Cylindrical leak in the pipeline system provokes secondary peaks caused by the leak depth. The results of simulation show that geometrical consideration can provide better physical information about the leak presence. This paper shows that *the simplicity of the leak location decreases with the leak depth dimension* because of the clarity of the wave reflected by the leak in the impedance spectrum at the end of the leaking pipe.

The results of numerical simulation were validated by (COVAS work 2005), they have shown a great concordance for *small pipe thicknesses.*

A difference appears only when the thickness increases. This difference is noted on the appearance of the secondary peaks related directly to the leak depth and allows leak location.

An empirical formulae was derived for leak locations in thick pipes, for other case and when there's a lack of secondary peaks a numerous analytical formulae were given and demonstrated in literatures.

Notations

The following symbols are used in this paper:

Q = Discharge

H = Head

C = Wave celerity

h = Head fluctuation

q = Discharge fluctuation

A = Pipe cross sectional area

A_ℓ = Area of cylindrical leak

f = Darcy-Weisbach friction factor

g = Acceleration due to gravity

x = Distance

L = Pipeline length

L_1 = Leak position from the tank

L_2 = Leak location from the valve

Z_U = Upstream impedance

Z_D = Downstream impedance

$\overline{H}$ Average of pressure head

$\overline{Q}$ Average of flow

Z_C = Characteristic impedance of main pipe

$Z_{C\ell}$ = Leak characteristic impedance.

D = Pipe diameter

D_ℓ = Leak diameter

e = Pipe thickness

t = Time

Z_{D1} = Impedance upstream the leak

Z_{U1} = The upstream impedance of leaking pipe.

Z_{U2} = Impedance downstream the leak

Z_{D2} = Impedance downstream the leaking pipe.

$n; j; k$ Integer

Greek symbols

ω = Frequency

ω_r = Normalized frequency

$\omega_{r\ell}$ = Normalized frequency at which appears new peaks

$\omega_{r,\mathrm{int}\,act\,pipe}$ = Frequency impedance period of no leaked pipe

$\Delta\omega_{Lr,(jK;Jk-1)}$ = Frequency period of the impedance at the downstream end (Leak near the valve).

$\Delta\omega_{\ell r,(K;k-1)}$ = Frequency period of the impedance at the downstream end (leak near the reservoir)

$\lambda = 4f$

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